



UNIVERSITI PUTRA MALAYSIA

**HIGH BREAKDOWN ESTIMATOR TO ROBUSTIFY PHASE II
CONTROL CHARTS**

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CONTROL CHARTS**

By

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Chair: Assoc. Prof. Habshah Midi, PhD

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Hotelling's T^2 chart is one of the most popular multivariate control charts for monitoring independently and identically distributed random vectors. This chart is able to detect many types of out-of-control signals, but it is not sensitive to a small shifts in the mean vector. Classical estimation methods for multivariate control charts will not yield efficient control limits if there is instability in the data sets.

The presence of outlying observations may influence standard measures such that for the points close to the center, the corresponding mahalanobis distances are improperly large and for the outliers are relatively small. Hence, the main emphasis in this thesis is to propose a robust control chart which is less sensitive to the presence of outliers. We propose a more efficient T^2 control charts based on the Re-weighted MVE (RMVE) and Re-weighted MCD (RMCD) estimators.

Since the distribution of the T_{RMVE}^2 and the T_{RMCD}^2 are intractable, we propose a closed form control limits based on the estimated nonlinear regression function whereby the $1 - \alpha\%$ quantile of the T_{RMVE}^2 and the T_{RMCD}^2 are regressed against

the sample size, n .

Our simulation studies showed that when the process is in-control, and the sample size is small, the best control chart is the standard T^2 , as noted in the literatures. Nonetheless, for a large sample size, the performance of the T_{RMVE}^2 and the T_{RMCD}^2 are reasonably close to the classical T^2 chart. On the other hand, when there are outliers in phase I, the T_{RMVE}^2 and the T_{RMCD}^2 charts are more effective than the standard T^2 and the ordinary MVE and the MCD based charts.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

PENGANGGAR TITIK MUSNAH TINGGI BAGI MENEGUHKAN CARTA KAWALAN FASA II

Oleh

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Carta Kawalan Hotelling's T^2 paling popular di kalangan carta kawalan multivariat bagi memantau taburan vector rawak seiras dan merdeka. Carta ini berupaya untuk mengesan banyak jenis amaran di luar kawalan, tetapi tidak sensitif kepada anjakan kecil vector min. Kaedah penganggaran klasik bagi carta kawalan multivariat tidak akan menghasilkan had kawalan yang efisien jika data set tidak stabil. Kehadiran cerapan terpercil mungkin mempengaruhi ukuran piawai sedemikian hingga titik-titik yang hampir ke pusat, jarak mahalnobisnya tidak sepatutnya besar dan titik terpercil menghasilkan jarak yang agak kecil.

Oleh itu, penekanan dalam tesis ini ialah untuk mencadangkan carta kawalan teguh yang kurang peka kepada kehadiran titik terpercil. Kami mencadangkan carta kawalan teguh T^2 yang berasaskan penganggar *Re-weighted MVE (RMVE)* dan penganggar *Re-weighted MCD (RMCD)*. Oleh kerana taburan bagi T_{RMVE}^2 dan T_{RMCD}^2 tidak diketahui, kami mencadangkan had carta kawalan berasaskan penganggar fungsi regresi tak-linear sedemikian hingga kuantil $1 - \alpha\%$ bagi T_{RMVE}^2 dan T_{RMCD}^2 diregresikan melawan saiz sampel, n .

Kajian simulasi yang dijalankan menunjukkan bahawa apabila proses berada

dalam kawalan dan saiz sampel kecil, carta piawai T^2 adalah yang terbaik seperti yang dinyatakan dalam rujukan. Walaubagaimana pun, bagi saiz sampel besar, prestasi T_{RMVE}^2 dan T_{RMCD}^2 adalah sangat hampir kepada carta klasik T^2 . Sebaliknya, apabila terdapat titik terpencil dalam Fasa I, carta T_{RMVE}^2 dan T_{RMCD}^2 adalah lebih berkesan berbanding dengan carta piawai T^2 dan carta yang berasaskan MVE dan MCD biasa.



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I certify that a Thesis Examination Committee has met on 15 April 2011 to conduct the final examination of **Mandana Mohammadi** on her thesis entitled “**High Breakdown Estimator to Robustify Phase II Control Charts**” in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MANDANA MOHAMMADI

Date: 15 April 2011

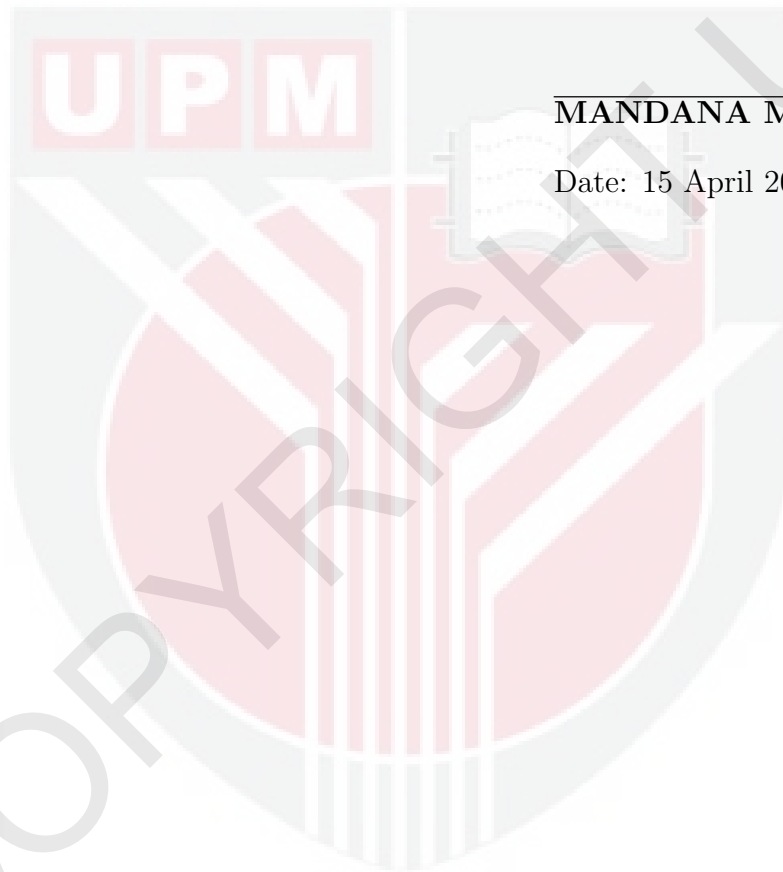


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