SOLVING SECOND ORDER BOUNDARY VALUE PROBLEM BY SHOOTING TECHNIQUE USING BACKWARD DIFFERENCE FORMULAE

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirement for the Degree of Master of Science

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First of all and before every thing, I submit in humility and gratitude to my beloved creator “Allah Subhana Wa Taala” for having blessing me this opportunity and protecting me from major obstacles in fulfilling this thesis.

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Finally, this thesis is dedicated to all those who believe in the richness of learning.
Abstract of thesis presented to the Senate of University Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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Chairman: Zanariah Abdul Majid, PhD
Institute: Institute for Mathematical Research

In this thesis, the direct implicit multistep method presented in the form of backward difference formula are developed for solving directly two point boundary value problems (BVPs). This method will solve the nonlinear second order BVPs by shooting technique using constant and variable step size. Newton’s method is considered as a procedure for solving the nonlinear equations and the convergence of the shooting technique.

Most of the existing researches involved BVPs will reduce the problem to a system of first order ordinary differential equations (ODEs). This approach is very well established but it obviously will enlarge the system of first order equations. However, the direct multistep method in this thesis will be utilized to obtain series solutions of the initial value problems directly without reducing to first order equations.
The coefficients of the developed method will be stored in the code in the form of backward difference formula. The codes were executed in UNIX operating system and the algorithms were written in C language.

The numerical results showed that the performance of the developed method is acceptable in terms of maximum error and number of iterations when solving second order nonlinear boundary value problem.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN MASALAH NILAI SEMPADAN TERBITAN KEDUA OLEH TEKNIK TEMBAKAN DENGAN MENGGUNAKAN FORMULASI BEZA KE BELAKANG

Oleh
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Kebanyakan penyelidik sedia ada yang terlibat dengan penyelesaian MNS akan menurunkan masalah tersebut kepada sistem persamaan peringkat pertama dalam bentuk persamaan peringkat biasa (PBB). Kaedah ini sangat berkesan tetapi secara jelas sistem persamaan peringkat pertama akan menjadi lebih besar. Walaubagaimanapun, di dalam
tesis ini kaedah multilangkah secara terus akan digunakan untuk mendapat penyelesaian siri bagi masalah nilai awal secara terus tanpa menurunkan kepada persamaan peringkat pertama.

Pekali-pekali bagi keadah yang dibina akan disimpan di dalam kod dalam bentuk formulasi beza ke belakang. Kod-kod ini dijalankan dalam sistem operasi UNIX dan algoritma ditulis dalam bahasa C.

Keputusan berangka menunjukkan pencapaian bagi kaedah yang dibangunkan memberikan keputusan yang boleh diterima dari segi ralat maksimum dan lelaran bagi menyelesaikan MNS terbitan kedua tak linear.
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I certify that a Thesis Examination Committee has met on 30 November 2010 to conduct the final examination of Mahanum Diana Binti Jafri on her thesis entitled “Solving Second Order Boundary Value Problem By Shooting Technique Using Backward Difference Formulae” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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Date: 24 March 2011
This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MAHANUM DIANA BINTI JAFRI

Date: 30 November 2010
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