UNIVERSITI PUTRA MALAYSIA

DIAGONAL QUASI-NEWTON'S METHOD FOR LARGE-SCALE SYSTEMS OF NONLINEAR EQUATIONS

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

December 2011

DEDICATIONS

Mum: Ramatu Mohammed

Dad : Mallam Mohammed

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the Degree of Doctor of Philosophy

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Chair: Associate Professor Leong Wah June, PhD

Faculty: Science

The famous and well known method for solving systems of nonlinear equations is the Newton's method. This method is simple to implement and converges rapidly. Nevertheless, an iteration of this method turns out to be computational expensive, this is due to the fact that the method requires the computation and storage of the Jacobian matrix, the floating point operations is $O(n)^3$ and as well as the solution of an n linear systems in each iteration. In addition the convergence may even be lost when the Jacobian is singular at a solution. These drawbacks/disadvantages is more noticeable when handling large-scale systems of nonlinear equations. This leads to the idea of this thesis.

This thesis focuses on some variants of Newton's method for solving systems of nonlinear equation, especially large-scale systems. Several diagonal Newton's methods for solving systems of nonlinear equations have been derived. Unlike the classi-

cal Newton's method, the proposed schemes require neither the computation and storage of the Jacobian matrix, nor the solution of a system of linear equations in each iteration. Thus, the floating point operations have been reduced to $O(n)$. This is made possible by approximating the Newton step via diagonal updating. The basic idea behind this novel approach is to approximate the Jacobian or its inverse to a nonsingular diagonal matrix which can be updated in each iteration.

The anticipation of the methods proposed in this thesis when applied to large-scale systems of nonlinear equation has been in the reduction of computational costs, storage requirements, floating points operations and processing time.

Extensive computational experiments are performed on standard problems to examine the impact of the proposed methods compared to other variants of Newton's method for solving systems of nonlinear equations. The results suggest that the proposed methods outperform the classical Newton's method and some of its variants in terms of matrix storage requirements, computational cost and processing time in seconds. In addition, the local convergence of the proposed methods have been proven.

We further apply the diagonal updating schemes proposed in this thesis to handle some well known real-life problems and compare their numerical performance with some Newton-like methods.

Finally, we discuss some possible extension of our work.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH KUASI-NEWTON PEPENJURU UNTUK SISTEM PERSAMAAN TAK-LINEAR BERSKALA-BESAR

Oleh

MOHAMMED WAZIRI YUSUF

Disember 2011

Pengerusi: Prof. Madya. Leong Wah June, PhD

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Kaedah yang terkenal untuk menyelesaikan sistem persamaan tak linear merupakan kaedah Newton. Kaedah ini adalah mudah untuk dilaksanakan dan menumpu dengan cepat . Walau bagaimanapun, lelaran kaedah ini ternyata mahal secara pengiraan , disebabkan oleh hakibatnya bahawa. kaedah ini memerlukan pengiraan dan penyimpanan matriks Jacobian, operasi titik apung ialah $O(n^3)$ serta penyelesaian sistem n persamaan linear dalam setiap lelaran. Di samping itu, penumpuannya mungkin akan terhilang apabila Jacobian adalah singular pada penyelesaian. Kelemahan keburukan tersebut adalah lebih ketara apabila digunakan untuk mengendalikan sistem persamaan tak linear secara besar-besaran. Ini membawa kepada idea bagi tesis ini.

Tesis ini memberi fokus kepada beberapa variasi kaedah Newton untuk menyelesaikan sistem persamaan tak linear khususnya sistem persamaan yang berskala

besar. Beberapa kaedah pepenjuru Newton untuk menyelesaikan sistem persamaan tak linear telah diperolehi. Berbeza daripada kaedah Newton klasik, skema yang dicadangkan tidak memerlukan pengiraan dan storan bagi matriks Jacobian, mahupun penyelesaian sistem persamaan linear dalam setiap lelaran. Oleh itu, operasi titik apung telah dikurangkan kepada $O(n)$. Ini dapat dilakukan dengan menghampirkan langkah Newton melalui rumus kemas kini pepenjuru . Idea asas di sebalik pendekatan baru ini adalah untuk menghampirkan Jacobian atau songsangannya sebagai satu matriks perpenjuru yang tak singular boleh dikemaskini dalam setiap lelaran.

Jangkaan kaedah yang dicadangkan dalam tesis ini sekiranya digunakan untuk sistem persamaan tak linear secara besar-besaran, adalah dalam pengurangan kos pengiraan, keperluan storan, operasi titik apung dan masa pemprosesan.

Ujikaji pengiraan yang mendalam telah dilaksanakan ke atas masalah piawai untuk mengkaji keberkesanan kaedah yang dicadangkan berbanding dengan variasi kaedah Newton yang lain bagi menyelesaikan sistem persamaan tak linear. Keputusannya menunjukkan bahawa kaedah yang dicadangkan adalah lebih baik daripada kaedah Newton klasik dan beberapa variasi dari segi keperluan storan matriks, pengiraan kos dan masa pemprosesan dalam saat. Di samping itu, penumpuan setempat kaedah yang dicadangkan telah dibuktikan .

Kami juga menggunakan skema pepenjuru kemas kini yang dicadangkan dalam tesis ini untuk menangani beberapa masalah terkenal yang sebenar dalam kehidupan dan membandingkan prestasi berangkanya dengan beberapa keadah Newton yang lain.

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I certify that a Thesis Examination Committee has met on (**14 December 2011**) to conduct the final examination of (**Mohammed Waziri Yusuf**) on his thesis entitled "**Diagonal Quasi-Newton's Method for Large- Scale Systems of Nonlinear Equations**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the (**Doctor of Philosophy**).

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of **Doctor of Philosophy**. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

(Signature)

MOHAMMED WAZIRI YUSUF

Date: 14 December 2011

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