UNIVERSITI PUTRA MALAYSIA

TWO - POINT BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

NOORAINI BINTI ZAINUDDIN

FS 2011 73

TWO - POINT BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

By

NOORAINI BINTI ZAINUDDIN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for Degree of Master of Science

October 2011

© COPYRIGHT UPM

DEDICATIONS

to

all of my family members

© COPYRIGHT UPM

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

TWO - POINT BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

By

NOORAINI BINTI ZAINUDDIN

October 2011

Chairman: Zarina Bibi Binti Ibrahim, PhD

Faculty: Science

This thesis focuses on solving higher order Ordinary Differential Equations (ODEs) directly using the Block Backward Differentiation Formula (BBDF) method. The BBDF method approximates the solution at two points concurrently. Implementation of this method is done by using equidistant stepsize on the set of stiff problems.

The first part of the thesis gives the derivation of the BBDF method for solving second order and third order stiff ODEs directly. The algorithms are written in C language and the numerical results of these methods are compared to that of reducing it to a system of first order ODEs and solves using the first order ODEs method.

© COPYRIGHT UPM

The subsequent part of the thesis discusses in detail the stability properties of the BBDF method which are given in the previous part. The stability properties justify the efficiency of the BBDF method as used in solving stiff problems. The illustrations of the stability region are provided.

Finally, this thesis zooms into the implementation of the BBDF method using the variable order algorithm for the solution of second order stiff ODEs directly. The variable order strategies for the BBDF method is elaborated and the numerical result of the variable order BBDF method is compared with the variable order method which is available in MATLAB.

In conclusion, the results show that BBDF method reduces the total number of steps and the time execution when compared to the nonblock first order ODEs method. Therefore, these new methods present significant alternatives for solving higher order ODEs directly.

© COPYRIGHT UPM

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

DUA - TITIK BLOK FORMULASI BEZA KE BELAKANG UNTUK PENYELESAI PERSAMAAN PEMBEZAAN BIASA PERINGKAT TINGGI

Oleh

NOORAINI BINTI ZAINUDDIN

Oktober 2011

Pengerusi: Zarina Bibi Binti Ibrahim, PhD

Fakulti: Sains

Tesis ini tertumpu kepada penyelesaian Persamaan Pembezaan Biasa (PPB) peringkat tinggi secara terus menggunakan kaedah Blok Formulasi Beza ke Belakang (BFBB). Kaedah BFBB menganggarkan penyelesaian pada dua titik secara serentak. Pelaksanaan kaedah ini dilakukan dengan menggunakan saiz langkah sama jarak ke atas set masalah kaku.

Bahagian pertama tesis ini memberikan penerbitan kaedah BFBB untuk menyelesaikan PPB kaku peringkat kedua dan ketiga secara terus. Algoritma ditulis dalam bahasa C dan keputusan berangka kaedah ini dibandingkan dengan penurunan

© COPYRIGHT UPM

kepada sistem PPB pada peringkat pertama dan diselesaikan dengan menggunakan kaedah PPB peringkat pertama.

Bahagian selanjutnya dalam tesis ini membincangkan secara terperinci ciri-ciri kestabilan oleh kaedah BFBB yang diberikan pada bahagian terdahulu. Ciri-ciri kestabilan mewajarkan keberkesanan kaedah BFBB untuk digunakan dalam menyelesaikan masalah kaku. Illustrasi kepada rantau kestabilan diberikan.

Akhirnya, tesis ini tertumpu kepada pelaksanaan kaedah BFBB menggunakan algoritma peringkat berubah untuk penyelesaian PPB kaku peringkat kedua secara terus. Strategi peringkat berubah untuk kaedah BFBB dihuraikan dan keputusan berangka untuk kaedah BFBB peringkat berubah dibandingkan dengan kaedah peringkat berubah yang terdapat di MATLAB.

Kesimpulannya, keputusan menunjukkan kaedah BBDF mengurangkan jumlah langkah dan masa pelaksanaan apabila dibandingkan dengan kaedah PPB peringkat pertama bukan blok. Oleh itu, kaedah-kaedah baru ini menjadi pilihan yang wajar untuk menyelesaikan PBB peringkat tinggi secara terus.

© COPYRIGHT UPM

ACKNOWLEDGEMENTS

In the Name of Allah

The Most Beneficent, The Most Merciful

First and foremost, I would like to express my deepest gratitude to the Chairman of the Supervisory Committee, Dr. Zarina Bibi bt. Ibrahim for her wise guidance, invaluable advice and patience throughout my research.

I am also very grateful to the member of Supervisory Committee, YBhg. Prof. Dato' Dr. Mohamed bin Suleiman and Associate Professor Dr. Khairil b. Othman for their kind support, suggestion and assistance in the completion of this research.

The financial support from Jabatan Perkhidmatan Awam (JPA) for the scholarship awarded that enables me to pursue this research. Many thanks also to my colleagues and friends for their support and help throughout the years spent in this research.

Lastly, my special thanks and deepest grateful goes to my family for their continuous understanding, caring, support and the love that has inspires me to excel in this life.

© COPYRIGHT UPM

I certify that a Thesis Examination Committee has met on 17 October 2011 to conduct the final examination of Nooraini binti Zainuddin on her thesis entitled "Two-Point Block Backward Differentiation Formula for Solving Higher Order Ordinary Differential Equations" in accordance with the Universities and University College Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The committee recommends that the student be awarded the Master of Science.

Members of the Thesis Examination Committee were as follows:

Idham Arif bin Hj. Alias, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Fudziah binti Ismail, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Leong Wah June, PhD
Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Daud Mohamad, PhD
Associate Professor
Faculty of Computer and Mathematical Science
Universiti Teknologi MARA
(External Examiner)

SEOW HENG FONG, PhD
Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 20 December 2011

© COPYRIGHT UPM

This thesis was submitted to the Senate of Universiti of Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of **Master of Science**. The members of the Supervisory Committee were as follows:

Zarina Bibi Binti Ibrahim, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Chairperson)

Dato' Mohamed Bin Suleiman, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Khairil Iskandar Bin Othman, PhD
Associate Professor
Faculty of Computer and Mathematical Sciences
Universiti Teknologi MARA
(Member)

_____

BUJANG BIN KIM HUAT, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

© COPYRIGHT UPM

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledge. I also declare that it has not been previously, and is not currently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

_____

NOORAINI BT. ZAINUDDIN

Date: 17 October 2011

© COPYRIGHT UPM

LIST OF TABLES

Table		Page
2.1	The values of $\delta_{j,m}$ with $j = 1, 2$ for 2BBDF method of constant step size with $x = x_{n+1}$	21
2.2	The values of $\gamma_{j,m}$ with $j = 1, 2$ for 2BBDF method of constant step size with $x = x_{n+2}$	23
2.3	Numerical Result for Problem 2.1	49
2.4	Numerical Result for Problem 2.2	49
2.5	Numerical Result for Problem 2.3	50
2.6	Numerical Result for Problem 2.4	51
2.7	Numerical Result for Problem 2.5	51
2.8	Numerical Result for Problem 2.6	52
2.9	Numerical Result for Problem 2.7	53
2.10	Numerical Result for Problem 2.8	53
2.11	Numerical Result for Problem 2.9	54
3.1	The values of $\delta_{j,m}$ with $j = 3$ for 3BBDF method of constant step size with $x = x_{n+1}$	63
3.2	The values of $\gamma_{j,m}$ with $j = 3$ for 3BBDF method of constant step size with $x = x_{n+2}$	64
3.3	Numerical Result for Problem 3.1	80
3.4	Numerical Result for Problem 3.2	80
3.5	Numerical Result for Problem 3.3	81
3.6	Numerical Result for Problem 3.4	81

© COPYRIGHT UPM

3.7	Numerical Result for Problem 3.5	82
3.8	Numerical Result for Problem 3.6	82
5.1	Numerical Result for Problem 5.1	118
5.2	Numerical Result for Problem 5.2	119
5.3	Numerical Result for Problem 5.3	119
5.4	Numerical Result for Problem 5.4	119
5.5	Numerical Result for Problem 5.5	120
5.6	Numerical Result for Problem 5.6	120
5.7	Numerical Result for Problem 5.7	120
5.8	Numerical Result for Problem 5.8	121

© COPYRIGHT UPM

LIST OF FIGURES

Figure		Page
1.1	Two - Point Block Method	10
4.1	Stability Region of the 2BBDF(3) Method	91
4.2	Stability Region of the 2BBDF(4) Method	95
4.3	Stability Region of the 2BBDF(5) Method	100
4.4	Stability Region of the 3BBDF Method	106

© COPYRIGHT UPM

LIST OF ABBREVIATIONS

ODEs	:	Ordinary Differential Equations
IVPs	:	Initial Value Problems
LMM	:	Linear Multistep Methods
BDF	:	Backward differentiation Formulas
2BBDF	:	Two-Point Block BDF methods for solving second order ODEs directly
3BBDF	:	Two-Point Block BDF methods for solving third order ODEs directly
2BBDF(3)	:	3^{rd} Order Two-Point Block BDF for solving second order ODEs directly
2BBDF(4)	:	4^{th} Order Two-Point Block BDF for solving second order ODEs directly
2BBDF(5)	:	5^{th} Order Two-Point Block BDF for solving second order ODEs directly
ode15s	:	A variable order method of Numerical Differentiation Formulas (NDFs) of order 1-5
ode23s	:	A fixed order method of new modified Rosenbrock (2,3) pair
VOBBDF	:	Variable order Two-Point Block BDF for solving second order ODEs directly

© COPYRIGHT UPM

TABLE OF CONTENTS

		Page
DEDICATION		ii
ABSTRACT		iii
ABSTRAK		v
ACKNOWLEDGEMENTS		vii
APPROVAL		viii
DECLARATION		x
LIST OF TABLES		xi
LIST OF FIGURES		xiii
LIST OF ABBREVIATIONS		xiv
CHAPTER		
1	**INTRODUCTION**	
	1.1 Introduction	1
	1.2 Literature Review	2
	1.3 Objectives of the Thesis	5
	1.4 Outline of the Thesis	6
	1.5 Problem to be Solved	7
	1.6 Stiff Ordinary Differential Equations	8
	1.7 Block Method	10
	1.8 Linear Multistep Method	11
2	**TWO – POINT BLOCK BACKWARD DIFFERENTIATION FORMULAE OF DIFFERENT ORDERS FOR SOLVING STIFF SECOND ORDER ODEs DIRECTLY**	
	2.1 Introduction	15
	2.2 Derivation of Two-Point Block BDF Method for Solving Second Order ODEs Directly (2BBDF)	16
	2.3 Derivation of the Predictors	25
	2.4 Order of the Methods	27
	2.5 Implementation of the Methods	34
	2.6 Problems Tested	40
	2.7 Numerical Results	46
	2.8 Discussion and Conclusion	55
3	**TWO – POINT BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING STIFF THIRD ORDER ODEs DIRECTLY**	
	3.1 Introduction	57
	3.2 Derivation of Two-Point Block BDF Method for Solving Third Order ODEs Directly (3BBDF)	58

© COPYRIGHT UPM

3.3 Derivation of the Predictors 65
3.4 Implementation of the Method 68
3.5 Problems Tested 74
3.6 Numerical Results 79
3.7 Discussion and Conclusion 83

4 STABILITY TWO – POINT BLOCK BACKWARD DIFFERENT FORMULA FOR SOLVING HIGHER ORDER ODEs DIRECTLY
4.1 Introduction 85
4.2 Stability of the 2BBDF Methods for Solving Second Order ODEs Directly 86
4.2.1 Third Order 2BBDF (2BBDF(3)) 88
4.2.2 Fourth Order 2BBDF (2BBDF(4)) 92
4.2.3 Fifth Order 2BBDF (2BBDF(5)) 96
4.3 Stability of the 3BBDF Method for Solving Third Order ODEs Directly 101

5 VARIABLE ORDER TWO – POINT BLOCK BACKWARD DIFFERENTIATION FORMULA FOR SOLVING SECOND ORDER ODEs DIRECTLY
5.1 Introduction 108
5.2 Strategy of the Variable Order 2BBDF (VOBBDF) Method 109
5.3 Problems Tested 111
5.4 Numerical Results 118
5.5 Discussion and Conclusion 121

6 CONCLUSION
6.1 Summary 123
6.2 Future Research 124

REFERENCES 126
APPENDIX A: CODE FOR TWO – POINT BLOCK BACKWARD DIFFERENTIATION FORMULA 130
BIODATA OF STUDENT 139
LIST OF PUBLICATIONS 140

© COPYRIGHT UPM