



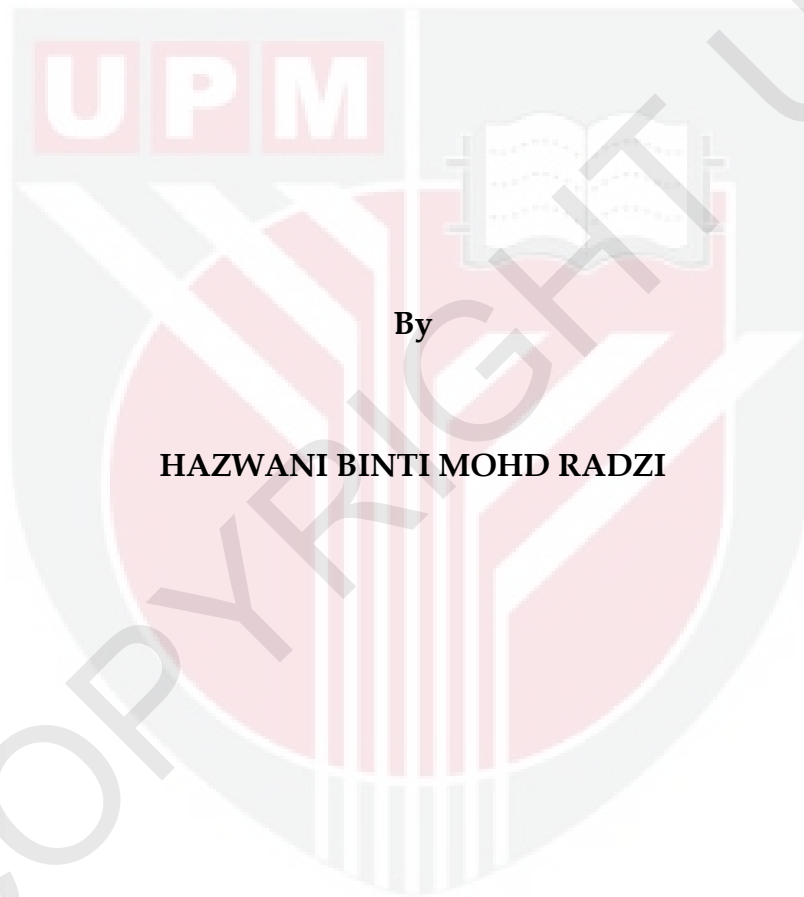
UNIVERSITI PUTRA MALAYSIA

**ONE-STEP BLOCK METHODS FOR SOLVING ORDINARY AND
DELAY DIFFERENTIAL EQUATIONS**

HAZWANI BINTI MOHD RADZI

FS 2011 76

**ONE-STEP BLOCK METHODS FOR SOLVING ORDINARY AND
DELAY DIFFERENTIAL EQUATIONS**



By

HAZWANI BINTI MOHD RADZI

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in
Fulfilment of the Requirements for the
Degree of Master of Science**

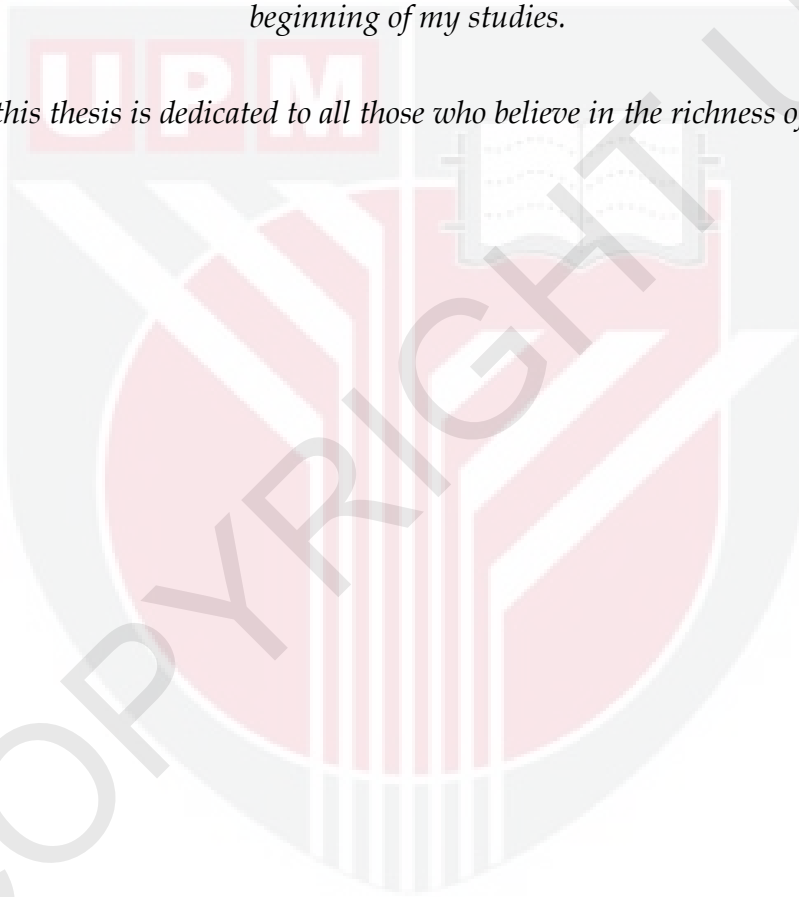
October 2011

DEDICATION

*I would like to dedicate this thesis to **my beloved parents, Mr. Mohd Radzi bin Ishak and Ku Mastura binti Ku Meh** who have taught me that even the largest task can be accomplished if it is done one step at a time.*

*To **my beloved family, lecturers and friends** who have supported me all the way since the beginning of my studies.*

Finally, this thesis is dedicated to all those who believe in the richness of learning.



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
fulfilment of the requirement for the degree of Master of Science

**ONE-STEP BLOCK METHODS FOR SOLVING ORDINARY AND
DELAY DIFFERENTIAL EQUATIONS**

By

HAZWANI BINTI MOHD RADZI

October 2011

Chair: Zanariah Abdul Majid, PhD

Faculty: Science

This thesis describes the implementation of one step block methods of Runge-Kutta type for solving first order Ordinary Differential Equations (ODEs) and Delay Differential Equations (DDEs). A new modified algorithm from the existing program for solving ODEs was developed. The algorithm which is based on the predictor-corrector scheme of Adams type was then extended from two up to six point one step block method. In the program, the solutions were approximated simultaneously based on equidistant block method and each of these values were calculated independently of each other.

The derivation of the two point up to six point block methods based on the Newton backward divided difference formulae were presented. The stability regions were obtained and illustrated in the thesis. The numerical results showed that the developed methods are better in terms of number of total steps, failure steps and function when compared with the existing methods in most problems tested.

Subsequently, the codes were then adapted to solve first order DDEs and the delay term is approximated using Neville's interpolation. In this research, we solve DDEs of the retarded type (RDE) with single delay. The stability aspects of the one step block method when applied to DDEs were investigated and the P- and Q- stability regions were presented. In general, all the proposed methods give less number of successful steps and number of functions compared to the existing method.

The numerical results have shown that the block methods proposed in this study are suitable for solving ordinary and delay differential equations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

**KAEDAH BLOK SATU-LANGKAH BAGI MENYELESAIKAN PERSAMAAN
PEMBEZAAN BIASA DAN PERSAMAAN
PEMBEZAAN LENGAH**

Oleh

HAZWANI BINTI MOHD RADZI

Oktober 2011

Pengerusi: Zanariah Abdul Majid, PhD

Fakulti: Sains

Tesis ini menerangkan tentang pelaksanaan kaedah blok satu langkah jenis Runge-Kutta bagi menyelesaikan persamaan pembezaan biasa (PPB) dan persamaan pembezaan lengah (PPL) peringkat pertama. Algoritma baru yang diubahsuai dari program yang sedia ada telah dibangunkan untuk menyelesaikan PPB. Seterusnya, algoritma yang berasaskan kepada skema peramal-pembetul jenis Adams ini dilanjutkan daripada dua sehingga enam titik kaedah blok satu langkah. Dalam program ini, penyelesaian akan dianggarkan secara serentak berdasarkan kaedah blok sama jarak dan setiap nilai dikira tanpa bergantung pada satu sama lain.

Penerbitan dua sehingga enam titik kaedah blok berdasarkan formulasi pembezaan terbahagi ke belakang Newton diterangkan. Rantau kestabilan diperolehi dan

diilustrasikan di dalam tesis ini. Keputusan berangka menunjukkan bahawa kaedah yang dibangunkan adalah lebih baik dari segi jumlah langkah, langkah kegagalan dan fungsi apabila dibandingkan dengan kaedah yang sedia ada di dalam masalah yang diuji.

Seterusnya, kod itu disesuaikan bagi menyelesaikan PPL peringkat pertama dan sebutan lengahnya dianggarkan menggunakan interpolasi Neville. Dalam kajian ini, kami menyelesaikan PPL jenis terencat (PPT) dengan kelewatan tunggal. Aspek kestabilan kaedah blok satu langkah apabila diaplikasikan kepada PPL dikaji dan rantau kestabilan P dan Q diberikan. Secara amnya, kaedah yang dicadangkan memberi kurang bilangan langkah berjaya dan fungsi berbanding kaedah yang sedia ada.

Hasil berangka menunjukkan kaedah blok yang dicadangkan dalam kajian ini sesuai bagi menyelesaikan persamaan pembezaan biasa dan persamaan pembezaan lengah.

ACKNOWLEDGEMENTS

In the name of Allah, the Most Gracious and the Most Merciful

Alhamdulillah, all praises to Allah for the strengths and His blessing in completing this thesis. It is impossible to thank everyone who has helped me over the years. However, two people must be singled out. I am forever grateful to my remarkable supervisor, Associate Professor Dr. Zanariah Abdul Majid for her constant encouragement and invaluable suggestions. I also thank my committee member, Associate Professor Dr. Fudziah Ismail for her help and assistance during my studies. To those who directly or indirectly contributed in this research, especially my graduate colleagues, your boundless support, encouragement, and practical assistance means a lot to me.

I am indebted to the Government of Malaysia particularly Universiti Putra Malaysia for granting the scholarship to pursue my studies. Last but not least, my deepest gratitude goes to my beloved parents; Mr. Mohd Radzi B. Ishak and Mrs. Ku Mastura Bt. Ku Meh and also to my brother and sisters for their endless love, prayers and encouragement. Finally, my sincere appreciation goes to my husband, Mohammad Nasseer B. Saad whose persistence and encouragement gave me the strength to survive and further pursue my graduate education at Universiti Putra Malaysia.

I certify that a Thesis Examination Committee has met on **3 October 2011** to conduct the final examination of **Hazwani Binti Mohd Radzi** on her thesis entitled “**One-Step Block Methods for Solving Ordinary and Delay Differential Equations**” in accordance with the Universities and University College Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **degree of Master of Science**.

Members of the Thesis Examination Committee were as follows:

Mohd Rizam Abu Bakar, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Zarina Bibi Ibrahim, PhD

Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Lee Lai Soon, PhD

Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Internal Examiner)

Ahmad Izani Md. Ismail, PhD

Professor
School of Mathematical Sciences
Universiti Sains Malaysia
(External Examiner)

SEOW HENG FONG, PhD

Professor and Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of **Master of Science**. The members of the Supervisory Committee were as follows:

Zanariah Abdul Majid, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Fudziah Ismail, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

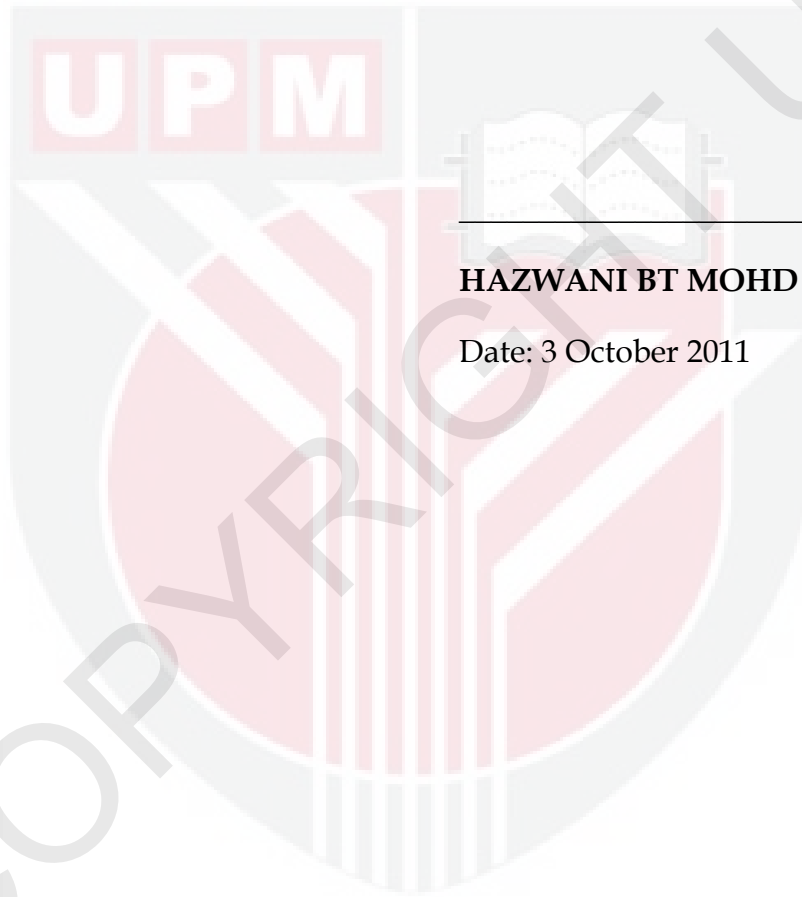
BUJANG BIN KIM HUAT, PhD

Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



HAZWANI BT MOHD RADZI

Date: 3 October 2011

TABLE OF CONTENTS

	Page
DEDICATION	ii
ABSTRACT	iii
ABSTRAK	v
ACKNOWLEDGEMENTS	vii
APPROVAL	viii
DECLARATION	x
LIST OF TABLES	xiii
LIST OF FIGURES	xv
LIST OF ABBREVIATIONS	xvi
CHAPTER	
1	
INTRODUCTION	1
1.1 Introduction	1
1.2 Objective of the thesis	3
1.3 Ordinary Differential Equation	4
1.4 Delay Differential Equation	5
1.5 Scope and Limitations	6
1.6 Outline of the Thesis	7
2	
LITERATURE REVIEW	9
2.1 Introduction	9
2.2 The Initial Value Problem	9
2.3 Assumptions and Definitions	10
2.4 Predictor and Corrector Method	14
2.5 Review of Previous Research	15
3	
DERIVATION AND ORDER OF ONE-STEP BLOCK METHODS OF RUNGE KUTTA TYPE	21
3.1 Introduction	21
3.2 Newton Backward Difference Formula	21
3.3 Derivation of One-Step Block Method	22
3.4 Order of the Method	30
3.4.1 Two Point One-Step Block Method	31
3.4.2 Three Point One-Step Block Method	33
3.4.3 Four Point, Five Point and Six Point One-Step Block Method	36

4	ONE-STEP BLOCK METHODS WITH VARIABLE STEP SIZE FOR ORDINARY DIFFERENTIAL EQUATIONS	39
	4.1 Introduction	39
	4.2 Modification of the Subprogram for Two Point One-Step Block Method	39
	4.3 Algorithm 2PBODE	46
	4.4 Absolute Stability	49
	4.4.1 Two Point One-Step Block Method	50
	4.4.2 Three Point One-Step Block Method	55
	4.4.3 Four Point, Five Point and Six Point One-Step Block Method	60
	4.5 Problems Tested	70
	4.6 Numerical Results	71
	4.7 Discussions	78
5	ONE-STEP BLOCK METHODS WITH VARIABLE STEP SIZE FOR DELAY DIFFERENTIAL EQUATIONS	81
	5.1 Introduction	81
	5.2 Neville's Interpolation	82
	5.3 Numerical Treatment of DDEs	84
	5.4 Algorithm of 2PBDDE	85
	5.5 Stability Properties of Numerical Method for Solving DDEs	89
	5.5.1 Two Point One-Step Block Method for Solving DDEs	91
	5.5.2 P-Stability for One-Step Block Method	92
	5.5.3 Q-Stability for One-Step Block Method	95
	5.6 Problems Tested	99
	5.7 Numerical Results	101
	5.8 Discussions	108
6	CONCLUSION AND FUTURE RESEARCH	110
	6.1 Conclusion	110
	6.2 Future Work	112
	BIBLIOGRAPHY	114
	BIODATA OF THE STUDENT	118
	LIST OF PUBLICATIONS	119