



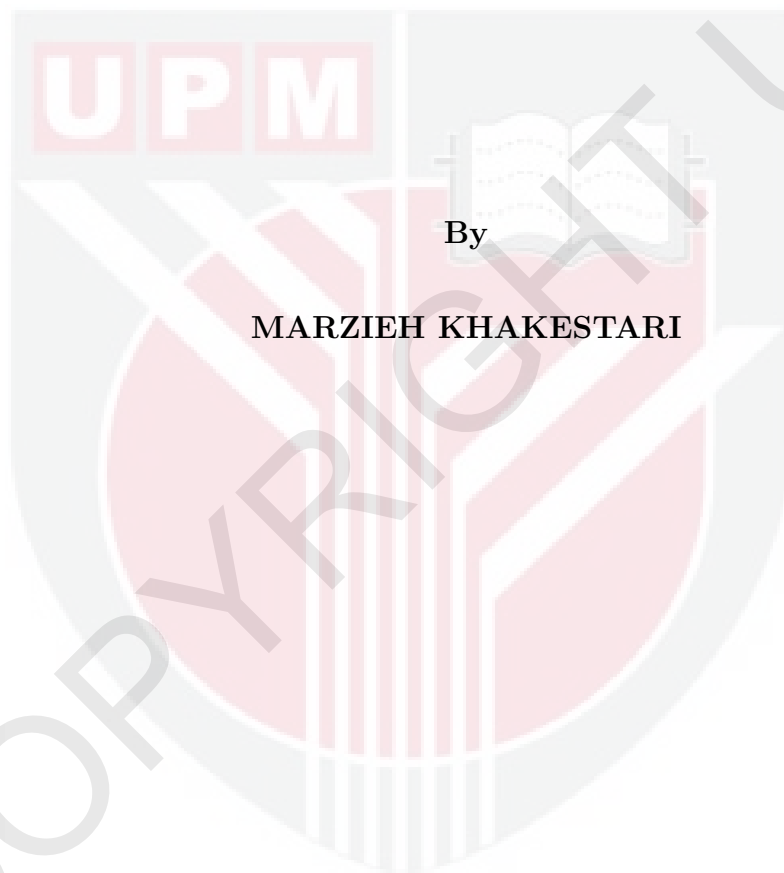
**UNIVERSITI PUTRA MALAYSIA**

**LINEAR PURSUIT-EVASION DIFFERENTIAL GAMES WITH  
INTEGRAL CONSTRAINTS ON CONTROL FUNCTIONS**

**MARZIEH KHAKESTARI**

**FS 2011 69**

**LINEAR PURSUIT-EVASION DIFFERENTIAL GAMES WITH  
INTEGRAL CONSTRAINTS ON CONTROL FUNCTIONS**



By

**MARZIEH KHAKESTARI**

Thesis Submitted to the School of Graduate Studies, Universiti Putra  
Malaysia, in Fulfilment of the Requirements for the Degree of Doctor  
of Philosophy

November 2011

## DEDICATION

To

My Mother and My Father

My Dear Husband

and

My Brothers and My Sisters

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in  
fulfilment of the requirement for the degree of Doctor of Philosophy

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**November 2011**

**Chair: Gafurjan Ibragimov, PhD**

**Faculty: Faculty of Science**

Recently use of decision-making in modern life has extensively increased. This lead to review subject of Pursuit-Evasion (PE) differential games. A differential game models a situation where two or more players operate in a same environment with conflicting goals. In this work, we attempt to solve general linear PE games in time-varying systems with continuous time. Most studies related to PE games in the current literature concentrate on two-player games with a single Pursuer and a single Evader and the results for general multi-player PE games are still largely sparse. The purpose of this study is to provide a theoretical foundation for linear PE games with integral constraints under the theory of the differential game and optimal control theory.

The results of this study contain of four parts, in the first part, the linear pursuit-evasion game by using optimal control techniques which is based on structured

controls of the players has been solved. We obtain a formula for the optimal pursuit time and construct the optimal strategies for the players when the control resource of the Pursuer is greater than the Evader. In addition, a new method for solving of the evasion problem is proposed where the control resources of the Pursuer are less than or equal to the Evader.

Secondly, the more general linear pursuit-evasion game in the case where the terminal set closed and convex has been solved. For this case, we construct the set of attainability which is an ellipsoid. Some conditions on capturability are also discussed. The construction of the optimal pursuit time and optimal strategies for the players are the main objectives of this part.

The third part deals with the study of differential game of optimal approach with many Pursuers and one Evader, which can be considered as the generalized case of a pursuit-evasion game with one Pursuer and one Evader. This part is devoted to the problem of capture of one Evader by many Pursuers. The case of integral constrains is considered and the strategies for the players are constructed. Conditions are obtained for the existence of solutions for a multi-Pursuer game. In order to estimate the value of the game, we obtain several lemmas and theorems.

In the fourth part, the optimal control is obtained by using the method of the maximum principle of Pontryagin, where only a special case is studied. The result

shows an applications of Pontryagin's maximum principle in a linear quadratic differential game (LQDG) with integral constraints.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PENGEJAR-PENGELAK PERMAINAN PEMBEZAAN LINEAR  
DENGAN KEKANGAN KAMIRAN PADA FUNGSI KAWALAN**

Oleh

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Dalam kehidupan harian yang moden kini, kekerapan membuat keputusan telah meningkat. Hal ini menyebabkan kajian semula subjek Kejar-Mengelak (KM) permainan pembezaan. Permainan pembezaan memodelkan situasi di mana dua atau lebih pemain beroperasi di persekitaran yang sama, dengan tujuan yang saling bertentangan. Di bahagian ini usaha dilakukan untuk menyelesaikan permainan umum KM linear dengan sistem masa yang berbeza-beza dalam masa yang berterusan. Kebanyakan berhubung kajian permainan KM di dalam literatur semasa memfokus kepada permainan dua pemain dengan satu pengejar dan satu pengelak. Hasil bagi permainan multi pemain adalah sedikit sekali. Objektif kajian ini adalah untuk memanjangkan teori permainan pembezaan kepada permainan pembezaan linear KM dengan kamiran yang tidak dipisahkan dalam kawalan.

Pada peringkat permulaan, di selesaikan masalah ketentuan dengan menggunakan teknik kawalan optimum berdasarkan kawalan berstruktur pemain. Satu formula dihasilkan untuk masa mengejar dan membina strategi yang optimum untuk pemain, apabila sumber pengejar melebihi pengelak. Sebagai tambahan, suatu kaedah baru bagi menyelesaikan masalah pengelak dihasilkan, yang mana sumber terkawal bagi pengejar adalah kurang atau sama dengan pengelak.

Hasil daripada kajian ini terbahagi kepada empat bahagian, peringkat seterusnya adalah penyelesaian masalah KM yang lebih umum dalam kes di mana set terminal tertutup dan cembung. Untuk kes ini, dibina set kecapaian yang ellipsoid. Beberapa syarat penangkapan juga dibincangkan. Objektif utama dalam bahagian ini ialah untuk mendapatkan masa optimum pengejar dan strategi optimum pemain.

Bahagian ketiga membincangkan kajian permainan pembezaan dengan pendekatan ramai pengejar dengan satu pengelak. Ia boleh dipertimbangkan sebagai pengiktirafan kes KM dengan satu pengejar dan satu pengelak. Bahagian ini memfokus kepada masalah penangkapan satu pengelak oleh ramai pengejar. Kes kekanakan kamiran juga dikaji dan strategi-strategi pemain dibentuk. Syarat-syarat dihasilkan untuk kewujudan penyelesaian bagi permainan multi pengejar. Bagi menganggarkan nilai suatu permainan, beberapa lemma dan teorem telah dihasilkan.



Bahagian keempat, kawalan optimum dihasilkan dengan menggunakan kaedah maksimum prinsip Pontryagin. Kaedah khas sahaja yang dikaji. Hasil kajian menunjukkan penggunaan kaedah maksimum prinsip Pontryagin dalam permainan pembezaan linear kuadrat dengan kekangan kamiran.



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I certify that a Thesis Examination Committee has met on 4 November 2011 to conduct the final examination of Marzieh Khakestari on her thesis entitled Linear Pursuit-Evasion Differential Games with Integral Constraints on Control Functions in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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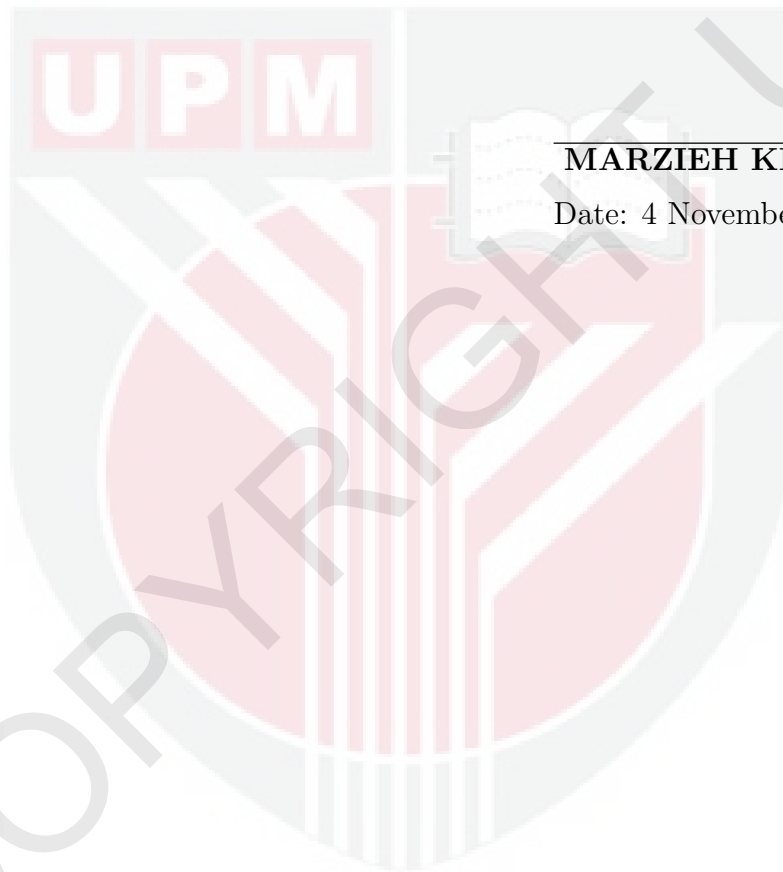
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## DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



MARZIEH KHAKESTARI

Date: 4 November 2011

## TABLE OF CONTENTS

	<b>Page</b>
<b>ABSTRACT</b>	iii
<b>ABSTRAK</b>	vi
<b>ACKNOWLEDGEMENTS</b>	ix
<b>APPROVAL</b>	x
<b>DECLARATION</b>	xii
<b>LIST OF ABBREVIATIONS</b>	xvi
<b>CHAPTER</b>	
<b>1 INTRODUCTION</b>	<b>1</b>
1.1 Dynamical Systems	1
1.1.1 Time-Varying Linear Systems	2
1.1.1.1 The Set of Attainability	4
1.2 Linear Quadratic and the Maximum Principle	5
1.2.1 Linear Quadratic Problem Definition	5
1.2.1.1 Transversality Conditions	6
1.2.1.2 Time-Varying Linear Quadratic	6
1.2.2 The Maximum Principle with Transversality Condi- tions for Fixed and Free Time Interval Problems	7
1.3 Mathematical Preliminaries	11
1.4 Objectives	14
1.5 Motivation	15
1.6 Dissertation Outline	17
<b>2 LITERATURE REVIEW</b>	<b>20</b>
2.1 Introduction	20
2.2 Differential Games	28
2.2.1 Deterministic Two-Player Pursuit-Evasion Differen- tial Games	30
2.2.1.1 Deterministic Game Formulation	30
2.2.1.2 Existence of the Value Function	33
2.2.1.3 The Differential Games with Constraints on the Control Functions	34
2.2.1.4 Example	35
2.2.2 Deterministic Multi-Player Pursuit-Evasion Differen- tial Games	37
2.2.2.1 Deterministic Game Formulation	37
2.2.2.2 Programmed Maximin Function	39
<b>3 A SOLUTION OF A LINEAR PURSUIT-EVASION GAME WITH INTEGRAL CONSTRAINTS</b>	<b>40</b>
3.1 Introduction	40
3.2 Statement of the Problem	41
3.3 Normal Form of Differential Games with Integral con- straints	41
3.3.1 General Normalization Scheme	41
3.3.2 Trajectories Generated by Strategies	44
3.3.3 Linear Differential Games with Integral Constraints	46

3.4	Auxiliary Results and Scheme of the Method	47
3.5	Formulation of Main Results	48
3.6	Conclusion	61
<b>4</b>	<b>SOLUTION OF A LINEAR PURSUIT-EVASION DIFFERENTIAL GAME WITH CLOSED AND CONVEX TERMINAL SET</b>	<b>62</b>
4.1	Introduction	62
4.2	Statement of the Problem	62
4.3	Auxiliary Results	64
4.3.1	Construction of the Attainability Set	66
4.3.2	Construction of a Control	67
4.4	Main Results	68
4.5	Conclusion	73
<b>5</b>	<b>ON A LINEAR DIFFERENTIAL GAME OF OPTIMAL APPROACH WITH MANY PURSUERS AND ONE EVADER</b>	<b>74</b>
5.1	Introduction	74
5.2	Statement of the Problem	75
5.3	Auxiliary Results	77
5.4	Main Results	84
5.5	Example	90
5.6	Conclusion	91
<b>6</b>	<b>LINEAR-QUADRATIC DIFFERENTIAL GAME WITH INTEGRAL CONSTRAINTS</b>	<b>92</b>
6.1	Introduction	92
6.2	Statement of the Problem	92
6.3	Conditions and Scheme of the Method	93
6.4	Main Result	94
6.5	Conclusion	96
<b>7</b>	<b>DISSERTATION SUMMARY AND FUTURE WORK</b>	<b>97</b>
7.1	Dissertation Summary	97
7.2	Future Work	99
	<b>BIBLIOGRAPHY</b>	<b>101</b>
	<b>BIODATA OF STUDENT</b>	<b>109</b>
	<b>LIST OF PUBLICATIONS</b>	<b>110</b>