



**UNIVERSITI PUTRA MALAYSIA**

**DIFFERENTIAL GAMES WITH MANY PURSUERS  
AND INTEGRAL CONSTRAINTS ON CONTROLS OF  
PLAYERS**

**FATEH ABDO ALI ALLAHABI**

**FS 2011 62**

**DIFFERENTIAL GAMES WITH MANY PURSUERS  
AND INTEGRAL CONSTRAINTS ON CONTROLS OF  
PLAYERS**

By

**FATEH ABDO ALI ALLAHABI**

Thesis Submitted to the School of Graduate Studies, Universiti Putra  
Malaysia, in Fulfilment of the Requirements for the Degree of  
Doctor of Philosophy

November 2011

## DEDICATIONS

*To my family.*



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in  
fulfilment of the requirement for the degree of Doctor of Philosophy

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**Chair: Ibragimov Gafurjan, PhD**

**Faculty: Science**

Control and differential game problems, with dynamics described by parabolic and hyperbolic partial differential equations attract the attention of many researchers. Some of these problems can be reduced to the one described by infinite systems of ordinary differential equations by using the decomposition method.

The main purpose of this thesis is to study the differential game problems described by an infinite system of 2-systems of second order differential equations, and it is extension to multi-player pursuit-evasion differential game problems, with various constraints, on control functions of players.

The existence and uniqueness theorem in the space  $C(0, T; l_r^2)$  is proved. Built on this, an optimal control for the control system described by an infinite system of differential equations with integral constraint is presented. The optimal control result is extended to study a pursuit differential game problem with the integral

constrains on the controls of players. The goal of the Pursuer is to force the system and its velocity to the origin on the spaces  $l_{r+1}^2$  and  $l_r^2$  respectively, and the Evader exactly tries to avoid this.

In addition to this, a theorem on pursuit with mixed constraints is proved, where Pursuers control is subjected to integral constraint and geometric constraint is imposed on Evaders control. Moreover, we established the sufficient conditions for which evasion is possible in the game considered, with geometric constraints on the control of players. Furthermore, control of the Evader is constructed in an explicit form.

Finally, a pursuit-evasion game with  $m$  Pursuer and one Evader are studied. In the pursuit game we present sufficient condition for which the Pursuers can bring the state of the system and its velocity into the origin for a finite time. For the evasion game we state and prove a theorem for which evasion is possible from any initial position.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PERMAINAN PEMBEZAAN RAMAI PEMANGSA DAN  
KEKANGAN KAMIRAN KE ATAS KAWALAN PARA PEMAIN**

Oleh

**FATEH ABDO ALI ALLAHABI**

November 2011

**Pengerusi: Ibragimov Gafurjan, PhD**

**Fakulti: Sains**

Kawalan dan permasalahan permainan pembezaan, dengan dinamik yang digambarkan oleh persamaan pembezaan separa parabola dan hiperbola menarik perhatian ramai penyelidik. Sebahagian daripada masalah ini boleh diturunkan kepada sistem persamaan pembezaan biasa tak terhingga dengan menggunakan kaedah penguraian.

Tujuan utama tesis ini adalah untuk mengkaji masalah permainan pembezaan menggunakan sistem tidak terhingga 2-sistem peringkat kedua persamaan pembezaan, dan lanjutan kepada multi-pemain mangsa-pemangsa dalam masalah permainan pembezaan, dengan pelbagai kekangan ke atas fungsi kawalan pemain.

Teorem kewujudan dan keunikan dalam ruang  $C(0, T; l_r^2)$  dibuktikan. Seterusnya, kawalan optimum bagi sistem kawalan berdasarkan sistem persamaan pembezaan tak terhingga dengan kekangan kamiran dipersembahkan. Keputusan kawalan optimum dilanjutkan untuk mengkaji permasalahan permainan pembezaan kejaran

dengan kekangan kamiran ke atas kawalan pemain. Matlamat pemangsa adalah untuk memaksa sistem dan halajunya ke asalan atas ruang  $l_{r+1}^2$  dan  $l_r^2$  masing-masing, dan mangsa cuba untuk mengelakkan keadaan berlaku ini daripada.

Di samping itu, teorem pengejaran dengan kekangan bercampur dibuktikan, dengan kawalan pemangsa tertakluk kepada kekangan kamiran dan kekangan geometri yang dikenakan ke atas kawalan mangsa. Selain itu, kami menubuhkan syarat cukup untuk pengelakan adalah mungkin dalam permainan yang dipertimbangkan, dengan kekangan geometri ke atas kawalan pemain. Selanjutnya, kawalan ke atas mangsa dibina dalam bentuk eksplisit.

Akhir sekali, permainan mangsa-pemangsa dengan  $m$  pemangsa dan satu mangsa dikaji. Dalam permainan pengejaran, kami persembahkan syarat cukup untuk pemangsa boleh membawa sistem dan halaju ke asalan dalam tempoh masa terhingga. Untuk permainan pengelakan kami nyatakan dan buktikan teorem yang pengelakan adalah mungkin dari sebarang kedudukan awal.

## ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my Supervisor, Associate Professor Dr. Ibragimov Gafurjan for his invaluable help, constant guidance, and encouragement during my graduate studies at the Department of Mathematics at Universiti Putra Malaysia.

I am also grateful to the members of the supervisory committee, Professor Dr. Malik Hj Abu Hassan and Associate Professor Dr. Norihan Md Arifin, for their supervision, advice, guidance and support.

Many thanks to all members of Department of Mathematics, Faculty of Science, Universiti Putra Malaysia, particularly, Head of Department, Associate Professor Dr. Fudziah Ismail, Professor Dr. Mohammed Suleiman for their help.

I would like to express my love and gratitude to my family, who supported and encouraged me during my studies.

Special thanks to my friends especially, Abbas Ja'afaru Badakaya, Dr. Mohammed Abdulkawi Mahiub and Risman Mat Hasim for their friendship and help.



I certify that a Thesis Examination Committee has met on 24 November 2011 to conduct the final examination of Fateh Abdo Ali Allahabi on his thesis entitled “**Differential Games with Many Pursuers and Integral Constraints on Controls of Players**” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **Doctor of Philosophy**.

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## DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

(Signature)

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**FATEH ABDO ALI ALLAHABI**

Date: 24 November 2011

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