UNIVERSITI PUTRA MALAYSIA

CLASSIFICATION OF NON-LIE COMPLEX FILIFORM LEIBNIZ ALGEBRAS FOR LOW DIMENSIONS

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FS 2011 57

CLASSIFICATION OF NON-LIE COMPLEX FILIFORM LEIBNIZ ALGEBRAS FOR LOW DIMENSIONS

By

SHARIFAH KARTINI BINTI SAID HUSAIN

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

May 2011

DEDICATION

To

My husband and my daughters

Syed Ahmad Thani bin Tuan Hadi @ Syed Mahadi, Sharifah Maisara,
Sharifah Habibah and Sharifah Nour El-Aqsa.

For their great patience

My Lovely Mother, Tuan Nong binti Syed Abdullah,
my brothers and my sisters.

For their encouragement

and

My Dear Teachers

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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Chair: Associate Professor Isamiddin S. Rakhimov, PhD

Faculty: Faculty of Science

The thesis is concerned with the structural properties of Leibniz algebras. These algebras satisfy certain identity that was suggested by J.-L.Loday (1993). When he used the tensor product instead of external product in the definition of the n-th cochain, in order to prove the differential property defined on cochains, it suffices to replace the anticommutativity and Jacobi identity by the Leibniz identity.

The algebras satisfying the Leibniz identity are called Leibniz algebras. We will investigate filiform Leibniz algebras. It is known that filiform Leibniz algebras arise from two sources. The first source is a naturally graded non-Lie filiform Leibniz algebras and the other one is a naturally graded filiform Lie algebras. Here we consider the class of filiform Leibniz algebras arising from the naturally graded non-Lie filiform Leibniz algebra.

In 2001, Ayupov and Omirov divided this class into two subclasses. However, isomorphism problems within these classes are yet to be investigated. The classes

in dimension n over a field k are denoted by $FLeib_n(k)$ and $SLeib_n(k)$, respectively. Bekbaev and Rakhimov (2006) suggested an algebraic approach to the description of isomorphism classes of filiform Leibniz algebras in terms of algebraic invariants. The main purpose of this thesis is to apply this method and find the complete classification and invariants of low dimensional complex filiform Leibniz algebras. The main result is the complete classification of complex filiform Leibniz algebras arising from the naturally graded non-Lie filiform Leibniz algebras from dimensions 5 to 8.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENGELASAN ALJABAR LEIBNIZ FILIFORM KOMPLEKS BUKAN LIE BAGI DIMENSI YANG RENDAH

Oleh

SHARIFAH KARTINI BINTI SAID HUSAIN

Mei 2011

Pengerusi: Profesor Madya Isamiddin S. Rakhimov, PhD

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Tesis ini memperkenalkan ciri-ciri struktur aljabar Leibniz. Aljabar ini memenuhi identiti-identiti tertentu yang telah diperkenalkan oleh J.-L.Loday (1993). Ketika beliau menggunakan hasildarab tensor sebagai ganti hasildarab luaran dalam takrifan korantai ke$-n$, untuk membuktikan sifat pembeza yang tertakrif bagi korantai, adalah memadai menggantikan sifat anti kalis tukar-terbit dan identiti Jacobi, dengan identiti Leibniz.

Aljabar yang memenuhi identiti Leibniz dikenali sebagai aljabar Leibniz. Kami akan mengkaji aljabar Leibniz filiform. Diketahui bahawa aljabar Leibniz filiform digarap daripada dua sumber. Sumber pertama ialah aljabar Leibniz filiform bukan Lie tergred semulajadi dan yang satu lagi ialah aljabar Lie filiform tergred semulajadi. Di sini kami mempertimbangkan kelas aljabar Leibniz filiform yang muncul daripada aljabar Leibniz filiform bukan Lie tergred semulajadi.

Pada 2001, Ayupov dan Omirov membahagikan kelas ini kepada dua subkelas. Namun begitu permasalahan isomorfisma dalam kelas tersebut masih belum dis-

elidiki sepenuhnya. Kelas-kelas ini yang berdimensi n terhadap satu medan k, masing-masing diberi tatatanda $FLeib_n(k)$ dan $SLeib_n(k)$. Bekbaev dan Rakhimov (2006) telah mencadangkan satu pendekatan bersifat aljabar untuk menerangkan masalah kelas isomorfisma bagi aljabar Leibniz filiform dalam sebutan tak variant. Tujuan utama tesis ini adalah untuk menggunakan kaedah tersebut dan mendapatkan pengkelasan yang lengkap dan sebutan tak variant selengkapnya, bagi aljabar Leibniz filiform berdimensi rendah terhadap medan kompleks. Hasil utama ialah satu pengkelasan lengkap aljabar Leibniz filiform terhadap medan kompleks yang muncul daripada aljabar Leibniz filiform bukan Lie tergred semulajadi dalam dimensi 5 hingga 8.

ACKNOWLEDGEMENTS

First and foremost, all praises are to the almighty ALLAH for His blessings and mercy which enable me to learn and complete this thesis.

During the almost four challenging years of my work I had collaboratively worked with some people. I am very pleased to express my indebtedness to all of them.

First of all, I would like to thank my supervisor Associate Professor Dr. Isamiddin S. Rakhimov, for his countless suggestions, guidance and constant support throughout this research. In my endeavour to complete this thesis, he has been very generous and patient in contributing his valuable time to correct and improve my thesis.

I also want to thank Associate Professor Dr. Ural D. Bekbeav for his guidance, idea contributions and motivations through my early learning years of the classification of filiform Leibniz algebras, prior to his retirement from Mathematics Department of Universiti Putra Malaysia. May Allah reward him for every single drop of contribution he gave me.

This acknowledgment will not be complete if I do not include my co supervisor Prof. Dr. Kamel Ariffin M. Atan, the director of Institute for Mathematical Research (INSPEM) for all his support and guidance. Also to Dr Mohammad Alinor bin Abdul Kadir who has generously contributed articles and books rarely found in Malaysia, and Associate Professor Dr. Mohamad Rushdan Md. Said as a member of supervisory committee for his cooperation. This thesis will not be possible without contributions from them.

My PhD study was supported by University Putra Malaysia and therefore, I

would like to take this opportunity to thank UPM for the financial support.

Last but not least, I am also indebted to my colleagues in the Mathematics Department of UPM, that are Seyed Jalal Langari and Munther Al-Hassan for all the assistance and discussions held.

When being away from the university, whom else would I depend on but a group of supporting beings at home who I call family. To my lovely angels Sara, Bibah and Nour, who have been very understanding when dear Umi had to allocate too much time staring at laptop screen. To my lovely hubby, who has been my 'home supervisor' as an English teacher. To Mak at home, your contribution for the past 36 years is more than noble; and you shower it with patient and love.

As my final note, everything that I have been doing is intended to uplift the image of muslim and let it be my humble contribution to the ummah.

I certify that a Thesis Examination Committee has met on the **26 May 2011** to conduct the final examination of Sharifah Kartini binti Said Husain on her thesis entitled "**Classifition of non-Lie complex filiform Leibniz algebras for low dimensions**", in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

Sharifah Kartini Binti Said Husain

Date: 26 May 2011

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