On the composition and neutrix composition of the delta function with the hyperbolic tangent and its inverse functions.

ABSTRACT

Let F be a distribution in D' and let f be a locally summable function. The composition F (f (x)) of F and f is said to exist and be equal to the distribution h (x) if the limit of the sequence {Fn (f (x)) } is equal to h (x), where Fn (x) = F (x) * δn (x) for n = 1,2,⋯ and { δn (x) } is a certain regular sequence converging to the Dirac delta function. It is proved that the neutrix composition δ(rs-1) ((tanh x+)1/r) exists and δ(rs-1) ((tanh x+)1/r) =√k=0 s-1√i=0 Kk ((- 1)k cs-2 i - 1, k (rs) !/2sk!) δ(k) (x) for r, s = 1,2,⋯ , where Kk is the integer part of (s - k - 1) / 2 and the constants c j,k are defined by the expansion (tanh - 1 x)k = {√i=0 ∞ (x 2i+1/(2 i + 1)) }k = √j=k ∞ c j, k xj, for k = 0,1, 2,⋯. Further results are also proved.

Keyword: Neutrix; Neutrix composition; Delta function.