

On the neutrix composition of the delta and inverse hyperbolic sine functions.

ABSTRACT

Let F be a distribution in D' and let f be a locally summable function. The composition $F(f(x))$ of F and f is said to exist and be equal to the distribution $h(x)$ if the limit of the sequence $\{F_n(f(x))\}$ is equal to $h(x)$, where $F_n(x) = F(x) * \delta_n(x)$ for $n = 1, 2, \dots$ and $\{\delta_n(x)\}$ is a certain regular sequence converging to the Dirac delta function. In the ordinary sense, the composition $\delta(s) [(\sinh^{-1} x)^r]$ does not exist. In this study, it is proved that the neutrix composition $\delta(s) [(\sinh^{-1} x)^r]$ exists and is given by $\delta(s) [(\sinh^{-1} x)^r] = \sum_{k=0}^{s+r-1} \binom{s+r-1}{k} \binom{k}{i} (-1)^k c_{s,k,i} / 2^{k+1} k! \delta(k)(x)$, for $s = 0, 1, 2, \dots$ and $r = 1, 2, \dots$, where $c_{s,k,i} = (-1)^s s! [(k-2i+1)^{r-1} + (k-2i-1)^{r-1}] / (2^{r+s-1})!$. Further results are also proved.

Keyword: Neutrix; Distributions.