On the neutrix composition of the delta and inverse hyperbolic sine functions.

ABSTRACT

Let $F$ be a distribution in $D'$ and let $f$ be a locally summable function. The composition $F(f(x))$ of $F$ and $f$ is said to exist and be equal to the distribution $h(x)$ if the limit of the sequence $\{F_n(f(x))\}$ is equal to $h(x)$, where $F_n(x) = F(x) * \delta_n(x)$ for $n = 1, 2, \ldots$ and $\{\delta_n(x)\}$ is a certain regular sequence converging to the Dirac delta function. In the ordinary sense, the composition $\delta(s) [(\text{sinh}^{-1} x)^r]$ does not exist. In this study, it is proved that the neutrix composition $\delta(s) [(\text{sinh}^{-1} x)^r]$ exists and is given by $\delta(s) [(\text{sinh}^{-1} x)^r] = \sum_{k=0}^{r+1} \sum_{i=0}^{k} (-1)^{k+i} \frac{(k)_i (s)_r}{r! (k+r)_{i+r}} \delta(k)(x)$ for $s = 0, 1, 2, \ldots$ and $r = 1, 2, \ldots$, where $c_{s,k,i} = (-1)^s \frac{(k-2i+1)s + (k-2i-1)s + r}{(2s+1)k!}$. Further results are also proved.

Keyword: Neutrix; Distributions.