Approximating Cauchy-type singular integral by an automatic quadrature scheme.

ABSTRACT

An automatic quadrature scheme is developed for the approximate evaluation of the product-type indefinite integral where \( K(t,c) = 1/(t-c) \) and \( f(t) \) is assumed to be a smooth function. In constructing an automatic quadrature scheme, we consider two cases: (1) \(-1 < x < y < 1\), and (2) \( x = -1, y = 1\). In both cases the density function \( f(t) \) is replaced by the truncated Chebyshev polynomial \( p_N(t) \) of the first kind of degree \( N \). The approximation \( p_N(t) \) yields an integration rule \( Q_N(f,x,y,c) \) to the integral \( Q(f,x,y,c) \). Interpolation conditions are imposed to determine the unknown coefficients of the Chebyshev polynomials \( p_N(t) \). Convergence problem of the approximate method is discussed in the classes of function \( C_{N+1, \alpha}[-1,1] \) and \( \mathcal{R}^{N+1, \alpha}[-1,1] \). Numerically, it is found that when the singular point \( c \) either lies in or outside the interval \((x,y)\) or comes closer to the end points of the interval \([-1,1]\), the proposed scheme gives a very good agreement with the exact solution. These results in the line of theoretical findings.

**Keyword:** Automatic quadrature scheme; Product integral; Singular integral; Clenshaw-curtis rules; Chebyshev polynomials; Indefinite integral; Recurrence relation.