Approximating Cauchy-type singular integral by an automatic quadrature scheme.

ABSTRACT

An automatic quadrature scheme is developed for the approximate evaluation of the product-type indefinite integral where View the MathML source, $K(t,c)=1/(t-c)$ and $f(t)$ is assumed to be a smooth function. In constructing an automatic quadrature scheme, we consider two cases: (1) $-1<x<y<1$, and (2) $x=-1,y=1$. In both cases the density function $f(t)$ is replaced by the truncated Chebyshev polynomial $pN(t)$ of the first kind of degree N. The approximation $pN(t)$ yields an integration rule $QN(f,x,y,c)$ to the integral $Q(f,x,y,c)$. Interpolation conditions are imposed to determine the unknown coefficients of the Chebyshev polynomials $pN(t)$. Convergence problem of the approximate method is discussed in the classes of function $CN+1,\alpha[-1,1]$ and View the MathML source. Numerically, it is found that when the singular point c either lies in or outside the interval (x,y) or comes closer to the end points of the interval $[-1,1]$, the proposed scheme gives a very good agreement with the exact solution. These results in the line of theoretical findings.

Keyword: Automatic quadrature scheme; Product integral; Singular integral; Clenshaw-curtis rules; Chebyshev polynomials; Indefinite integral; Recurrence relation.