

On summability of N -fold Fourier integrals corresponding to pseudodifferential operators

ABSTRACT

It is well known, that if $N \geq 3$, then spherical partial sums of N -fold Fourier integrals (eigenfunction expansions of Laplace operator) of the characteristic function of the unit ball diverge at the origin. Note, here level surface of Laplace operator and the surface of discontinuity of the considered piecewise smooth function are both spheres. It was first noted by Pinsky and Taylor, that if we consider nonspherical partial sums (eigenfunction expansions of elliptic pseudodifferential operators), then to obtain the same effect, we should change the above second sphere with the dual set to level surface of the pseudodifferential operator. Namely, nonspherical partial sums of a piecewise smooth function, supported inside the dual surface converge everywhere except the origin. In this paper we investigate summability of these expansions by Riesz method and show that the order $s > (N - 3)/2$ of Riesz means guarantees convergence everywhere, where the function is smooth. Since piecewise smooth functions are in Nikolskii class $H^{1,1}(\mathbb{R}^N)$, we also establish necessary and sufficient conditions for uniform convergence of expansions of H^p -functions.

Keyword: N -fold Fourier integrals; Nikolskii spaces; Pseudodifferential operators; Spectral expansions; Uniformly convergence