UNIVERSITI PUTRA MALAYSIA

APPROXIMATE SOLUTION OF THE SYSTEM OF NONLINEAR INTEGRAL EQUATIONS

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By

ODAY SHAFIQ HAZAIMEH

Thesis Submitted to the School of Graduate studies, Universiti Putra Malaysia, in Fulfillment of the Requirement for the Degree of Master of Science

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DEDICATION

This thesis dedicated to

all my family members

especially

my dear father Shafiq

and my lovely mother Fatima

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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ODAY SHAFIQ HAZAIMEH

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Chairman: Zainnidin Eshkuvatov, PhD

Faculty : Institute for Mathematical Research

Integral equations are used as mathematical models for many physical situations and applied mathematics. The numerical solutions of such integral equations have been highly studied by many authors. In this thesis we deal with the system of nonlinear integral equations (NIEs) of the form

$$\begin{cases} x(t) - \int_{y(t)}^{t} H(t,\tau)x^n(\tau)d\tau = 0, \\ \int_{y(t)}^{t} K(t,\tau)x^n(\tau)d\tau = f(t). \end{cases} \qquad n \geq 2 \qquad (1)$$

where $0 < t_0 \leq t \leq T$, $y(t) < t$, and the given functions $H(t,\tau),\ K(t,\tau) \in C_{[0,\infty]\times[t_0,\infty]}$, $f(t) \in C_{[t_0,\infty]}$. The aim of the work is to find the unknown functions $x(t) \in C^1_{[t_0,\infty]}$, $y(t) \in C^1_{[t_0,\infty]}$ in (1). To this end, we introduce the operator function

$$P(X) = (P_1(X), P_2(X)) = (0,0), \qquad X = (x(t), y(t)), \qquad (2)$$

and hence (1) can be expressed in the operator form

$$\begin{cases} P_1(x(t), y(t)) = x(t) - \int_{y(t)}^{t} H(t,\tau)x^n(\tau)d\tau, \\ P_2(x(t), y(t)) = f(t) - \int_{y(t)}^{t} K(t,\tau)x^n(\tau)d\tau. \end{cases}$$

We solve (2) by the modified Newton-Kantorovich method

$$P'(X_0)(X - X_0) + P(X_0) = 0\,,\; X_0 = (x_0(t), y_0(t)). \qquad (3)$$

Substituting the first derivatives in (3), we have

$$\left.\begin{aligned}
&\Delta x(t) - \int_{y_0(t)}^{t} H(t,\tau) n x_0^{n-1}(\tau)\Delta x(\tau) d\tau + H(t, y_0(t)) x_0^n(y_0(t))\Delta y(t) \\
&\qquad = \int_{y_0(t)}^{t} H(t,\tau) x_0^n(\tau) d\tau - x_0(t), \\
&- \int_{y_0(t)}^{t} K(t,\tau) n x_0^{n-1}(\tau)\Delta x(\tau) d\tau + K(t, y_0(t)) x_0^n(y_0(t))\Delta y(t) \\
&\qquad = \int_{y_0(t)}^{t} K(t,\tau) x_0^n(\tau) d\tau - f(t).
\end{aligned}\right\} \qquad (4)$$

where $\Delta x(t) = x_1(t) - x_0(t)$, $\Delta y(t) = y_1(t) - y_0(t)$. Solving (4) in terms of $(\Delta x(t), \Delta y(t))$ we obtain $(x_1(t), y_1(t))$, by continuing this process, we arrive to the sequence of approximate solutions $(x_m(t), y_m(t))$ from

$$\begin{cases}
\Delta x_m(t) - n \displaystyle\int_{y_0(t)}^{t} K_1(t,\tau) x_0^{n-1}(\tau)\Delta x_m(\tau) d\tau = F_{m-1}(t), \\
\Delta y_m(t) = \dfrac{1}{H(t, y_0(t)) x_0^n(y_0(t))}\left[\begin{aligned} & n \int_{y_0(t)}^{t} H(t,\tau) x_0^{n-1}(\tau)\Delta x_m(\tau) d\tau - \Delta x_m(t) \\ & + \int_{y_{m-1}(t)}^{t} H(t,\tau) x_{m-1}^n(\tau) d\tau - x_{m-1}(t) \end{aligned}\right]
\end{cases} \qquad (5)$$

where $\Delta x_m(t) = x_m(t) - x_{m-1}(t)$ and $\Delta y_m(t) = y_m(t) - y_{m-1}(t)$, m=2, 3…

In discretization process the modified trapezoidal rule is applied for Eq. (5).

In this thesis we have proved the existence and the uniqueness of the solution of Eq. (1). Moreover, the rate of convergence of modified Newton-Kontorovich method for Eq. (2) is established. Finally, FORTRAN code is developed to obtain numerical results which are in line with the theoretical findings.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN HAMPIRAN SISTEM PERSAMAAN KAMIRAN TAK LINEAR MENGGUNAKAN

Oleh

ODAY SHAFIQ HAZAIMEH

Ogos 2010

Pengerusi: **Zainnidin Eshkuvatov, PhD**

Fakulti : Institute Penyelidikan Matematik

Persamaan kamiran digunakan sebagai model matematik untuk pelbagai situasi fizikal dan matematik gunaan. Penyelesaian berangka bagi persamaan kamiran telah dikaji secara mendalam oleh ramai pengarang. Dalam tesis ini, kami berurusan dengan sistem persamaan kamiran tak linear (PKTL) dalam bentuk

$$\begin{cases} x(t) - \int_{y(t)}^{t} H(t,\tau)x^n(\tau)d\tau = 0, \\ \int_{y(t)}^{t} K(t,\tau)x^n(\tau)d\tau = f(t). \end{cases} \quad n > 2 \qquad (1)$$

di mana $0 < t_0 \le t \le T,\ y(t) < t$ dengan fungsi yang di beri, $H(t,\tau),\ K(t,\tau) \in C_{[0,\infty]\times[t_0,\infty]},\ f(t) \in C_{[t_0,\infty]}$. Matlamat kerja ini adalah untuk mencari fungsi anu $x(t) \in C^1_{[t_0,\infty]}$, $y(t) \in C^1_{[t_0,\infty]}$ dalam (1).

Untuk mencapai matlamat ini, kami mempekenalkan satu operator

$$P(X) = (P_1(X), P_2(X)) = (0,0), \qquad X = (x(t), y(t)) \qquad (2)$$

dan maka boleh digambarkan (1) dalam bentuk operator

$$\begin{cases} P_1(x(t), y(t)) = x(t) - \int_{y(t)}^{t} H(t,\tau)x^n(\tau)d\tau, \\ P_2(x(t), y(t)) = f(t) - \int_{y(t)}^{t} K(t,\tau)x^n(\tau)d\tau. \end{cases}$$

Kami menyelesaikan (2) dengan menggunakan kaedah Newton-Kantorovich terubahsuai

$$P'(X_0)(X - X_0) + P(X_0) = 0\,,\; X_0 = (x_0(t), y_0(t))\,. \qquad (3)$$

Menggantikan derivatif pertama dalam (3), kami mempunyai

$$\left.\begin{aligned}
&\Delta x(t) - \int_{y_0(t)}^{t} H(t,\tau) n x_0^{n-1}(\tau)\Delta x(\tau)d\tau + H(t, y_0(t)) x_0^{n}(y_0(t))\Delta y(t)\\
&\qquad = \int_{y_0(t)}^{t} H(t,\tau) x_0^{n}(\tau)d\tau - x_0(t)\,,\\
&-\int_{y_0(t)}^{t} K(t,\tau) n x_0^{n-1}(\tau)\Delta x(\tau)d\tau + K(t, y_0(t)) x_0^{n}(y_0(t))\Delta y(t)\\
&\qquad = \int_{y_0(t)}^{t} K(t,\tau) x_0^{n}(\tau)d\tau - f(t)\,.
\end{aligned}\right\} \qquad (4)$$

di mana $\Delta x(t) = x_1(t) - x_0(t)$, $\Delta y(t) = y_1(t) - y_0(t)$. Selesaikan (4) dalam ungkapan $(\Delta x(t), \Delta y(t))$ kami memperoleh $(x_1(t), y_1(t))$, dan dengan meneruskan proses ini, kami akan sampai kepada urutan penyelesaian hampiran $(x_m(t), y_m(t))$ daripada

$$\begin{cases}
\Delta x_m(t) - n\displaystyle\int_{y_0(t)}^{t} K_1(t,\tau) x_0^{n-1}(\tau)\Delta x_m(\tau)d\tau = F_{m-1}(t),\\[2ex]
\Delta y_m(t) = \dfrac{1}{H(t, y_0(t)) x_0^{n}(y_0(t))}\left[\begin{aligned} &n\int_{y_0(t)}^{t} H(t,\tau) x_0^{n-1}(\tau)\Delta x_m(\tau)d\tau - \Delta x_m(t)\\ &+\int_{y_{m-1}(t)}^{t} H(t,\tau) x_{m-1}^{n}(\tau)d\tau - x_{m-1}(t)\end{aligned}\right]
\end{cases} \qquad (5)$$

di mana $\Delta x_m(t) = x_m(t) - x_{m-1}(t)$ dan $\Delta y_m(t) = y_m(t) - y_{m-1}(t)$, m=2, 3…

Dalam proses pendiskretan, petua trapezium terubahsuai diaplikasikan kepada persamaan (5).

Dalam tesis ini, kami telah buktikan kewujudan dan keunikan bagi penyelesaian persamaan (1). Selain itu, kadar penumpuan kaedah Newton-Kontorovich terubahsuai untuk persamaan (2) telah diwujudkan. Akhir sekali, kod FORTRAN dibina bagi mendapatkan keputusan berangka di mana ianya adalah selari dengan keputusan secara teorinya.

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My deepest gratitude and love to my parents, brothers, and sisters for their support, patience and prayers for my success.

I certify that a Thesis Examination committee has met on 20 August 2010 to conduct the final examination of Oday Shafiq Muhammad Hazaimeh on his thesis entitled “Approximate Solution for System of Nonlinear Integral Equations” in accordance with the Universitis and University College Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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Date: 21 October 2010

DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or other institutions.

ODAY SHAFIQ HAZAIMEH

Date: 3/09/2010

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