



UNIVERSITI PUTRA MALAYSIA

**SOME MODIFICATIONS ON INTERVAL SYMMETRIC SINGLE-STEP
PROCEDURE FOR SIMULTANEOUS INCLUSION OF REAL ZEROS OF
POLYNOMIALS**

SYAIDA FADHILAH BT MOHAMMAD RUSLI

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**SOME MODIFICATIONS ON INTERVAL SYMMETRIC SINGLE-STEP
PROCEDURE FOR SIMULTANEOUS INCLUSION OF REAL ZEROS OF
POLYNOMIALS**

By

SYAIDA FADHILAH BT MOHAMMAD RUSLI

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in
Fulfilment of the Requirements for the Degree of Master of Science**

October 2011

DEDICATION

to

Nujaidi Israr Abdul Hadi,

Mardhiyah Haidir,

Mohammad Rusli Fahaya,

Lina Sofiyah Mohammad Rusli,

Eiffah Rodhiyah Mohammad Rusli,

Ezzah Rufsaidah Mohammad Rusli,

and

all my family members...

Abstract of thesis presented to the senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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PROCEDURE FOR SIMULTANEOUS INCLUSION OF
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SYAIDA FADHILAH BT MOHAMMAD RUSLI

October 2011

Chair : Mansor Bin Monsi, PhD

Faculty: Faculty of Sciences

In this thesis, we discuss about the interval iterative procedures of bounding real zeros of polynomials simultaneously. We concentrate on the procedure that has been proposed by Monsi in 1988 that is the interval symmetric single-step procedure *ISSI* and do some modifications on the procedure and come out with three modified procedures.

For these procedures, we start with suitably chosen initial disjoint intervals where each interval contains a zero of a polynomial. These procedures will produce successively smaller intervals that are guaranteed to still contain the zeros.

In order to assure that the procedures are promising, we analyze the R -order of convergence of the procedures and compare them with the original procedure *ISSI*. We include the analysis of inclusions to certify the convergences of the procedures.

The coding for the algorithms of these procedures are developed and implemented using the MATLAB R2007a in co-operated with the Intlab V5.5 toolbox for interval arithmetic developed by Rump. These three new modified procedures are proved to have better rate of convergences and this is supported by lesser CPU times and lesser number of iterations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk Ijazah Master Sains

BEBERAPA PENGUBAHSUAIAN PADA PROSEDUR SELANG LANGKAH-TUNGGAL BERSIMMETRI DALAM MEMERANGKAP SUATU PENSIFAR NYATA POLINOMIAL SECARA SERENTAK

Oleh

SYAIDA FADHILAH BT MOHAMMAD RUSLI

Oktober 2011

Pengerusi: Mansor Monsi, PhD

Fakulti : Fakulti Sains

Dalam tesis ini, kami membincangkan tentang prosedur selang berlelar yang menghadkan punca nyata polinomial secara serentak. Kami menumpukan kajian kepada prosedur yang telah dicadangkan oleh Monsi pada 1988 iaitu prosedur selang langkah-tunggal bersimetri *ISSI* dan melakukan beberapa pengubahsuaian ke atas prosedur *ISSI* yang menghasilkan tiga prosedur terubahsuai.

Bagi kaedah ini, kami bermula dengan selang permulaan tak bercantum di mana setiap selang tersebut mengandungi punca polinomial. Prosedur-prosedur ini akan menjana selang yang lebih kecil secara berturutan yang dijamin masih mengandungi punca-punca.

Bagi memastikan prosedur-prosedur ini meyakinkan, kami menganalisa kadar penumpuan prosedur bagi peringkat R dan membandingkannya dengan prosedur asal *ISSI*. Kami juga menyertakan analisa rangkuman bagi menjamin penumpuan prosedur tersebut.

Pengekodan algoritma prosedur-prosedur ini dibina dan diimplimentasikan dengan menggunakan “MATLAB R2007a” dengan kerjasama “Intlab V5.5 toolbox” untuk aritmetik selang yang dibina oleh Rump. Ketiga-tiga prosedur terubahsuai ini telah terbukti mempunyai kadar penumpuan yg lebih baik dan di sokong dengan masa CPU yang lebih rendah dan juga jumlah lelaran yang lebih kecil.

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I certify that a Thesis Examination Committee has met on 18 October 2011 to conduct the final examination of Syaida Fadhilah Bt Mohammad Rusli on her thesis entitled “**Some Modifications On Interval Symmetric Single-Step Procedure For Simultaneous Inclusion of Real Zeros of Polynomials**” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the degree of Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and not concurrently, submitted for any other degree at Universiti Putra Malaysia or any other institution.

SYAIDA FADHILAH BT MOHAMMAD RUSLI

Date: 18 October 2011

TABLE OF CONTENTS

	Page
DEDICATION	ii
ABSTRACT	iii
ABSTRAK	v
ACKNOWLEDGEMENTS	vii
APPROVAL	x
DECLARATION	xi
LIST OF TABLES	xiv
LIST OF FIGURES	xvi
LIST OF ABBREVIATIONS	xvii
CHAPTER	
1 INTRODUCTION	
1.1. Scope of the Problem	1
1.2. Objective of the Research	3
1.3. Thesis Outline	4
1.4. Concept of Interval Computations	5
1.4.1. Operations of Interval Computations	6
1.4.2. Properties of Interval Computations	10
1.4.3. Interval Evaluation and Range of Real Function	16
2 LITERATURE REVIEW	
2.1. Overview of Techniques in Solving the Zeros of Polynomials	20
2.2. The Interval Iterative Procedures	25
3 THE INTERVAL MIDPOINT SYMMETRIC SINGLE-STEP PROCEDURE <i>IMSSI</i>	
3.1. The Algorithm <i>IMSSI</i>	29
3.2. The <i>R</i> -Order of Convergence	30
3.3. Analysis of <i>R</i> -Order of Convergence of <i>MISSI</i>	31
3.4. Numerical Results	43
3.5. Discussion	49

4	THE INTERVAL ZORO SYMMETRIC SINGLE-STEP PROCEDURE <i>IZSSI</i>	
4.1.	The Algorithm <i>IZSSI</i>	51
4.2.	Analysis of <i>R</i> -Order of Convergence of <i>IZSSI</i>	52
4.3.	Numerical Results	69
4.4.	Discussion	76
5	THE INTERVAL MIDPOINT ZORO SYMMETRIC SINGLE-STEP PROCEDURE <i>IMZSSI</i>	
5.1.	The Algorithm <i>IMZSSI</i>	78
5.2.	Analysis of <i>R</i> -Order of Convergence of <i>IMZSSI</i>	80
5.3.	Numerical Results	83
5.4.	Discussion	90
6	CONCLUSION AND RECOMMENDATION FOR FUTURE RESEARCH	
6.1.	Summary of the Numerical Results	92
6.2.	Conclusion	99
6.3.	Recommendation for Future Research	100
	REFERENCES	103
	APPENDIX A	106
	BIODATA OF STUDENT	112
	LIST OF PUBLICATIONS	113