



UNIVERSITI PUTRA MALAYSIA

**PROPERTIES AND ESTIMATION FOR FRACTIONALLY
INTEGRATED SPATIAL MODELS AND NON-NEGATIVE
INTEGER-VALUED AUTOREGRESSIVE SPATIAL MODELS**

ALI REZA GHODSI

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INTEGRATED SPATIAL MODELS AND NON-NEGATIVE
INTEGER-VALUED AUTOREGRESSIVE SPATIAL MODELS**



By

ALI REZA GHODSI

Thesis Submitted to the School of Graduate Studies, Universiti
Putra Malaysia, in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy

November 2011

DEDICATION

To all those

Who educated me, encouraged me, love me and Whom I love

specially

My Parents; My Wife, Toktam and My Son, Mohsen

And

In loving memory of my grandmother, Shahrbanoo Shakeri

May Allah bless her soul

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

PROPERTIES AND ESTIMATION FOR FRACTIONALLY INTEGRATED SPATIAL MODELS AND NON-NEGATIVE INTEGER-VALUED AUTOREGRESSIVE SPATIAL MODELS

By

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November 2011

Chair: Associate Professor Mahendran Shitan, PhD

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Spatial modelling has its applications in many fields like geostatistics, geology, geography, agriculture, meteorology, biology, epidemiology, etc. Spatial data can be classified as geostatistical data, lattice data, or point patterns. This research concentrates on lattice data observed on a regular grid. Examples of spatial data include data collected on a regular grid from satellites (such as ocean temperature) and from agricultural field trials. Many models have been suggested in modelling spatial dependence like the Simultaneous Autoregressive (SAR), Conditional Autoregressive (CAR), Moving Average (MA) and Autoregressive Moving Average (ARMA). There also exist a class of spatial models that are known as separable models where its correlation structure can be expressed as a product of correlations. In some cases spatial data may exhibit a long memory structure where their autocorrelation function decays rather slowly which can be modelled by fractionally integrated ARMA models.

The aim of this research is to introduce and investigate some types of spatial models which have many applications.

We first focus on estimation of the memory parameters of the fractionally integrated spatial models. The estimation of the memory parameters by two different methods, namely the regression method and Whittle's method are discussed.

Next we consider the Fractionally Integrated Separable Spatial ARMA (FISSARMA) models. The asymptotic properties of the normalised periodogram of the FISSARMA model such as the asymptotic mean and the asymptotic second-order moments of the normalised fourier coefficients and the asymptotic distribution of the normalised periodogram are established.

The third objective of this research is to develop a non-separable counterpart of the FISSAR(1,1) model. We term this model as the first-order Fractionally Integrated Non-Separable Spatial Autoregressive (FINSSAR(1,1)) model. The theoretical autocovariance function and the spectral function of the model are obtained and some numerical results are presented.

Finally, as spatial data may have non-negative integer values, there is a need to introduce non-Gaussian integer-valued spatial models. In this research the first-order Spatial Integer-valued Autoregressive SINAR(1,1) model with discrete marginal distribution is introduced. Some properties of this model (mean, variance and autocorrelation functions) are established. The Yule-Walker estimator of the parameters of the model is also introduced and the strong consistency of the Yule-Walker estimators of the parameters of the model are also established.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**SIFAT DAN ANGGARAN BAGI MODEL RERUANG
KESEPADUAN PECAHAN DAN MODEL RERUANG
AUTORESIF INTEGER BERNILAI BUKAN NEGATIF**

Oleh

ALI REZA GHODSI

November 2011

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Permodelan ruang mempunyai aplikasi dalam banyak bidang seperti geostatistik, geologi, geografi, meteorologi, biologi, epidemiologi dan lain-lain. Data ruang boleh dikelaskan sebagai data geostatistik, data *lattice*, atau polar titik. Kajian ini tertumpu kepada data *lattice* yang dicerap di atas grid sekata. Contoh data ruang termasuklah data yang dikumpulkan di atas grid sekata daripada satelit (seperti suhu lautan) dan daripada ujian ladang pertanian. Banyak model telah dicadangkan untuk memodelkan kebersandaran riang seperti Autoregresif Serentak (SAR), Autoregresif Bersyarat (CAR), Purata Bergerak (MA) dan Autoregresif Purata Bergerak (ARMA). Terdapat pula kelas model ruang yang dikenali sebagai model terpisahan yang boleh dinyatakan sebagai hasil darab korelasi. Dalam kes tertentu, data ruang boleh menunjukkan data memori yang panjang yang menyusut secara perlahan yang boleh dimodelkan dengan model kesepaduan pecahan ARMA.

Tujuan kajian ini ialah untuk memperkenalkan dan menyelidiki beberapa jenis

model ruang yang mempunyai banyak aplikasi.

Kami mulakan dengan anggaran parameter memori bagi kesepaduan pecahan model ruang. Penganggaran parameter memori dengan dua kaedah iaitu kaedah regresi dan kaedah Whittle dibincangkan.

Kemudian, kami mempertimbangkan model kesepaduan ruang pecahan terpisahkan. Sifat asimptot periodogram yang ternormal model FISSARMA seperti min asimptot dan momen pekali Fourier ternormal peringkat kedua periodogram ternormal ditunjukkan hasilnya.

Objektif ketiga kajian ini ialah untuk membangunkan model tidak terpisahkan setara FISSAR(1,1). Kami namakan model ini sebagai model Ruang Autoregresif pecahan kesepaduan tidak terpisahkan. Fungsi autocovarians yang secaran teori dan fungsi model spectrum yang diperoleh serta keputusan berangka disertakan.

Akhir sekali, oleh kerana data ruang boleh mengambil integer bukan-negatif, maka ada keperluan untuk memperkenalkan model bukan-gaussian bernilai-integer. Dalam kajian ini model autoregresi ruang bernilai-integer tertib pertama bersama taburanut diskrit diperkenalkan. Beberapa sifat model ini (min, varians dan fungsi autokorelasi) dinyatakan. Penganggar Yule-Walker bagi parameter model tersebut turut diperkenalkan dan ketekalan yang tinggi bagi penganggar parameter bagi model tersebut turut dimantapkan.

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I certify that a Thesis Examination Committee has met on 30th November 2011 to conduct the final examination of Ali Reza Ghodsi on his thesis entitled " Properties and Estimation for Fractionally Integrated Spatial Models and Non-Negative Integer-Valued Autoregressive Spatial Models" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



ALI REZA GHODSI

Date: 30th November 2011

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