UNIVERSITI PUTRA MALAYSIA

INTEGRAL SOLUTIONS TO THE EQUATION $X^2 + 2^a \cdot 7^b = y^n$

YOW KAI SIONG

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INTEGRAL SOLUTIONS TO THE EQUATION $x^2 + 2^a \cdot 7^b = y^n$

By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirements for the Degree of Master of Science

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

INTEGRAL SOLUTIONS TO THE EQUATION $x^2 + 2^a \cdot 7^b = y^n$

By

YOW KAI SIONG

October 2011

Chairperson : Siti Hasana Binti Sapar, PhD

Faculty : Institute for Mathematical Research

Diophantine equation is an equation in which solutions to it are from some predetermined classes and it is one of the oldest branches of number theory. There are many types of diophantine equations, for instance linear diophantine equation, exponential diophantine equation and others.

In this research, we will investigate and find the integral solutions to the diophantine equation $x^2 + 2^a \cdot 7^b = y^n$ where a and b are positive integers and n is even. By fixing $n = 2r$, we determine the generators of x and y^r for $1 \leq a \leq 6$ with any values of b. Then, we investigate the necessary conditions to obtain integral solutions of x and y under each value of a if there is any.

The approach is by looking at the possible combinations for the product $2^a \cdot 7^b$ and solving the equations simultaneously. Then, from the results obtained, we substitute the values of a followed by b to get integer values of x and y^r under each category.

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After that, the equations are grouped according to the pattern that emerged and a geometric progression formula is applied to create the general formulae for the generators of solutions to the equation examined.

Besides that, we have to identify the range of i, the number of non-negative integral solutions associated with each b for different values of a. When b is even, we find some special cases of determining the generators of solutions for x and y^r with a certain condition.

From our investigation, we find that there is no integral solution of x and y^r to the diophantine equation $x^2 + 2^a \cdot 7^b = y^n$, when n is even and $a = 1$. It is found that the number of generators to determine the integral solutions to the equation depend on the values of a. Values of y are determined by taking the r-th root of y^r for certain values of r.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN INTEGER KEPADA PERSAMAAN $x^2 + 2^a \cdot 7^b = y^n$

Oleh

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Persamaan diofantus merupakan persamaan di mana penyelesaiannya terdiri daripada kelas-kelas yang telah ditentukan dan ia merupakan salah satu cabang yang paling lama dalam teori nombor. Terdapat banyak jenis persamaan diofantus seperti persamaan diofantus linear, persamaan diofantus eksponen dan sebagainya.

Dalam kajian ini, kita akan mengkaji dan mencari penyelesaian integer bagi persamaan diofantus $x^2 + 2^a \cdot 7^b = y^n$ di mana a, b integer positif dan n adalah genap. Dengan menetapkan $n = 2r$, kita menentukan penjana bagi x dan y^r bagi $1 \leq a \leq 6$ dengan sebarang nilai b. Kemudian, kita akan mengkaji syarat perlu untuk memperolehi penyelesaian integer bagi x dan y bagi setiap nilai a jika wujud.

Pendekatannya adalah dengan mencari kombinasi yang mungkin wujud bagi hasil darab $2^a \cdot 7^b$ dan menyelesaikannya secara serentak. Daripada keputusan yang diperolehi, kita menggantikan nilai a diikuti dengan b untuk mendapatkan nilai

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integer bagi x dan y^r dalam setiap kategori. Selepas itu, kumpulkan persamaan tersebut mengikut pola yang muncul dan menggunakan rumus janjang geometri untuk membentuk rumus umum yang menghasilkan penjana-panjana bagi penyelesaian dalam persamaan yang diperiksa.

Selain itu, kita juga akan mengenalpasti julat bagi i, bilangan penyelesaian integer tak negatif yang disekutukan dengan setiap b bagi nilai a yang berlainan. Sekiranya b adalah genap, kita dapati beberapa kes khusus bagi menentukan penjana-penjana penyelesaian untuk x dan y^r dengan syarat tertentu.

Daripada kajian ini, kita dapati bahawa tiada penyelesaian integer untuk x dan y^r bagi persamaan diofantus $x^2 + 2^a \cdot 7^b = y^n$ dengan n genap dan $a = 1$. Kemudian, bilangan panjana untuk menentukan penyelesaian integer bagi persamaan adalah bergantung kepada nilai a. Nilai bagi y ditentukan dengan mengambil punca kuasa dua r bagi y^r, dengan nilai tertentu bagi r sahaja.

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I certify that a Thesis Examination Committee has met on **28 October 2011** to conduct the final examination of Yow Kai Siong on his thesis entitled " **Integral Solutions to the Equation $x^2 + 2^a \cdot 7^b = y^n$** " in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P. U. (A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

_____

YOW KAI SIONG

Date: 28 October 2011

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