



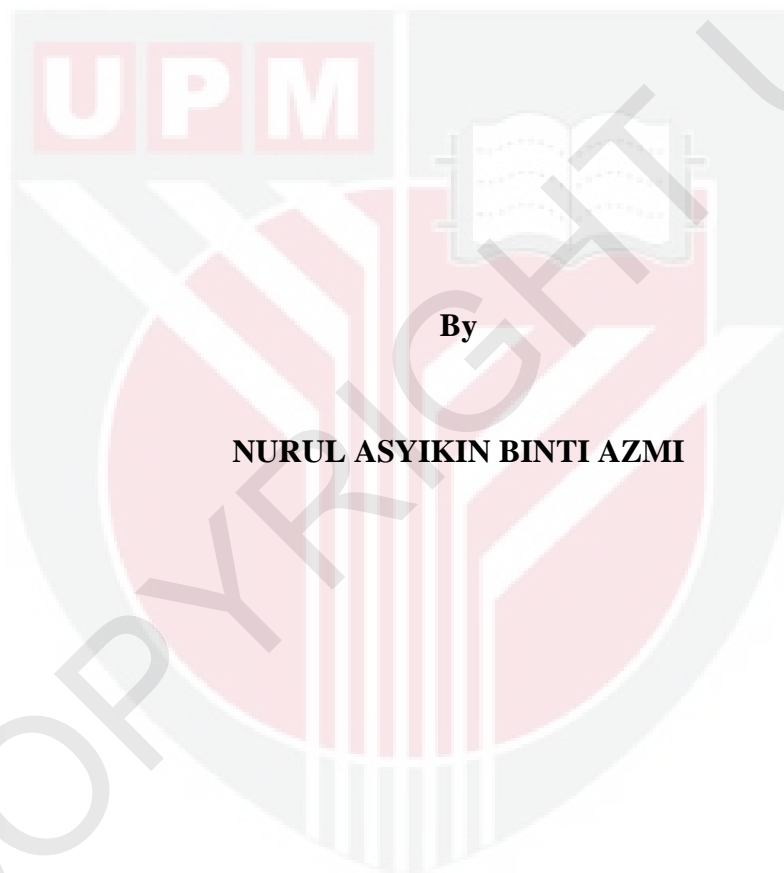
UNIVERSITI PUTRA MALAYSIA

**DIRECT INTEGRATION BLOCK METHOD FOR SOLVING HIGHER
ORDER ORDINARY DIFFERENTIAL EQUATIONS**

NURUL ASYIKIN BINTI AZMI

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ORDER ORDINARY DIFFERENTIAL EQUATIONS**



By

NURUL ASYIKIN BINTI AZMI

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirements for the Degree of Master of Science**

March 2010

Abstract of thesis presented to the Senate of University Putra Malaysia in fulfilment
of the requirement for the degree of Master of Science

**DIRECT INTEGRATION BLOCK METHOD FOR SOLVING HIGHER
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NURUL ASYIKIN BT AZMI

March 2010

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Faculty: Institute for Mathematical Research (INSPEM)

In this thesis, the implicit block methods presented as in the simple form of Adams Moulton method are developed for solving higher order systems of Ordinary Differential Equations (ODEs). This method will solve the Initial Value Problems (IVPs) of second and third order ODEs using variable step size (VS) and variable step or variable order (VSVO) techniques. The proposed block methods will approximate the solutions at two distinct points on the x -axis simultaneously in a block.

A system of higher order can also be reduced to a system of first order equations and then solved using any numerical method. This approach is very well established but it obviously will enlarge the dimension of the equations. However, the developed block method will solve the system of higher order ODEs directly without reducing it to first order.

The formulae of the block method involve Lagrange's interpolation formulae in order to compute the integration coefficients. All the coefficients will be stored in the code and there will be no computation involved for the integration coefficients. The codes were executed on UNIX operating system and the algorithms were written in C language.

The numerical results showed that the performance of the developed methods gave better results in terms of total number of steps, maximum error, and total function calls compared to the existing block methods.

In conclusion, the proposed implicit block methods in this thesis are appropriate for solving the second and third order ODEs.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

**KAMIRAN TERUS KAEDAH BLOK BAGI MENYELESAIKAN
PERSAMAAN PEMBEZAAN BIASA PERINGKAT TINGGI**

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Dalam tesis ini, kaedah blok tersirat yang hadir berasaskan kaedah Adams Moulton telah dibangunkan bagi menyelesaikan sistem Persamaan Pembezaan Biasa (PPB) peringkat tinggi. Kaedah ini akan menyelesaikan Masalah Nilai Awal (MNA) bagi PPB peringkat dua dan tiga dengan menggunakan panjang langkah berubah dan juga peringkat atau langkah berubah. Kaedah blok yang dicadangkan ini akan menganggar penyelesaian pada dua titik nyata pada paksi- x secara serentak dalam blok.

Sistem peringkat tinggi juga boleh diturunkan kepada sistem persamaan peringkat pertama dan kemudiannya diselesaikan menggunakan sebarang kaedah berangka.

Pendekatan ini sangat mapan tetapi ia nyata akan membesarkan lagi dimensi persamaan tersebut. Walau bagaimanapun, kaedah blok yang dibangunkan ini akan menyelesaikan sistem PPB peringkat tinggi secara terus tanpa perlu menurunkannya kepada peringkat pertama.

Formula bagi membangunkan kaedah blok melibatkan formula penyisipan Lagrange yang digunakan untuk mengira nilai bagi pekali kamiran. Seterusnya, kesemua pekali akan disimpan di dalam kod dan tidak ada lagi pengiraan yang terlibat untuk mendapatkan pekali kamiran. Kod-kod baru ini dilaksanakan menggunakan sistem operasi UNIX dan algoritma ditulis menggunakan bahasa C.

Keputusan berangka menunjukkan pelaksanaan kaedah blok yang baru dibangunkan ini memberi keputusan yang baik dari segi jumlah bilangan langkah, ralat maksima dan jumlah fungsi panggilan jika dibandingkan dengan kaedah blok yang dibangunkan sebelum ini.

Secara kesimpulannya, kaedah blok tersirat yang dibangunkan dalam tesis ini adalah bersesuaian untuk menyelesaikan PPB peringkat dua dan tiga.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee were as follows:

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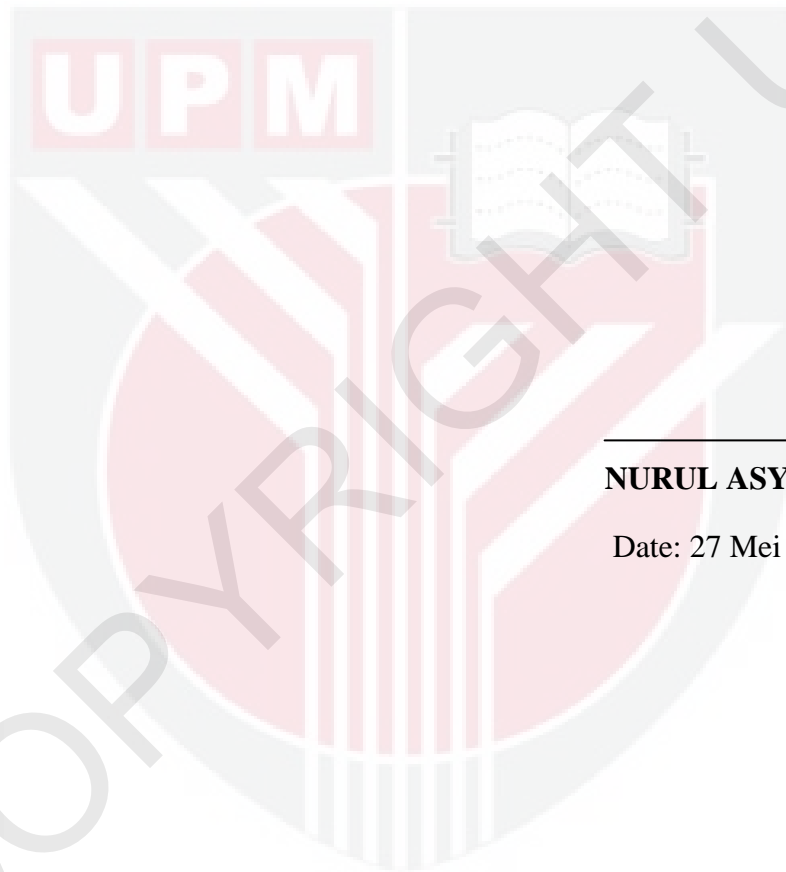
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Date: 15 July 2010

DECLARATION

I declare that the thesis is my original work except quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



NURUL ASYIKIN AZMI

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