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IMPROVED MULTIVARIATE CONTROL CHARTS WITH ROBUST METHODS

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By

ASHKAN SHABBAK

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATIONS

- To the best two people in the word that I owe my life to them, my parents.
- To my beloved wife that I owe my happiness in life
- To my sisters for their kinds

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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Chairman: Habshah Midi, PhD

Faculty: Institute for Mathematical Research

In real life, usually more than one important quality characteristic of a products and services are considered. In this situation, a multivariate control chart is appropriate to simultaneously monitor more than one quality characteristics. Three of the most popular multivariate statistical quality control charts are Hotelling's T^2, the MCUSUM (Multivariate Cumulative Sum) and MEWMA (Multivariate Exponentially Weighted Moving Average). Unfortunately, it is now evident that all these classical charts are easily affected by multiple outliers which may appear as a scatter outliers or sustained shift in the drawn dataset. To remedy this drawback, only recently robust methods such as the Minimum Volume Ellipsoid (MVE) or the

Minimum Covariance Determinant (MCD), be applied to multivariate control process applications. In this thesis, the performance of robust T^2 control charts, based on the MVE and MCD, for individual observations is extensively studied when there is a sustained shift in the dataset. The results of the study reveal that the robust T^2 control charts is more efficient in detecting the sustained shift than the classical T^2 charts.

The existing study on robust control charts are mainly focussed on individual observation. However, in real situation, we often encountered more than one observation in each variable of each subgroup. Hence, in this situation, we propose a robust T^2 control charts based on the MVE and MCD estimators, in the case of scatter outliers. Our findings show that both the proposed robust T^2 control charts outperform the classical chart, in the situation of more than one observation in each variable of each subgroup in the presence of scatter outliers.

We also propose applying the robust median and MAD (Med-Mad) cut-off points as control limits to the MVE-based and the MCD –based control charts. The performance of this new proposed empirical control limits are investigated when a sustained shift exists in the mean vector. The real examples and simulation studies signify that the proposed control limits are better than the existing control limits for detecting the sustained shift in a dataset.

The signal probability is usually used to evaluate the performance of control charts, in the phase I. Due to misleading results that occur by using this criterion when the proposed robust control limits (Med-Mad) and the existing control limits are applied for scatter outliers' situation, we propose another criterion for assessing different control charts. This new criterion is based on the number of true detected outliers with respect to the correct position of the out of control observations in the Historical Data Set (HDS). In this thesis, a new robust control chart based on the Diagnostic Robust Generalized Potential (DRGP) procedure is also proposed to rectify the swamping effect that happens due to applying the proposed control limit (Med-Mad) and the existing control limits for scatter outliers' situation. The results of the study indicate that applying this new approach can improve the performance of detection of real scatter outliers.

This thesis also addresses the problem of detecting small or moderate changes in the HDS, since even small or moderate changes in the HDS can result in creating serious problems in an analysis and inferences about future status of a process. The MCUSUM and MEWMA charts are effective alternatives for T^2 charts when detection of small or moderate shifts in a process is of interest. In this research, we attempt to increase the probability of detecting small or moderate changes in the HDS by developing a new robust version of the MCUSUM and MEWMA control charts by integrating the MVE or the MCD estimators in the formulation of the robust chart, in the step shifts and also scatter outliers' situations.

Even though the performance of the T^2 control charts based on the MVE and the MCD is better than the classical T^2 control charts, the findings of this research show that their performances decrease as the number of quality characteristics (variables) increases for a fixed number of observations. To remedy this problem, an attempt is made to develop a new robust multivariate control chart by incorporating the MVE, the MCD and the Principal Components Analysis (PCA) approach in the development of the new robust T^2 control charts. The performance of the proposed robust T^2control charts based on the PCA, MVE and MCD estimators has shown improvement compared to the robust T^2control charts based only on the MVE and MCD estimators.

All procedures, in this thesis, are examined by using numerical examples and Monte Carlo simulation studies and all programs are written in S-PLUS language. The comparative studies among the classical and proposed robust methods reveal that all the proposed robust methods successfully perform better than the classical and existing methods.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENAMBAHBAIKAN CARTA KAWALAN MULTIVARIAT DENGAN KAEDAH TEGUH

Oleh

ASHKAN SHABBAK

November 2011

Pengerusi: Habshah Midi, PhD

Fakulti: Institut Penyelidikan Matematik

Dalam kehidupan sebenar, biasanya terdapat lebih daripada satu ciri kualiti yang penting bagi produk dan perkhidmatan yang dipertimbangkan. Dalam situasi begini, kawalan kualiti multivariat sesuai digunakan bagi mengawal lebih daripada satu ciri kualiti, serentak. Tiga carta kawalan berstatistik multivariat yang paling popular ialah Hotelling's T^2, *MCUSUM* (Hasiltambah Kumulatif Multivariat) dan *MEWMA* (Purata Bergerak Multivariat Berpemperat Ekponen). Malangnya, telah terbukti bahawa kesemua carta kawalan klasik ini mudah dipengaruhi oleh titik terpencil berganda yang mungkin timbul sebagai titik terpencil berselerak atau anjakan tetap pada data yang di pungut. Bagi mengatasi kelemahan ini, kaedah teguh seperti

Minimum Volume Ellipsoid (MVE) atau *the Minimum Covariance Determinant (MCD),* baru saja diketengah untuk diapplikasikan keatas penggunaan kawalan proses multivariat. Dalam tesis ini, prestasi carta kawalan teguh T^2 berasaskan *MVE* dan *MCD*, bagi cerapan individu dikaji secara mendalam apabila terdapat anjakan tetap pada set data. Hasil dapatan kajian menunjukkan bahawa carta kawalan teguh T^2 adalah lebih efisien daripada carta klasik T^2 , bagi mengenalpasti anjakan tetap.

Kajian pada masa sekarang yang berkaitan kawalan kualiti teguh banyak tertumpu pada cerapan individu. Walau bagaimana pun bagi situasi sebenar, kita biasa berdepan dengan lebih daripada satu cerapan dalam setiap pembolehubah bagi setiap sub-kumpulan. Oleh itu, dalam situasi begini, kami mencadangkan carta kawalan teguh T^2 berasaskan penganggar *MVE* dan *MCD* apabila titik terpencil berselerak wujud. Dapatan kami menunjukkan bahawa kedua-dua carta kawalan teguh T^2 yang dicadangkan, prestasinya lebih baik daripada carta kawalan klasik.

Kami juga mencadang mengapplikasikan titik genting teguh median dan *MAD (Med-Mad)* sebagai had kawalan kepada carta kawalan yang berasaskan *MVE-based* dan *MCD –based.* Prestasi had kawalan empirik yang dicadangkan di selidik apabila anjakan tetap wujud dalam vektor min. Contoh sebenar dan kajian simulasi menunjukkan bahawa had kawalan yang dicadangkan adalah lebih baik dari had kawalan sedia ada bagi mengenalpasti anjakan tetap dalam suatu set data.

Kebarangkalian amaran biasanya digunakan untuk menilai prestasi carta kawalan dalam Fasa I. Kami mencadangkan kriteria lain bagi menilai prestasi carta kawalan yang berlainan disebabkan wujudnya kekeliruan keputusan apabila kriteria lama digunakan dengan had kawalan yang dicadangkan (*Med-Mad*) ke atas situasi titik terpencil berselerak. Kriteria baharu ini berasaskan bilangan titik terpencil sebenar yang dikenalpasti mengikut kedudukan mereka yang betul dalam *HDS*. Dalam kajian ini, satu carta kawalan teguh baharu berasaskan prosedur *Diagnostic Robust Generalized Potential (DRGP)* juga di cadangkan untuk mengatasi masalah kesan *swamping* yang berlaku disebabkan mengapplikasikan had kawalan sedia ada dan cadangan had kawalan (*Med-Mad)* bagi situasi titik terpencil berselerak. Keputusan kajian menunjukkan dengan mengapplikasikan pendekatan baharu ini mampu meningkatkan prestasi pengenalpastian dengan betul titik terpencil berselerak.

Oleh kerana perubahan kecil atau sederhana dalam *HDS* boleh menghasilkan masalah serious dalam analisis dan pentakbiran berkaitan status proses akan datang, kajian ini juga menyebut masalah pegenalpastian perubahan kecil atau sederhana dalam *HDS.* Carta *MCUSUM* dan carta *MEWMA* merupakan alternatif berkesan bagi carta T^2 apabila pengenalpastian anjakan kecil dan sederhana diperlukan. Dalam kajian ini, kami cuba untuk mempertingkatkan kebarangkalian mengesan perubahan kecil dan sederhana dalam *HDS* dengan membangunkan carta kawalan teguh versi baru *MCUSUM* dan *MEWMA* dengan menggabungkan penganggar *MVE* dan *MCD* bagi memformulasikan carta teguh dalam situasi anjakan tetap dan juga titik terpencil berselerak.

Walau pun prestasi carta kawalan teguh T^2 berasaskan *MVE* dan *MCD* lebih baik daripada carta kawalan klasik T^2 , dapatan kajian ini menunjukkan bahawa prestasi mereka menurun apabila bilangan ciri kualiti (pembolehubah) meningkat, bagi bilangan cerapan yang tetap. Bagi mengatasi masalah ini, cubaan telah dilakukan untuk membangunkan carta kawalan multivariat yang baharu dengan menggabungkan pendekatan *MVE*, *MCD* dan *Principal Components Analysis (PCA)* dalam pembinaan carta teguh baharu T^2. Cadangan carta kawalan teguh T^2 berasaskan *PCA, MVE* dan *MCD* telah menunjukkan peningkatan prestasi berbanding dengan carta kawalan teguh T^2 yang hanya berasaskan penganggar *MVE* dan *MCD* .

Semua prosedur dalam tesis ini dikaji dengan menggunakan contoh berangka dan kajian simulasi Monte Carlo dan semua pengaturcaraan ditulis menggunakan bahasa S-PLUS. Perbandingan antara kaedah klasik dan kaedah teguh yang dicadangkan, menunjukkan bahawa semua kaedah teguh yang dicadangkan lebih baik daripada kaedah klasik dan kaedah sedia ada.

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I certify that a Thesis Examination Committee has met on 9/11 /2011 to conduct the final examination of Ashkan Shabbak on his thesis entitled " Improved Multivariate Control Charts With Robust Methods" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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This thesis was submitted to the Senate of Univirsiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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Ashkan Shabbak

Date:9 November 2011

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