



UNIVERSITI PUTRA MALAYSIA

**PRECONDITIONING SUBSPACE QUASI-NEWTON METHOD
FOR LARGE SCALE UNCONSTRAINED OPTIMIZATION**

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By

SIM HONG SENG

Thesis Submitted to the School of Graduate Studies, Universiti Putra
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Chairman: Leong Wah June , PhD

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Subspace quasi-Newton (SQN) method has been widely used in large scale unconstrained optimization problem. Its popularity is due to the fact that this method can construct subproblems in low dimensions so that the storage requirement as well as the computation cost can be reduced. Besides of this, it also can offer a possible way to handle large scale optimization problems and yet it has vast applications in almost every branch of science and technology such as tomography, signal and image deionizing with Basis Pursuit, pattern recognition with Support Vector Machine, and many others. This method can be implemented extremely fast when the objective function is a combination of mappings with computationally cheap non-linear functions for example, quadratics functions. However, the main drawback of the SQN method is that it can be very slow on certain type of nonlinear problem such as ill-conditioned problems. Thus, the focus of this thesis is to overcome this deficiency via preconditioning on the SQN method.

In practise, preconditioners can be often adopted to speed up the convergence of the quasi-Newton methods. Hence, we propose a preconditioned SQN method which is generally more effective than the SQN method. For this purpose, we construct a

preconditioner which is computationally cheap and is a good approximation to the actual Hessian since the evaluation of actual Hessian is considered as impractical and costly. In order to do this, we propose to use a diagonal updating matrix that has been derived based on the weak quasi-Newton relation instead of using the identity matrix to approximate the initial inverse Hessian.

Numerical experiments are performed on quadratics test problems to compare the efficiency and performance of the preconditioned SQN method with the standard SQN method. Our computational results show that the proposed preconditioned SQN method performs better than SQN method that without preconditioning.

In addition, the convergence of this method is also presented. Finally, some possible future extensions are to be given to conclude this thesis.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**KAEDAH SUBRUANG KUASI-NEWTON DENGAN
PRAPENSYARAT BAGI PENGOPTIMUMAN TAK
BERKEKANGAN BERSKALA BESAR**

Oleh

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Kaedah subruang kuasi-Newton (SQN) telah diaplikasikan secara luas dalam penyelesaian masalah pengoptimuman tak berkekangan berskala besar. Kaedah ini begitu popular disebabkan keupayaan kaedah ini boleh menerbit sub-masalah berdimensi kecil untuk mengurangkan keperluan kapasiti penyimpanan dan kos pengiraan. Selain itu, kaedah ini juga boleh digunakan untuk menyelesaikan masalah pengoptimuman berskala besar dan mempunyai aplikasi yang luas dalam bidang sains dan teknologi seperti tomografi, penyahionan isyarat dan imej dengan asas mengejar, pola pembezaan dengan mesin vektor sokongan dan sebagainya. Kaedah ini boleh dilaksanakan dengan pantas apabila fungsi objektif adalah gabungan pemetaan dengan fungsi tak linear berkost rendah seperti fungsi

kuadratik. Namun, kaedah ini menjadi perlahan terhadap masalah tak linear tertentu seperti masalah syarat tak sempurna merupakan kekurangan utama. Fokus utama tesis ini adalah mengatasi kekurangan tersebut dengan mencari prapensyarat pepenjuru untuk kaedah SQN.

Secara pratikal, prasyarat sering diadopsi untuk mempercepatkan proses penumpuan kaedah kuasi-Newton. Oleh yang demikian, kami mencadangkan prapensyaran kaedah SQN yang lebih berkesan daripada SQN piawai. Untuk tujuan ini, kami membina prapensyarat yang kos pengiraannya rendah dan juga merupakan penganggaran yang baik bagi Hessian sebenar kerana penghitungan Hessian sebenar dikatakan tak praktikal dan berkos tinggi. Daripada penggunaan matriks identiti sebagai penganggaran bagi songsangan Hessian, kami mencadangkan penggunaan matriks pepenjuru terkemuka yang diterbit berdasarkan kepada hubungan kuasi-Newton lemah

Ujikaji berangka telah dijalankan ke atas masalah ujian kuadratik untuk membandingkan kecekapan kaedah SQN bersyarat dengan kaedah SQN piawai. Keputusan pengiraan menunjukkan kaedah SQN berprapensyarat adalah lebih baik daripada kaedah SQN tanpa sebarang prapensyarat.

Tambahan pula, penumpuan kaedah ini juga ditunjukkan. Cadangan untuk penyelidikan lanjutan juga diberi bagi mengakhiri tesis ini.

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I certify that a Thesis Examination Committee has met on **14 November 2011** to conduct the final examination of **Sim Hong Seng** on his thesis entitled "**Preconditioning Subspace Quasi-Newton Method for Large Scale Unconstrained Optimization** " in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the **Master of Science**.

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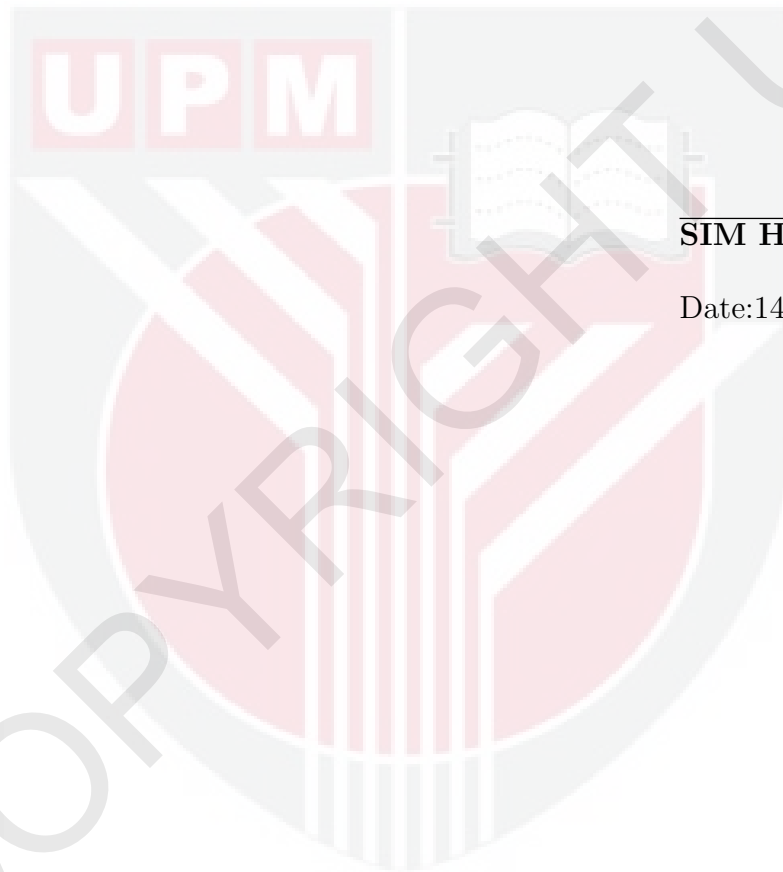
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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



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Date:14 November 2011

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