Quadratic Programming for Two Dimensional Case

Ismail Bin Mohd

Faculty of Science and Technology
Kolej Universiti Sains dan Teknologi Malaysia (KUSTEM)
Mengabang Telipot
21030 Kuala Terengganu
Malaysia

E-mail of Corresponding Author: ismailmd@kustem.edu.my

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Introduction

The theory of quadratic programming problem is concerned with problems of constrained minimization where the constraint functions are linear and the objective function is positive definite quadratic function. As we know, if the optimal solution of linear programming problem exists, then it is either an extreme point of the feasible region or a convex combination of such extreme points. However, the optimal solution of quadratic programming problem can be interior point or a boundary point of the feasible region. In this work, we would like to propose an efficient method for seeking the optimal solution of the minimization quadratic programming problem which is located on the boundary of the feasible region can be obtained through searching the equality constraints which are violated by the unconstrained minimization of the objective function of the problem.

Materials and Methods

In this work we minimize an objective function given by a positive definite quadratic function $f(x) = (1/2)x^TAx + b^Tx$ subject to the constraint system of the form $Bx \leq c$ and $x_i \geq 0$ $(i=1,\ldots,n)$ where $A = (a_{ij})_{n\times n}$ is a simetric matrix, $b = (b_1,\ldots,b_n)^T$, $B = (b_{ij})_{m\times n}$ and $c = (c_1,\ldots,c_m)^T$. We have used (i) the contours (surfaces along which function f is constant) of a function, (ii) the equality constraints which are violated by the unconstrained minimization of the objective function of the problem and (iii) closed path idea, to locate or to show the way for obtaining or locating the optimal solution of quadratic programming problem.

Results and Discussion

The results and discussion of this work can be obtained from the following theorems.

Theorem 1 : If (1) $x^{(0)}$ is the infeasible unconstrained minimum of f(x), (2) $S = \{j : B_j^Tx > c_j, j \in \{1,\ldots,m\}\} \neq \emptyset$ (empty set), then the minimizer x^* is located on the equality constraints where their indexes are in S.♦

Theorem 2 : Suppose that the hypotheses of Theorem 1 are valid. If x_j^* for one $j \in S$ is feasible, then all other x_k^* ($k \in S$ and $k \neq j$) are infeasible constrained minimum f(x) subject to the k-th equality constraint with $k \neq j$.♦

Theorem 3 : Suppose that the hypotheses ofTheorem 1 are valid. If one of the x_j^* constrained minimum of f(x) subject to the j-th equality constraint for $j \in S$, is feasible, then x_j^* is optimal solution of the quadratic programming problem.♦

Conclusions

In this work, we have seen the capability of the methods for solving the quadratic programming problem without using any additional information such as slack, surplus and artificial variables.

Benefits from the study

Benefits from the study indicated that both labour costs and time consumption can be reduced, and problems can be solved easily.

Literature cited in the text

None.

Project Publications in Refereed Journals

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Graduate Research

None.

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