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NUMERICAL SOLUTION OF INFINITE AND FINITE BOUNDARY INTEGRAL EQUATIONS BY POLYNOMIALS

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By

MOHAMMAD HASAN BIN ABDUL SATHAR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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DEDICATION

This thesis is dedicated to

all my family members

especially

my beloved father Abdul Sathar

my dear mother Khadijah

my lovely wife Aznira

including my lectures

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

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Chair : Zainnidin Eshkuvatov, PhD

Faculty : Institute for Mathematical Research

The Fredholm Integral equation of the form

$$g(s) = f(s) + \lambda \int_a^b k(s,t) g(t) dt, \qquad (1)$$

where $k(s,t)$ is a regular kernel on $D = \{(s,t) : a \le s, t \le b\}$ and $f(s)$ is a continuous function defined on $[a,b]$ and the unknown function $g(s)$ is to be determined, are encountered in many problems of ordinary differential equations and mathematical physics. Unfortunately in many cases the equation (1) cannot be solved in the closed form. Therefore, numerical method is applied to solve the integral equation (1).

To approximate the integral equation (IE) (1), we usually choose a finite dimensional family of functions that is believed to contain a function $g_n(s)$ close to the true solution $g(s)$. The desired approximate solution $g_n(s)$ is selected by forcing it to satisfy equation (1). There are various means in which $g_n(s)$ can be said to satisfy equation (1) approximately, and this leads to different type of methods. The most popular and powerful tools are Collocation and Galerkin methods.

If the limit of integration in (1) is infinite and corresponding functions belong to certain class of functions then equation (1) is called infinite boundary integral equations (IBIEs).

Many problems of electromagnetic, scattering problems and boundary integral equations lead to IBIEs of the second kind,

$$g(s) = f(s) + \lambda \int_{0}^{\infty} k(s,t) g(t) dt, \tag{2}$$

$$g(s) = f(s) + \lambda \int_{-\infty}^{\infty} k(s,t) g(t) dt. \tag{3}$$

In this thesis, we have developed Galerkin method with Laguerre polynomials for Eq. (2) on the interval [0,∞) and Galekin method with Hermite polynomials for Eq. (3) on the interval (-∞,∞) to get the approximate solution. We have also solved equation (1) numerically using Collocation method based on Legendre polynomials on the interval [-1,1]. Existence of the solution for Eq. (2) and (3) and exactness of the approximate method for Eq. (1) are given. These developments gave good fit (even for small *n*) to the exact solution $g(s)$ for finite and IBIEs (1)-(3). Maple software 11 is used to obtain the approximate solution for Eq. (1)-(3) and the results show good convergence for finite and infinite interval for small *n*.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

PENYELESAIAN BERANGKA BAGI PERSAMAAN KAMIRAN TIDAK TERHAD DAN TERHAD DENGAN MENGGUNAKAN POLINOMIAL

Oleh

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Persamaan Kamiran Fredholm dalam bentuk

$$g(s) = f(s) + \lambda \int_a^b k(s,t) g(t) dt, \tag{1}$$

di mana $k(s,t)$ adalah kernel yang lazim pada $D = \{(s,t) : a \le s,t \le b\}$ dan $f(s)$ adalah fungsi selanjar dalam selang $[a,b]$ dan fungsi yang tidak diketahui $g(s)$ perlu ditentukan, ditemui dalam banyak masalah persamaan pembezaan biasa dan fizik bermatematik. Malangnya dalam beberapa kes persamaan (1) tidak boleh diselesaikan dalam bentuk tertutup. Oleh yang demikian kaedah berangka digunakan untuk menyelesaikan persamaan kamiran (1).

Bagi penghampiran persamaan kamiran (PK) (1), kami memilih fungsi famili dimensi yang terhingga yang dipercayai mengandungi fungsi $g_n(s)$ yang hampir dengan penyelesain sebenar $g(s)$. Penyelesaian hampiran yang dikehendaki $g_n(s)$ dipilih bagi memenuhi persaman (1). Terdapat pelbagai definisi di mana $g_n(s)$ boleh dikatakan dapat memenuhi hampiran persamaan (1), dan ini menjurus kepada

pelbagai kaedah yang berlainan. Kaedah yang paling popular dan kuat adalah kaedah Collocation and kaedah Galerkin.

Jika had pengamiran dalam (1) adalah tidak terhingga dan menggunakan fungsi yang sesuai di dalam kelas fungsi tertentu maka persamaan (1) dikenali sebagai persamaan kamiran tidak terhad (PKTT).

Kebanyakan masalah dalam elektromagnetik, masalah penyebaran dan persamaan kamiran termasuk dalam PKTT jenis kedua.

$$g(s) = f(s) + \lambda \int_0^{\infty} k(s,t) g(t) dt, \tag{2}$$

$$g(s) = f(s) + \lambda \int_{-\infty}^{\infty} k(s,t) g(t) dt. \tag{3}$$

Dalam penyelidikan ini, kami telah menbangunkan kaedah Galerkin dengan polinomial Laguerre untuk persamaan (2) dalam selang (0,∞) dan kaedah Galerkin dengan polinomial Hermite untuk persamaan (3) dalam selang (-∞,∞) bagi mendapatkan penyelesaian hampiran. Kami juga telah menyelesaikan persamaan (1) secara berangka dengan menggunakan kaedah Collocation berasaskan kepada polinomial Lagendre dalam selang [-1,1]. Kewujudan penyelesaian bagi persamaan (2) dan (3) dan ketepatan kaedah hampiran bagi persamaan (1) dapat ditunjukkan. Pembagunan ini memberi padanan yang sesuai (walaupun untuk nilai n yang kecil) bagi peyelesaian tepat $g(s)$ untuk PKTT dan terhingga (1)-(3). Perisian Maple 11 digunakan untuk mendapatkan penyelesaian hampiran untuk persamaan (1)-(3) dan hasilnya menunjukkan penumpuan yang baik bagi interval terhingga dan tak terhingga untuk nilai *n* yang kecil.

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I certify that a Thesis Examination committee has met on **29 November 2010** to conduct the final examination of **Mohammad Hasan Bin Abdul Sathar** on his thesis entitled **“Numerical Solution of Infinite Boundary Integral Equations by Polynomials”** in accordance with Universities and University Colleges Act 1971 and the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1988. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institutions.

MOHAMMAD HASAN BIN ABDUL SATHAR

Date: 29 November 2010

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