ROBUST DIAGNOSTICS AND ESTIMATION IN HETEROSCEDASTIC REGRESSION MODEL IN THE PRESENCE OF OUTLIERS

MD. SOHEL RANA

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ROBUST DIAGNOSTICS AND ESTIMATION IN HETEROSCEDASTIC REGRESSION MODEL IN THE PRESENCE OF OUTLIERS

By

MD. SOHEL RANA

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

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Dedication

- To the best memory of my parents who passed away and wanted to see my dreams come true
- To my family, having unconditional love for me
- To my one beloved teacher who uplifted my life
Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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Chairman: Habshah Midi, PhD

Faculty: Institute for Mathematical Research

The violation of the assumption of homoscedasticity in OLS method, usually called heteroscedasticity, gravely misleads the inferential statistics. The current study has considered the situation when outliers occur in heteroscedastic data. Hence, the main focus of this research is to take remedial measures on the violation of the assumption of homoscedasticity in the presence of outliers. This thesis also concerns on the normality assumption of the errors of regression model in the presence of outliers. It is now evident that outliers have great impact on the existing normality tests, heteroscedasticity tests, and the estimators for heteroscedastic model. We propose the Robust Rescaled Moment (RRM) test for testing the normality of the regression residuals when there is an evidence of outlier(s). The results of the study signify that the RRM test offers substantial improvements over other existing tests in the presence of outliers. For the detection of heteroscedasticity in the presence of outliers, a modified version of
the classical Goldfeld-Quandt (MGQ) test is proposed which is most powerful than the classical tests of heteroscedasticity. Most statistics practitioners assume that the forms of the heteroscedastic error structures are known which may lead to inefficient estimates if it is not correctly specified. In this respect, a Leverage Based Near-Neighbor (LBNN) method is proposed, where prior information on the structure of the heteroscedastic error is not required. The findings indicate that the LBNN is very efficient for correcting the problem of heteroscedastic errors with unknown structure. We also examine the effect of outliers on the existing remedial measures of heteroscedasticity. Hence, in this thesis, a one step M-type of Robust Weighted Least Squares Method (RWLS) and the Two-Step Robust Weighted Least Squares (TSRWLS) are developed. Finally, the new robust wild bootstrap techniques which are resistant to outliers are proposed. The proposed techniques are based on the weighted residuals which incorporated the MM estimator, robust location, robust scale and the bootstrap sampling schemes of Wu (1986) and Liu (1988). All procedures, in this thesis, are examined by using real data and Monte Carlo simulation studies. The comparative studies among the classical and proposed robust methods reveal that all the proposed robust methods outperform the classical methods.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

DIAGNOSTIK TEGUH DAN PENGANGGARAN MODEL REGRESI BER HETEROSKEDASTIK DENGAN KEHADIRAN TITIK TERPENCIL

Oleh

MD. SOHEL RANA

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Andaian homoskedastik yang tidak dipatuhi oleh kaedah OLS, biasanya dinamakan heteroskedastik, memburukkan pentakbiran statistik. Kajian terkini mempertimbangkan situasi titik terpencil berada dalam data berheteroskedastik. Oleh itu, fokus utama dalam kajian ini adalah untuk mengambil langkah yang sewajarnya bagi menangani andaian homoskedastik yang tidak dipenuhi dengan kehadiran titik terpencil. Tesis ini juga mempertimbangkan andaian normal ralat bagi model regresi dengan kehadiran titik terpencil. Bukti terkini menunjukkan bahawa titik terpencil memberikan kesan buruk keatas ujian kenormalan, ujian heteroskedastik dan penganggar bagi model heteroskedastik. Kami mencadangkan ujian Teguh Momen Berskala (RRM) bagi menguji kenormalan reja regresi apabila terdapat bukti kehadiran titik terpencil. Keputusan kajian menunjukkan bahawa ujian RRM lebih baik daripada kaedah sedia ada dalam kehadiran titik terpencil. Bagi pengenalpastian heteroskedastik dengan kehadiran titik terpencil, pengubahsuaian ujian Goldfeld klasik Quandt (MGQ) dicadangkan yang
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I certify that a thesis Examination Committee has met on 22-10-2010 to conduct the final examination of Md. Sohel Rana on his thesis entitled “Robust Diagnostics and Estimation in Heteroscedastic Regression Model in the Presence of Outliers” in accordance with Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy of Statistics.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

______________________________
MD. SOHEL RANA

Date: 22 October 2010
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATIONS</td>
<td>ii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>vi</td>
</tr>
<tr>
<td>APPROVAL</td>
<td>ix</td>
</tr>
<tr>
<td>DECLARATION</td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xvi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xix</td>
</tr>
<tr>
<td>LIST OF APPENDICES</td>
<td>xxiii</td>
</tr>
<tr>
<td>LIST OF ABBREVIATIONS</td>
<td>xxiv</td>
</tr>
</tbody>
</table>

## CHAPTER

### 1 INTRODUCTION

1.1 Background of the Study  
1.2 Importance and Motivation of the Study  
1.3 Objectives of the Study  
1.4 Plan of the Study  

### 2 REVIEW OF LITERATURE

2.1 Introduction  
2.2 Outliers in Linear Regression  
2.2.1 Identification of Outliers  
2.3 Least Squares Regression and the Violation of its Homogeneity Assumption  
2.4 Overview of the Detection of Heteroscedasticity and Test of Normality  
2.5 Overview of the Remedial Measures of Heteroscedasticity  
2.5.1 Weighted Least Squares (WLS) Regression  
2.5.2 Variance Stabilizing Transformation  
2.5.3 Heteroscedasticity Consistence Covariance Matrix (HCCM)  
2.5.4 Near-Neighbor (NN) Estimators  
2.5.5 Wild Bootstrap Estimators  
2.6 Robust Regression Estimators  
2.7 Review of Robust Test of Normality and Robust Heteroscedastic Regression (Test and Remedial Measures)  
2.8 Conclusion  

xii
3
ROBUST TEST FOR NORMALITY IN REGRESSION
3.1 Introduction 49
3.2 The Robust Rescaled Moment Test 52
3.3 Numerical Results 58
  3.3.1 Belgian Road Accident data 58
  3.3.2 Shelf-Stocking Data 60
  3.3.3 Housing Expenditures Data: A Heteroscedastic Data Set 62
3.4 Power Simulations 65
3.5 Conclusion 73

4
DETECTION OF HETEROSEDASTICITY IN THE PRESENCE OF OUTLIERS
4.1 Introduction 74
4.2 Methods of Detecting Heteroscedasticity 76
  4.2.1 Graphical Methods 77
  4.2.2 Analytical Methods 79
4.3 Weakness of Classical Heteroscedasticity Tests 86
4.4 Robust Tests of Heteroscedasticity 86
  4.4.1 Robust Graphical Test: The Deletion Residuals-Deletion Fits (DR-DF) Plot 86
  4.4.2 Modified Goldfeld-Quandt Test: A Proposed Robust Detection Technique of Heteroscedasticity 87
  4.4.3 Why the Modification of Breusch-Pagan Test and White Test are Not Helpful? 89
4.5 Examples 90
  4.5.1 Housing Expenditures Data 90
  4.5.2 Consumption Expenditure Data 92
  4.5.3 Restaurant Food Sales Data 94
4.6 Simulation Results 96
4.7 Conclusion 99

5
LEVERAGE BASED NEAR-NEIGHBORS METHOD: A REMEDIAL MEASURE OF HETEROSEDASTIC MULTIPLE REGRESSION MODEL
5.1 Introduction 100
5.2 Remedial Measures of Heteroscedasticity 104
  5.2.1 Error Variances $\sigma_i^2$ is Known 105
  5.2.2 Error Variances $\sigma_i^2$ is Unknown 106
5.3 The Leverage Based Near-Neighbor (LBNN) Method 110
5.4 Examples 114
  5.4.1 Restaurant Food Sales Data 114
5.4.2 Education Expenditure Data 117
5.5 Monte Carlo Simulation 121
5.6 Conclusion 125

6 ROBUST WEIGHTED LEAST SQUARES FOR SIMPLE LINEAR HETEROSEDASTIC MODEL IN THE PRESENCE OF OUTLIERS
6.1 Introduction 127
6.2 Robust Weighted Least Squares (RWLS) 128
6.3 Numerical Examples 131
   6.3.1 Restaurant Food Sales Data 131
   6.3.2 Simulated High Leverage Data with Heterogeneous Variances 138
6.4 Monte Carlo Simulation Results 142
6.5 Conclusion 151

7 TWO-STEP ROBUST ESTIMATOR: A ROBUST REMEDIAL MEASURE FOR MULTIPLE REGRESSION MODEL WITH HETEROSEDASTIC ERRORS IN THE PRESENCE OF OUTLIERS
7.1 Introduction 152
7.2 Two-Step Robust Weighted Least Squares (TSRWLS) 154
7.3 Numerical Evaluation 157
7.4 Simulations 163
7.5 Further Evaluation of TSRWS Estimator Based on Real Data
   7.5.1 Bootstrap and Monte Carlo Simulation 188
7.6 Conclusion 195

8 ROBUST WILD BOOTSTRAP FOR STABILIZING THE VARIANCE IN HETEROSEDASTIC REGRESSION MODEL
8.1 Introduction 197
8.2 Classical Wild Bootstrap Techniques 199
   8.2.1 Limitations of Classical Wild Bootstraps 203
8.3 Newly Proposed Robust Wild Bootstrap Techniques 203
8.4 Numerical Example 207
8.5 Simulation Study 214
8.6 Concluding Remarks 219
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDIES

9.1 Introduction 220
9.2 Summary 220

9.2.1 Robust Rescaled Moments (RRM) Test for Testing the Normality of Regression Residuals in the Presence of Outliers 221
9.2.2 Modified Goldfeld-Quandt (MGQ) Test for Detecting Heteroscedasticity in the Presence of Outliers 221
9.2.3 Leverage Based Near-Neighbors (LBNN) Method for Remedial Measures of Heteroscedasticity in Multiple Linear Regression 222
9.2.4 Robust Weighted Least Squares (RWLS) for Simple Linear Heteroscedastic Model in the Presence of Outliers 223
9.2.5 Two-Step Robust Weighted Least Squares (TSRWLS) for Heteroscedastic Multiple Regression Model in the Presence of Outliers 224
9.2.6 Robust Wild Bootstrap for Stabilizing the Variance in Heteroscedastic Regression Model 224

9.3 Conclusions 225
9.4 Areas of Further Research 226

REFERENCES 228
APPENDICES 240
BIODATA OF STUDENT 262
LIST OF PUBLICATIONS 263
AWARDS 265