UNIVERSITI PUTRA MALAYSIA

NEW QUARTER-SWEEP-BASED ACCELERATED OVER-RELAXATION ITERATIVE ALGORITHMS AND THEIR PARALLEL IMPLEMENTATIONS IN SOLVING THE 2D POISSON EQUATION

SHUKHRAT RAKHIMOV

IPM 2010 12

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By

SHUKHRAT RAKHIMOV

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia in Fulfilment of the Requirements for the Degree of Master of Science

November 2010

DEDICATION

To my parents Isamiddin Rakhimov and Elena Rakhimova,

To my sister Irina and my little nephew Timur

To my fiancée Marina Paskina

Abstract of thesis presented to the Senate of Universiti Putra Malaysia
in fulfilment of the requirement for the degree of Master of Science

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Chair: Mohamed Othman, PhD

Faculty: Institute for Mathematical Research

This thesis deals with iterative methods for solving the Poisson equation, which is a representative of partial differential equations. The research considers different techniques and strategies in over-relaxation theory. The over-relaxation methods are easy to implement on a computer and flexible in management of the rate of convergence. Recent research in this area is related to different variations and applications of Successive Over-Relaxation (SOR) and Accelerated Over-Relaxation (AOR) methods.

Three types of finite-difference schemes are in the base of the full-sweep (FS), half-sweep (HS), and quarter-sweep (QS) approaches, considered in this research. Among them, the QS approach is shown to be the fastest and the most economical, achieving satisfactory result with less number of operations. Another approach to speed up the convergence is grouping of iteration points into a single iteration unit. Implemented with the finite-difference schemes mentioned above, this approach produces Explicit Group (EG), Explicit Decoupled Group (EDG),

and Modified Explicit Group (MEG) methods. While all the above mentioned methods were implemented with SOR, among them, the QS point and MEG methods have never been implemented with AOR before.

The main objective of the thesis is to develop new sequential and parallel iterative methods that will be faster and more efficient as compared to the existing methods. Eventually, new AOR QS and AOR MEG iterative methods are proposed. The experimental results and numerical complexity analysis have shown the new methods to be much faster than the existing counterparts. With respect to the AOR EDG method, which is the fastest counterpart, the total improvement in terms of execution time is about 74%.

Parallel implementations of these methods are very important, since high performance computing has become main supportive technology of scientific research. Newly developed parallel AOR QS and AOR MEG methods for distributed memory parallel machine are shown to be efficient for large sparse matrices, which occur in large sizes of problem. The parallel strategies used in the new algorithms are based on the message latency minimization and processor-independent iterations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

ALGORITMA LELARAN PENGENDURAN BERLEBIHAN TERPECUT BERDASARKAN SUKU SAPUAN BAHARU DAN PELAKSANAAN SELARINYA UNTUK MENYELESAIKAN PERSAMAAN POISSON DUA DIMENSI

Oleh

SHUKHRAT RAKHIMOV

November 2010

Pengerusi: Mohamed Othman, PhD

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Tesis ini menghuraikan kaedah lelaran untuk menyelesaikan persamaan Poisson yang merupakan sebahagian daripada persamaan pembezaan separa. Kajian yang dilakukan mempertimbangkan teknik dan strategi berbeza dalam teori pengenduran berlebihan. Kaedah pengenduran berlebihan mudah dilaksanakan pada komputer dan pelarasan kadar penumpuannya pula bersifat fleksibel. Kajian terkini dalam bidang ini adalah berkaitan dengan variasi yang berbeza dan penggunaan kaedah pengenduran berlebihan berturut-turut (PBB) dan kaedah pengenduran berlebihan terpecut (PBT).

Terdapat tiga jenis skim perbezaan terhingga yang berasaskan pendekatan sapuan penuh (SP), separuh sapuan (SeS) dan suku sapuan (SuS) yang digunakan dalam kajian ini. Antara ketiga-tiga pendekatan tersebut, pendekatan SuS didapati paling laju dan menjimatkan, malah menghasilkan keputusan yang memuaskan dengan jumlah pengendalian yang sedikit. Pendekatan lain untuk meningkatkan kadar penumpuan adalah dengan mengumpulkan titik lelaran kepada unit lelaran

tunggal. Apabila dilaksanakan dengan skim perbezaan terhingga yang disebutkan di atas, pendekatan ini menghasilkan kaedah Kumpulan Tak Tersirat (KTT), Kumpulan Tak Tersirat Nyah-Pasangan (KTTNP), dan Kumpulan Tak Tersirat Terubahsuai (KTTT). Walaupun semua kaedah tersebut dilaksanakan dengan kaedah PBB, namun titik SuS dan kaedah KTTT belum pernah dilaksanakan dengan kaedah PBT.

Objektif utama tesis ini adalah untuk membangunkan kaedah lelaran berjujukan dan selari baharu yang lebih laju dan cekap jika dibandingkan dengan kaedah yang sedia ada. Oleh yang demikian, kaedah lelaran PBT SuS dan PBT KTTT yang baharu dicadangkan. Keputusan eksperimen dan analisis kompleks berangka menunjukkan bahawa kaedah baharu tersebut lebih laju berbanding kaedah lain yang sedia ada. Bagi kaedah PBT KTTNP yang merupakan kaedah paling laju, peningkatan keseluruhan dari segi masa perlaksanaan dicatatkan sebagai lebih kurang 74%.

Pelaksanaan selari bagi kaedah-kaedah ini adalah sangat penting memandangkan pengiraan berprestasi tinggi sudah menjadi teknologi tulang belakang utama dalam bidang penyelidikan saintifik. Kaedah PBT SuS dan PBT KTTT selari yang baru dibangunkan untuk mesin selari ingatan teragih didapati lebih cekap bagi matriks jarang besar, yang wujud dalam bentuk masalah yang bersaiz besar. Strategi selari yang digunakan dalam algoritma baharu ini adalah berdasarkan peminimuman pemendaman mesej dan lelaran tak bersandar pemproses.

ACKNOWLEDGEMENTS

Pursuing Master of Science degree for me would not be possible without help of many people. First and foremost, I owe my deepest gratitude to the Chairman of the Supervisory Committee, Professor Dr. Mohamed Othman for his guidance, advice, support and encouragement. His attention to details, hard work, and persistence have set an example I hope to match some day.

I am very grateful to Professor Dato' Mohamed Suleiman for his valuable advice and motivation. His course of differential equations was very helpful for my research.

I want to express my deep gratitude to my family for support and understanding, especially, to my father, Assoc. Professor Dr. Isamiddin Rakhimov, who not only supported me morally and financially as father, but also shared his scientific experience with me.

I appreciate the assistance Ngoh Jin Yee, another Master student, who helped me with technical details of the Sun Fire V1280 Server and in solving the problems related to architecture. I would like to thank Sharifah Kartini for her valuable help in translations from English to Malay and back.

My special thanks and deepest appreciation to my close friends Timur Aitov and my fiancée Marina Paskina. Though they were far from me, the conversations over the Internet encouraged me a lot and set me in an optimistic mood.

This research would not be possible without help of Malaysian Government with the support by Fundamental Research Grant Scheme (FRGS), Grant 02-10-07-321FR under the Ministry of High Education of Malaysia.

I certify that a Thesis Examination Committee has met on **November 3, 2010** to conduct the final examination of Shukhrat Rakhimov on his (or her) thesis entitled "**New Quarter Sweep Based Accelerated Over-Relaxation Iterative Algorithms and Their Parallel Implementations in Solving the 2D Poisson Equation**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the University Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

SHUKHRAT RAKHIMOV

Date: 3 November 2010

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