UNIVERSITI PUTRA MALAYSIA

ESTIMATION OF EXPONENTIAL SUMS USING P-ADIC METHODS AND NEWTON POLYHEDRON TECHNIQUE

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UPM

ESTIMATION OF EXPONENTIAL SUMS USING P-ADIC METHODS AND NEWTON POLYHEDRON TECHNIQUE

By

YAP HONG KEAT

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ESTIMATION OF EXPONENTIAL SUMS USING P-ADIC METHODS AND NEWTON POLYHEDRON TECHNIQUE

By

YAP HONG KEAT

December 2010

Chairman : Kamel Ariffin bin Mohd Atan, PhD

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Let p be a prime and $f(x,y)$ be a polynomial in $Z_p[x,y]$. For $\alpha > 1$, the exponential sums associated with f modulo a prime p^α is defined as $S(f;p^\alpha) = \sum_{x,y \bmod p^\alpha} e_{p^\alpha}(f(x,y))$. Estimation of $S(f;p^\alpha)$ has been shown to depend on the number and p-adic sizes of common roots of the partial derivative polynomials of f. The objective of this research is to arrive at such estimations associated with a quadratic and cubic polynomials $f(x,y)$.

To achieve this objective we employ the p-adic methods and Newton polyhedron technique to estimate the p-adic sizes of common zeros of partial derivative polynomials associated with quadratic and cubic forms. The combination of indicator diagrams associated with the polynomials are examined and analyzed especially on cases where p-adic sizes of common zeros occur at the overlapping segments of the indicator diagrams. Cases involving p-adic sizes of common zeros

that occur at simple points of intersection and the vertices have been investigated by earlier researchers.

The information obtained above is then applied to estimate the cardinality of the set $V(f_x, f_y; p^\alpha)$. This estimation is then applied in turn to arrive at the estimation of exponential sums for quadratic and cubic polynomials.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

PENGANGGARAN HASIL TAMBAH EKSPONEN DENGAN KAEDAH P-ADIC DAN TEKNIK POLIHEDRON NEWTON

Oleh

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Katakan p suatu nombor perdana dan $f(x,y)$ suatu polinomial dalam $Z_p[x,y]$. Untuk $\alpha > 1$, hasil tambah eksponen yang disekutukan dengan f modulo p^α ditakrifkan sebagai $S(f;p^\alpha) = \sum_{x,y \bmod p^\alpha} e_{p^\alpha}(f(x,y))$ yang dinilaikan bagi semua x dan y di dalam set reja lengkap modulo p^α. Penganggaran $S(f;p^\alpha)$ telah ditunjukkan bersandar kepada bilangan dan saiz *p*-adic pensifar sepunya polinomial terbitan separa f. Objektif kajian ini adalah untuk mendapatkan penganggaran hasil tambah eksponen disekutukan dengan polinomial $f(x,y)$ berbentuk kuadratik dan kubik.

Untuk mencapai objektif di atas kami menggunakan kaedah *p*-adic dan teknik polihedron Newton untuk menganggarkan saiz *p*-adic pensifar sepunya polinomial terbitan separa yang disekutukan dengan polinomial kuadratik dan kubik. Kombinasi

gambar rajah penunjuk yang disekutukan dengan polinomial di atas diperiksa dan dianalisis terutama bagi kes saiz *p*-adic pensifar sepunya yang berlaku di tembereng bertindih gambar rajah penunjuk. Kes saiz *p*-adic pensifar sepunya berlaku pada titik persilangan mudah dan bucu telah disiasat oleh pengkaji dahulu.

Keputusan yang diperolehi digunakan untuk menganggarkan kekardinalan bagi set $V(f_x, f_y; p^\alpha)$. Penganggaran tersebut kemudiannya digunakan untuk mendapatkan penganggaran hasil tambah eksponen yang disekutukan dengan polinomial kuadratik dan kubik.

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I certify that a Thesis Examination Committee has met on 2 December 2010 to conduct the final examination of Yap Hong Keat on his thesis entitled "Estimation of Exponential Sums using P-Adic Methods and Newton Polyhedron Technique" in accordance with the Universities and University College Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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YAP HONG KEAT

Date: 2 December 2010

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