COMPUTATIONAL ANALYSIS OF GAS KINETIC BHATNAGAR-GROSS-KROOK SCHEME FOR INVISCID COMPRESSIBLE FLOW

By

ONG JIUNN CHIT

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

January 2004

Dedicated

To

My beloved Yen Yen,

My Parents,

Jiunn Heong and Wooi Boon

For all their love, support, motivation and understanding

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

COMPUTATIONAL ANALYSIS OF GAS KINETIC BHATNAGAR-GROSS-KROOK SCHEME FOR INVISCID COMPRESSIBLE FLOW

By

ONG JIUNN CHIT

January 2004

Chairman: Ashraf Ali Omar, Ph.D.

Faculty: Engineering

Many numerical schemes have been developed in the field of computational fluid dynamics to simulate inviscid, compressible flows. Among those most notable and successful are the Godunov-type schemes and flux vector splitting schemes. Besides these numerical schemes, schemes based on the gas kinetic theory have been developed in the past few years. Stemming from this approach, the gas kinetic Bhatnagar-Gross-Krook (BGK) scheme is realized. In this thesis, the BGK scheme based on the BGK model of the approximate Boltzmann equation has been fully analyzed and developed accordingly. The numerical algorithms for the BGK scheme are first developed for simulating one-dimensional flow, and then follow by the-two dimensional flow realms. Higher-order spatial accuracy of the scheme is achieved through the reconstruction of the flow variables via the Monotone Upstream-Centered Schemes for Conservation Laws (MUSCL) approach. For time integration method, an explicit method is adopted for the first-order schemes in both one and two-dimensional flow problems. The classical Runge-kutta multistage method is employed only for schemes with higher-order of

accuracy. In addition, an implicit time integration method known as the Approximate Factorization-Alternating Direction Implicit (AF-ADI) would be employed when dealing with two-dimensional flow problems in higher-order. In order to investigate the computational characteristics of the BGK scheme in detail, several cases of shock-shock interaction problem have been numerically analyzed. Developed code for the one-dimensional flow is validated with three typical test cases, namely, quasi-one-dimensional supersonic-subsonic nozzle flow, shock tube, and two interacting blast waves. Likewise, four typical two-dimensional test cases that are found in the literatures are used to validate the developed code for the two-dimensional flow. They are regular shock reflection, supersonic flow over a wedge, channel with a fifteen-degree ramp, and flow past a cylinder. From these validation cases, computed results are compared with the available exact solutions and with other computational results obtained by using some well known numerical discretization schemes. In comparison, the BGK scheme exhibits the most accurate shock resolution capabilities, least diffusiveness, least oscillatory, and great robustness.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

ANALISIS PERKOMPUTERAN SKIM GAS KINETIK BHATNAGAR-GROSS-KROOK UNTUK ALIRAN LIKAT BERMAMPAT

Oleh

ONG JIUNN CHIT

Januari 2004

Pengerusi: **Ashraf Ali Omar, Ph.D.**

Fakulti: **Kejuruteraan**

Dalam bidang dinamik bendalir pengiraan, terdapat banyak skim berangka yang telah dihasilkan semata-matanya untuk simulasi aliran likat bermampat. Di antara skim-skim yang dihormati dan berjaya dalam bidang tersebut adalah skim jenis "Godunov" dan skim "Flux Vector Splitting". Selain daripada skim-skim ini yang dimaksudkan, skim yang berdasarkan kepada teori kinetik gas telah dikembangkan sejak kebelakangan ini. Bercabang daripada teori ini, skim kinetik gas "Bhatnagar-Groos-Krook" atau lebih dikenali sebagai skim BGK berjaya ditakrifkan. Dalam tesis ini, skim berangka yang berasaskan kepada kaedah BGK yang merupakan suatu penghampiran kepada persamaan "Bolztmann" telah berjaya dianalisa dan dibangunkan secara berperingkat. Algoritma-algoritma berangka untuk skim BGK ini akan dihasilkan terlebih dahulu untuk simulasi aliran satu dimensi dan diikuti dengan aliran dua dimensi. Untuk memperolehi kejituan ruangan yang lebih tinggi untuk skim BGK, satu kaedah yang dikenali sebagai "Monotone Upstream-Centered Schemes for Conservation Laws" (MUSCL) akan digunakan untuk mengubahsuai pembolehubah-pembolehubah yang terdapat dalam

aliran. Untuk kaedah pengkamiran masa, satu kaedah tak-tersirat akan digunakan untuk skim-skim yang mempunyai kejituan tertib pertama untuk menyelesaikan masalah-masalah dalam aliran satu dan dua dimensi. Manakala, kaedah "Runge-Kutta" berperingkat hanya akan digunakan untuk skim-skim yang mempunyai kejituan yang lebih daripada tertib pertama. Selain daripada itu, satu kaedah tersirat untuk pengkamiran masa yang dikenali sebagai penghampiran pemfaktoran berulang-alik tersirat (AF-ADI) akan digunakan apabila berdepan dengan masalah-masalah aliran dua dimensi pada kejituan yang lebih tinggi. Untuk menyiasat ciri-ciri perkomputeran skim BGK dengan lebih terperinci, beberapa masalah berangka yang melibatkan interaksi di antara kejutan telah dianalisa secara berangka. Algoritma untuk aliran satu dimensi yang telah dihasilkan akan diuji dan disahkan dengan tiga kes uji yang tipikal. Kes-kes uji yang dimaksudkan ini terdiri daripada aliran muncung supersonic-subsonik mirip satu-matra, tiub kejutan, dan dua ombak meletup berinteraksi. Seterusnya, empat kes uji dua dimensi yang terdapat dalam kesusasteraan akan digunakan untuk mengesahkan algoritma-algoritma yang telah dibentuk untuk aliran dua dimensi. Kes-kes ini adalah pembalikan kejutan biasa, aliran supersonic mengelilingi baji, saluran dengan lima belas darjah lereng, dan aliran mengelilingi silinder. Melalui kes-kes pengesahan ini, hasil kiraan berangka akan dibandingkan dengan penyelesaian tepat yang boleh didapati dan dengan penyelesaian berangka daripada beberapa skim berangka yang dikenali. Secara perbandingan, skim BGK mempamerkan keputusan kejutan yang paling tepat, kurang melumuri, kurang getaran dan ketegapan yang paling memuaskan.

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude and sincere appreciation to my supervisor, Dr. Ashraf Ali Omar for his untiring support and seemingly unlimited belief in me which seem to be the only motivation and inspiration to me when all things fall short. If without him who I regarded as my mentor and friend, whose invaluable knowledge, guidance and suggestions, adversity would certainly find its place in my study. The other members of my supervisory committee; Associate Professor Dr. Waqar Asrar and Associate Professor Dr. Megat Mohamad Hamdan Megat Ahmad are not left out for their special roles that they played in ensuring the success of this study. I truly believed that without their help, intelligent guidance, and useful criticism. I would have never accomplished this study. Thus, special thanks and a heartfelt gratitude are conveyed to them.

In addition, I would like to thank the examination committee chairman; Dr. Ahmad Samsuri Mokhtar for his wise presiding and useful comments in making this study realizable. There are special people and friends that I must acknowledge due to their importance in my study. Among them, great appreciation is passed on to Mr. Mahmood K. Mawlood who has been so modest in sharing his knowledge, references, and ideas. I'm also grateful to the Ministry of Science, Technology and Environment of Malaysia for their financial support conceded to this research.

Any acknowledgements would not be complete if my parents' unconditional love and support went unrecognized. Thanks and loves are also extended to my brother and sister for their untiring understanding and encouragement to further my studies. Last but not least, I would like to take this opportunity to say thank you to my love; Yen Yen, for her untiring support, passion in sharing her life with me through sweet and bitter times, and most of all her unconditional love that grant me the strength to endure all the hardship of life.

I certify that an Examination Committee met on 7 January 2004 to conduct the final examination of Ong Jiunn Chit on his Master of Science thesis entitled "Computational Analysis of Gas Kinetic Bhatnagar-Gross-Krook Scheme for Inviscid Compressible Flow" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

Ahmad Samsuri Mokhtar, Ph.D.
Lecturer
Faculty of Engineering
University Putra Malaysia
(Chairman)

Ashraf Ali Omar, Ph.D.
Assistant Professor
Department of Mechanical Engineering
Faculty of Engineering
International Islamic University Malaysia
(Member)

Waqar Asrar, Ph.D.
Associate Professor
Department of Mechanical Engineering
Faculty of Engineering
International Islamic University Malaysia
(Member)

Megat Mohamad Hamdan Megat Ahmad, Ph.D.
Associate Professor
Faculty of Engineering
University Putra Malaysia
(Member)

GULAM RUSUL RAHMAT ALI, Ph.D.
Professor/Deputy Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

This thesis submitted to the Senate of Universiti Putra Malaysia has been accepted as fulfillment of the requirement for the degree of Master of Science. The members of the Supervisory Committee are as follows:

Ashraf Ali Omar, Ph.D.
Assistant Professor
Department of Mechanical Engineering
Faculty of Engineering
International Islamic University Malaysia
(Chairman)

Waqar Asrar, Ph.D.
Associate Professor
Department of Mechanical Engineering
Faculty of Engineering
International Islamic University Malaysia
(Member)

Megat Mohamad Hamdan Megat Ahmad, Ph.D.
Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Member)

____
AINI IDERIS, Ph.D.
Professor/Dean
School of Graduate Studies
Universiti Putra Malaysia

Date:

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

ONG JIUNN CHIT

Date:

TABLE OF CONTENTS