COMMUNICATION IN CHORDAL RING NETWORKS

By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Ful¯lment of the Requirements for Degree of Doctor of Philosophy
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TO MY FAMILY

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in ful¯lment of the requirement for the degree of Doctor of Philosophy

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The chordal ring networks have been the objects of a great deal of attention in recent years, and several parallel computers have con¯gurations based on the chordal ring topology. Common ways to improve the network performance are to increase its connectivity and decrease its diameter. Therefore, this thesis addresses the fundamental problems of communication in chordal ring of high degree and studies the degree diameter problem in such topology. In particular, we concentrate on Compact Routing, a family of routing methods that minimizes the space and time complexity. An e®ecient boolean routing scheme that has $O(1)$ time complexity and $O(log\ n)$ space complexity is introduced. Based on the existing results in [61] done by Narayanan and Opatrny, we propose a new algorithm for some families of chordal ring of de-

gree six graphs. New properties for this families of graphs have been introduced such as ¯nding the maximum number of nodes for a given diameter; it has been found that the chordal ring that has the maximum number of nodes for diameter k is G($4k2 + 2k + 1$; $2k + 1$; $2k2$). Moreover, a broadcasting scheme for this family of chordal rings of degree six has been constructed. It has been found that this scheme can broadcast the message to all nodes in the graph by time at most $k + 3$ where k is the diameter. The uniqueness property of the shortest path type between any two nodes in chordal rings of degree four and six has been introduced, this property helps us in deriving our results.

Abstrak tesis yang diserahkan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KOMUNIKASI DALAM RANGKAIAN CINCIN CHORDAL

Oleh

ATHEER ABBAS MATROUD

April 2006

Pengerusi : Profesor Madya Mohamed Othman, PhD

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Rangkaian cincin *chordal* telah memperolehi perhatian yang meluas sejak kebelakangan ini dan beberapa komputer selari telah dikon¯gurasikan dengan menggunakan topologi cincin *chordal*. Kebiasaannya untuk memperbaiki prestasi rangkaian adalah dengan memperbanyakkan sambungan dan mengurangkan garispusatnya. Oleh yang demikian, thesis ini menerangkan masalah asas bagi komunikasi cincin *chordal* darjah tertinggi dan kajian tentang permasalahan penetapan darjah garispusat bagi topologi tersebut.

Secara terperinci, kami menfokuskan kepada *Compact Routing*, iaitu satu kaedah penghalaan yang meminimumkan kompleksiti ruang dan masa. Algoritma pengha-

laan *boolean* efektif yang mempunyai kompleksiti masa $O(1)$ dan ruang $O(log\ n)$ diperkenalkan.

Berdasarkan keputusan daripada kajian yang dilakukan oleh Narayanan dan Opatrny [61], kami mencadangkan satu algoritma baru bagi beberapa keluarga cincin *chordal* darjah enam.

ceri-ceri terbaru keluarga ini diperkenalkan seperti pencarian bilangan nod maksimum bagi setiap garispusat yang diberi. Dalam mencari cincin *chordal* mempunyai bilangan nod maksimum bagi garispusat k ialah $G(4k2 + 2k + 1; 2k + 1; 2k2)$. Tambahan pula, kami telah membangunkan algoritma penyebaran bagi jenis cincin *chordal* darjah enam. Ianya didapati bahawa algoritma ini boleh menyebarkan mesej kepada semua nod dalam graf dengan masatidak melebaki $k + 3$ di mana k adalah garispusat. Keunikkan ciri jenis laluan terpendek antara mana-mana dua nod dalam cincin *chordal* darjah empat dan enam telah diperkenalkan. Justeru itu, ciri tersebut membantu kami dalam perolehi keputusan.

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ATHEER ABBAS MATROUD

I certify that an Examination Committee has met on 10*th* April 2006 to conduct the ¯nal examination of Atheer Abbas Matroud on his Doctor of Philosophy thesis entitled "Communication in Chordal Ring Networks" in accordance with Universiti Pertanian Malaysia (Higer Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

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ATHEER ABBAS MATROUD

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