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Interval Estimation for Parameters of a Bivariate Time Varying Covariate Model

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ABSTRACT

This paper investigates several asymptotic confidence interval estimates, based on the Wald, likelihood ratio and the score statistics for the parameters of a parallel two-component system model, with dependent failure and a time varying covariate, when data is censored. This model is an extension of the bivariate exponential model. The procedures are investigated via a coverage probability study using the simulated data. The results clearly indicate that the interval estimates, based on the likelihood ratio method, work better than any of the other two methods when dealing with the censored data.

Keywords: Bivariate, time varying, censoring, covariates, asymptotic, Wald, parallel, score

INTRODUCTION

The main limitation for most models, with censored data, is the fact that exact confidence intervals are impossible to compute. One alternative is to use large sample intervals, based on the asymptotic normality of the maximum likelihood estimates. However, there are some concerns over the use of the intervals which are based on asymptotic normality. Jeng and Meeker (2000) pointed out that its actual coverage probability could be significantly different from the nominal specification for a small to moderate number of failures, particularly for the one-sided confidence bounds. Cox and Hinkley (1979) mentioned that one of the disadvantages of using the Wald statistic is that it is not invariant under the transformation of the parameter of interest, unlike the methods which are based on the likelihood ratio and the score tests.

This paper investigates the interval estimates based on the Wald, likelihood ratio and the score methods, when they are applied to the parameters of the bivariate exponential model with dependent failure, time varying covariate and censored data. This is an extension of the bivariate exponential model by Freund (1961). Most of the studies, with parametric models involving time varying covariates, are in the area of political science and sociology. Works involving time varying covariates and duration dependence were done and discussed by authors such as Petersen (1986), Beck (1999), Bennet (1999), Box-Steffensmeier and Jones (1997), Tuma and Hannan (1984), Blossfield *et al*. (1989), Yamaguchi (1991), Courgeau and Lelièvre (1992), as well as Kalbfleisch and Prentice (1980).

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MATERIALS AND METHODS

Bivariate Model with Time Varying Covariate

Let T_1 and T_2 be random variables representing the lifetimes of two components, A and B from a parallel system with dependent failure. Similarly, m and r are the values of the random variables representing the total number of levels of the covariate in $[0, \min(t_1, t_2)]$ and $[0, \min(t_1, t_2)]$ respectively, with an initial value of m=1. The hazard rate of component A is $h=\exp(-\beta_0-\beta_1 x)$and the hazard rate of the component B is $g=\exp(-\omega_0-\omega_1 x)$, where $-\beta_0$, β_1, ω_0 and ω_1 are unknown parameters. If component A fails before component B, the hazard rate of component B then changes as a function of the covariates to $g' = \exp(-\omega^*_0-\omega^*_1 x)$, and similarly if component B fails first, then the hazard rate of component A changes to $h' = \exp(-\beta^*_0-\beta^*_1 x)$. The joint density function of T_1 and T_2 is:

$$f(t_1,t_2)=\begin{cases} h_m g_r^{'} \exp(-[(h_m+g_m)(t_1-a_{m-1})+\sum_{l=1}^{m-1}\ (h_l+g_l)(a_l-a_{l-1})]-[g_m^{'}(a_m-t_1) \\ +g_r^{'}(t_2-a_{r-1})+\sum_{l=m+1}^{r-1}\ g_l^{'}(a_l-a_{l-1})]) \\ for\ \ r>m>0\ \ and\ \ a_{m-1}<t_1\le a_m<t_2\le a_r, \\ h_m g_m^{'} \exp(-[(h_m+g_m)(t_1-a_{m-1})+\sum_{l=1}^{m-1}\ (h_l+g_l)(a_l-a_{l-1})]-g_m^{'}(t_2-t_1)) \\ for\ \ r=m>0\ \ and\ \ a_{m-1}<t_1<t_2\le a_m, \\ \\ g_m h_r^{'} \exp(-[(h_m+g_m)(t_2-a_{m-1})+\sum_{l=1}^{m-1}\ (h_l+g_l)(a_l-a_{l-1})]-[h_m^{'}(a_m-t_2) \\ +h_r^{'}(t_1-a_{r-1})+\sum_{l=m+1}^{r-1}\ h_l^{'}(a_l-a_{l-1})]) \\ for\ \ r>m>0\ \ and\ \ a_{m-1}<t_2\le a_m<t_1\le a_r, \\ g_m h_m^{'} \exp(-[(h_m+g_m)(t_2-a_{m-1})+\sum_{l=1}^{m-1}\ (h_l+g_l)(a_l-a_{l-1})]-h_m^{'}(t_1-t_2)) \\ for\ \ r=m>0\ \ and\ \ a_{m-1}<t_2<t_1\le a_m, \end{cases}$$

This paper is divided into 4 sections; Section 1 looks into some of the background studies related to the current work and the objectives of this research; Section 2 explores several asymptotic confidence interval estimates based on the Wald, likelihood ratio and the score statistics for the parameters of the time varying covariate model via coverage probability study using simulated data. Meanwhile, the result of the simulation study is discussed in Section 3, and finally Section 4 offers some concluding remarks and suggestions for future work.

Asymptotic Interval Estimation

The Wald Method

Let $\hat{\theta}$ be the maximum likelihood estimator for parameter and the log-likelihood function of θ. Under mild regularity conditions, $\hat{\theta}$ is asymptotically normally distributed with mean θ and covariance matrix $I^{-1}(\theta)$, where $I(\theta)$ is the Fisher information matrix, evaluated at the true value of the parameter θ, (Cox and Hinkley, 1979). The matrix $I(\theta)$ which is not available, can be replaced by the observed information matrix $I(\hat{\theta})$ whose $(j, k)^{th}$ element can be obtained from the second partial derivatives of the log-likelihood function evaluated at $\hat{\theta}$. If $z_{1-\frac{\alpha}{2}}$ is the $\left(1-\frac{\alpha}{2}\right)$quantile of the standard normal distribution, the $100(1-\alpha)\%$ confidence interval for θ_j is given by the following:

$$\hat{\theta}_j - z_{1-\frac{\alpha}{2}}\sqrt{I^{-1}(\hat{\theta}_{jj})} < \theta_j < \hat{\theta}_j + z_{1-\frac{\alpha}{2}}\sqrt{I^{-1}(\hat{\theta}_{jj})}.$$

The LR Method

The confidence interval, based on the likelihood ratio statistic (LR), has been described by many authors such as Cox and Hinkley (1979), Lawless (1982), Neslon (1990), Doganaksoy and Schmee (1993a) as well as Ostrouchov and Meeker (1988). Recently, there has been more emphasis on using the corrected version of the LR intervals such as the corrected signed square root of the likelihood ratio statistic by Diciccio (1988) and Bartlett's correction to the likelihood ratio statistic by Barndorff-Nielsen and Cox (1984).

For a scalar parameter of interest θ, the likelihood ratio statistic for testing the null hypothesis, $H_0: \theta = \theta_0$ versus $H_a: \theta \neq \theta_0$ is given as:

$$\Psi = -2[l(\theta_0, \tilde{\eta}) - l(\hat{\theta}, \hat{\eta})].$$

where l is the log-likelihood function, η is the vector of nuisance parameters, $(\hat{\theta}, \hat{\eta})$ is the maximum likelihood estimator of (θ, η), and $\tilde{\eta}$ is the restricted maximum likelihood estimator of η under H_0. For a large sample size, Ψ is approximately $\chi^2_{(1)}$ under H_0 and an approximate 100(1-α)% confidence interval for θ can be obtained by finding the two values of θ_0, for which H_0 is not rejected at the α level of significance, that is, the values that satisfy:

$$l(\theta_0, \tilde{\eta}) = l(\hat{\theta}, \hat{\eta}) - \tfrac{1}{2}\chi^2(1; 1-\alpha),$$

with the lower confidence limit, $\theta_L < \theta$ and the upper confidence limit, $\theta_U > \theta$. The general algorithm to obtain the LR confidence limits can be found in Venzon and Moolgavkar (1988).

The Score Method

The score statistic for testing hypothesis was introduced by Rao (1948). It was further discussed by Cox and Hinkley (1979). Let's reconsider the hypothesis, $H_0: \theta = \theta_0$ versus $H_a: \theta \neq \theta_0$ for testing a certain parameter of interest θ. The score statistic is asymptotically equivalent to the likelihood ratio statistic and is therefore obtained by considering the score vector at point θ_0, $S(\theta_0, \tilde{\eta})$. A score vector which is close to 0 at point θ_0 will indicate that is $\hat{\theta}$ also close to θ_0 and thus in favour of H_0. For a parameter of interest θ, the score statistic at point θ_0 is given as:

$$Q = S'(\theta_0, \tilde{\eta})[I^{-1}(\theta_0, \tilde{\eta})]S(\theta_0, \tilde{\eta})$$

where $\tilde{\eta}$ is the restricted maximum likelihood estimator of γ under H_0 and $I^{1}(\theta_0, \tilde{\eta})$ is the inverse of the observed information matrix evaluated at point $(\theta_0, \tilde{\eta})$. For a large sample, Q is approximately $\chi^2_{(1)}$ and an approximate 100(1-α)% confidence interval for θ can be obtained by finding the two values of θ_0, for which H_0 is not rejected at α level of significance, that is, the values that satisfy:

$$S'(\theta_0, \tilde{\eta})[I^{-1}(\theta_0, \tilde{\eta})]S(\theta_0, \tilde{\eta}) = \chi^2(1; 1-\alpha)$$

with the lower confidence limit, $\theta_L < \theta$ and the upper confidence limit, $\theta_U > \theta$. Under some regular conditions, the score has an asymptotic normal distribution with mean 0 and variance-covariance matrix equals to the information matrix.

Simulation and Coverage Probability Study

The simulation study was conducted using $N = 2000$ samples of size $n = 150, 200, 250, 300$ and 400 to compare the performance of the Wald, score and likelihood ratio confidence interval estimates. In this research, the maximum likelihood estimators of all the parameters were computed using the Newton Raphson iterative method, which was implemented using the FORTRAN programming language.

Consider an example with at most two levels of the covariate x, for each observation. These were simulated independently from the Standard Normal distribution. Let m and r be the values of the random variables representing the total number of the covariate levels in $[0, \min(t_1, t_2)]$ and $[0, \max(t_1, t_2)]$ respectively, with initial value of m=1. Recall that $0<m\leq r$, so m can take two possible values of 1 and 2. For each simulated observation, the number of covariate levels is determined by the values of m and r, where m is dependent on $(\min(t_1, t_2|x)$ and r is dependent on both $(\min(t_1, t_2|x)$ and $(\max(t_1, t_2|x))$. Since the covariate can only change its value once, then $r\leq 2$.

A total of 8 parameters were estimated. The values of 5, 1, 4.8 and 1.5 were chosen as the parameters of β_0, β_1, ω_0 and ω_1 whereas the values of 4.8, 1.5, 3.5 and 0.7 were chosen as the parameters of β_0^*, β_1^*, ω_0, ω_1^*. Three random numbers from the uniform distribution on the interval (0,1), u_{io}, u_{i1} and , were generated to produce t_{i1} and t_{i2}. Suppose there are both censored and uncensored lifetimes for subjects. Three types of data will be considered. The first is when both t_{i1} or t_{i2} are uncensored, while the second is when both t_{i1} and t_{i2} are censored and finally in the case when only t_{i1} and t_{i2} is censored.

First, generate a_{i1}~exp(v) where the value of v can be adjusted to obtain larger or smaller intervals of a_{i1}. In a real situation, the time intervals during which a covariate value remains constant, may vary or be fixed from one individual to another, depending on the covariate. If the covariate was, for example, the presence of a certain symptom, or status such as adult or juvenile, it would certainly be different among subjects. On the other hand, if the covariate was age at the beginning of the year, the intervals will then be the same for each individual. In this study, the intervals were assumed to vary between the individuals. Then,

$$t_{iA} = \begin{cases} -\dfrac{1}{h_{i1}}\log(1-u_{i0}) & \text{for} \quad u_{i0} \leq 1-e^{-h_{i1}a_{i1}}, \\ -\dfrac{1}{h_{i2}}\log(1-u_{i0}) + \left(1-\dfrac{h_{i1}}{h_{i2}}\right)a_{i1} & \text{otherwise.} \end{cases}$$

$$t_{iB} = \begin{cases} -\dfrac{1}{g_{i1}}\log(1-u_{i1}) & \text{for} \quad u_{i1} \leq 1-e^{-g_{i1}a_{i1}}, \\ -\dfrac{1}{g_{i2}}\log(1-u_{i1}) + \left(1-\dfrac{g_{i1}}{g_{i2}}\right)a_{i1} & \text{otherwise.} \end{cases}$$

Following that, if $t_{iA} < t_{iB}$, $t_{i1} = t_{iA}$. Then, if $u_{i0} \leq 1\text{-e}^{-hi1ai1}$,

$$t_{i2} = \begin{cases} -\dfrac{1}{g'_{i1}}\log(1-u_{i2})+t_{i1} & \text{for} \quad u_{i2} \le 1-e^{-g'_{i1}(a_{i1}-t_{i1})}, \\ -\dfrac{1}{g'_{i2}}\log(1-u_{i2})+\left(1+\dfrac{g'_{i1}}{g'_{i2}}\right)(a_{i1}-t_{i1})+t_{i1} & \text{otherwise.} \end{cases}$$

If $u_{i0} > 1\text{-}e^{-h_{i1}a_{i1}}$, $t_{i2} = -\dfrac{1}{g'_{i2}}\log(1-u_{i2})+t_{i1}$. Otherwise if $t_{iB} < t_{iA}$, $t_{i2} = t_{iB}$. Then, if $u_{i1} \le 1\text{-}e^{-g_{i1}a_{i1}}$

$$t_{i1} = \begin{cases} -\dfrac{1}{h'_{i1}}\log(1-u_{i2})+t_{i2} & \text{for} \quad u_{i2} \le 1-e^{-h'_{i1}(a_{i1}-t_{i2})}, \\ -\dfrac{1}{h'_{i2}}\log(1-u_{i2})+\left(1+\dfrac{h'_{i1}}{h'_{i2}}\right)(a_{i1}-t_{i2})+t_{i2} & \text{otherwise.} \end{cases}$$

If $u_{i1} > 1-e^{-g_{i1}a_{i1}}$, $t_{i1} = -\dfrac{1}{h'_{i2}}\log(1-u_{i2})+t_{i2}$.

The censoring time, $c_i \sim \exp(\mu)$, where the value of μ would be adjusted to obtain the desired approximate censoring proportion in the data of the present study. In this research, the two levels of approximate censoring proportions, cp=0.10 and cp=0.30, were used to see how they affected the performance of the interval estimates. The values of cp=0.10 and cp=0.30 were chosen to represent both low and high levels of censoring proportions, respectively. The coverage probability is the probability that an interval contains the true parameter value. The study was conducted by calculating the left and right estimated error probabilities for each of the parameter estimates. The estimated left (right) error probability was calculated by adding the number of times the left (right) endpoint was more (less) than the true parameter value, divided by the total number of samples, *N*.

Following Doganaksoy and Schmee (1993), if the total error probability is greater than α + 2.58 s.e ($\hat{\alpha}$), the method is then termed as anticonservative, and if it is lower than α-2.58 s.e ($\hat{\alpha}$), the method is termed as conservative. The estimated error probabilities are known as symmetric when the larger error probability is less than 1.5 times the smaller one.

RESULTS AND DISCUSSION

The summary of the simulation results, comparing the performances of the Wald, LR and score intervals, is given in Tables 1 and 2. These tables display the total number of anticonservative, conservative and asymmetrical intervals, generated by each of these methods at different nominal error probabilities, and censoring proportion. Tables 3 and 4 provide some of the more detailed results and show how the intervals performed at different sample sizes. *Figs. 1* through *4* give a graphical view of some of the coverage probabilities for each of the methods when α= 0.05 and cp=10%. Tables 1 and 2 show that all the intervals produced 1 anticonservative and 2 conservative intervals, but only when the nominal level, α is high.

The LR method clearly generates intervals which are more symmetrical than the other two methods. It only produced 1 asymmetrical interval when the censoring proportion in the data is low

TABLE 1
Summary of the number of interval estimates at $\alpha = 0.05$

Type of interval	Cp=0.10			Cp=0.30		
	Wald	LR	Score	Wald	LR	Score
Anticonservative(A)	0	0	0	0	0	0
Conservative(C)	0	0	0	0	0	0
Asymetrical	9	1	9	9	3	9

TABLE 2
Summary of the number of interval estimates at $\alpha = 0.10$

Type of interval	Cp=0.10			Cp=0.30		
	Wald	LR	Score	Wald	LR	Score
Anticonservative(A)	1	1	1	0	0	0
Conservative(C)	1	1	1	1	1	1
Asymetrical	6	1	5	4	0	4

and α is 0.05, whereas the Wald and score generated 9 asymmetrical intervals. The high censoring level in the data seems to affect the LR more than the other methods, but only when α is low where it starts to produce more asymmetrical intervals. However, this number is still much lower than the number of asymmetrical intervals generated by the Wald and score intervals.

Both Wald and score methods gave almost similar results, but the score method appears to be slightly more symmetrical than the Wald. However, the Wald has more intervals with the total error probability closer to the nominal level than the score intervals. The increase in the censoring proportion does not seem to affect the performances of the Wald and score intervals. When both α and censoring proportion is high, the performances of all the intervals seem to be slightly improved, where they produce fewer asymmetrical intervals, particularly the LR interval. The reason for this is probably the highly censored data that generates wider intervals because of the larger standard errors of the parameter estimates. This produces more intervals that include the true parameter value.

All the methods seem to generate more conservative and anticonservative intervals, but fewer asymmetrical intervals when α is high. They also seem to converge to the nominal level at almost the same rate, but the LR intervals perform slightly better than the other two when the size of the sample is lower.

CONCLUSIONS

Overall, the LR method appears to perform best since it has the least number of asymmetrical intervals. It should be the preferred method, specifically when the censoring level in the data is low. Although the high censoring level and low value of α seem to affect the LR in that it produces more asymmetrical intervals, the number of these asymmetrical intervals are still relatively low. Similarly, the LR intervals are still more symmetrical than the intervals produced by the other two methods.

TABLE 3
Estimated error probabilities at $\alpha = 0.05$, cp=0.10, A="Anticonservative", C="Conservative"

		Wald			LR			Score		
		Left Error	Right Error	Total Error	Left Error	Right Error	Total Error	Left Error	Right Error	Total Error
β_0	150	0.0235	0.0280	0.0515	0.0305	0.0250	0.0555	0.0240	0.0280	0.0520
	200	0.0215	0.0250	0.0465	0.0275	0.0230	0.0505	0.0225	0.0255	0.0480
	250	0.0250	0.0270	0.0520	0.0290	0.0220	0.0510	0.0250	0.0270	0.0520
	300	0.0225	0.0265	0.0490	0.0245	0.0230	0.0475	0.0225	0.0265	0.0490
	400	0.0265	0.0275	0.0540	0.0280	0.0225	0.0505	0.0270	0.0280	0.0550
β_1	150	0.0230	0.0300	0.0530	0.0230	0.0305	0.0535	0.0225	0.0300	0.0525
	200	0.0195	0.0280	0.0475	0.0195	0.0280	0.0475	0.0195	0.0275	0.0470
	250	0.0250	0.0200	0.0450	0.0245	0.0200	0.0445	0.0250	0.0200	0.0450
	300	0.0190	0.0260	0.0450	0.0190	0.0260	0.0450	0.0190	0.0260	0.0450
	400	0.0235	0.0255	0.0490	0.0225	0.0255	0.0480	0.0235	0.0255	0.0490
ω_0	150	0.0215	0.0300	0.0515	0.0270	0.0235	0.0505	0.0215	0.0305	0.0520
	200	0.0250	0.0330	0.0580	0.0295	0.0260	0.0555	0.0255	0.0330	0.0585
	250	0.0205	0.0365	0.0570	0.0270	0.0290	0.0560	0.0205	0.0370	0.0575
	300	0.0210	0.0290	0.0500	0.0275	0.0275	0.0550	0.0215	0.0300	0.0515
	400	0.0210	0.0285	0.0495	0.0275	0.0245	0.0520	0.0220	0.0290	0.0510
ω_1	150	0.0210	0.0245	0.0455	0.0215	0.0240	0.0455	0.0210	0.0245	0.0455
	200	0.0255	0.0275	0.0530	0.0255	0.0275	0.0530	0.0255	0.0280	0.0535
	250	0.0315	0.0200	0.0515	0.0315	0.0200	0.0515	0.0315	0.0205	0.0520
	300	0.0260	0.0190	0.0450	0.0255	0.0185	0.0440	0.0260	0.0195	0.0455
	400	0.0240	0.0235	0.0475	0.0240	0.0245	0.0485	0.0240	0.0235	0.0475
β_0^*	150	0.0210	0.0335	0.0545	0.0295	0.0285	0.0580	0.0215	0.0340	0.0555
	200	0.0170	0.0360	0.0530	0.0225	0.0300	0.0525	0.0170	0.0365	0.0535
	250	0.0155	0.0295	0.0450	0.0180	0.0250	0.0430	0.0160	0.0305	0.0465
	300	0.0190	0.0250	0.0440	0.0225	0.0240	0.0465	0.0190	0.0250	0.0440
	400	0.0180	0.0300	0.0480	0.0200	0.0255	0.0455	0.0190	0.0310	0.0500
β_1^*	150	0.0225	0.0240	0.0465	0.0245	0.0255	0.0500	0.0235	0.0240	0.0475
	200	0.0305	0.0275	0.0580	0.0305	0.0265	0.0570	0.0305	0.0275	0.0580
	250	0.0205	0.0225	0.0430	0.0210	0.0225	0.0435	0.0205	0.0225	0.0430
	300	0.0170	0.0230	0.0400	0.0170	0.0235	0.0405	0.0170	0.0230	0.0400
	400	0.0205	0.0300	0.0505	0.0205	0.0300	0.0505	0.0205	0.0305	0.0510
ω^*_0	150	0.0195	0.0370	0.0565	0.0235	0.0310	0.0545	0.0195	0.0380	0.0575
	200	0.0185	0.0340	0.0525	0.0205	0.0265	0.0470	0.0185	0.0345	0.0530
	250	0.0185	0.0345	0.0530	0.0215	0.0295	0.0510	0.0185	0.0350	0.0535
	300	0.0280	0.0305	0.0585	0.0310	0.0265	0.0575	0.0280	0.0315	0.0595
	400	0.0255	0.0335	0.0590	0.0290	0.0285	0.0575	0.0255	0.0335	0.0590
ω^*_0	150	0.0255	0.0315	0.0570	0.0245	0.0310	0.0555	0.0250	0.0315	0.0565
	200	0.0235	0.0275	0.0510	0.0240	0.0280	0.0520	0.0240	0.0280	0.0520
	250	0.0285	0.0215	0.0500	0.0280	0.0215	0.0495	0.0285	0.0215	0.0500
	300	0.0290	0.0235	0.0525	0.0290	0.0240	0.0530	0.0290	0.0235	0.0525
	400	0.0255	0.0325	0.0580	0.0255	0.0325	0.0580	0.0255	0.0325	0.0580

TABLE 4
Estimated error probabilities at $\alpha = 0.05$, cp=0.30, A="Anticonservative", C="Conservative"

		Wald			LR			Score		
		Left Error	Right Error	Total Error	Left Error	Right Error	Total Error	Left Error	Right Error	Total Error
β_0	150	0.0220	0.0330	0.0550	0.0320	0.0285	0.0605	0.0235	0.0335	0.0570
	200	0.0245	0.0300	0.0545	0.0305	0.0210	0.0515	0.0245	0.0305	0.0550
	250	0.0200	0.0230	0.0430	0.0245	0.0200	0.0445	0.0215	0.0230	0.0445
	300	0.0265	0.0285	0.0550	0.0325	0.0260	0.0585	0.0265	0.0295	0.0560
	400	0.0255	0.0285	0.0540	0.0315	0.0260	0.0575	0.0255	0.0285	0.0540
β_1	150	0.0210	0.0225	0.0435	0.0220	0.0225	0.0445	0.0205	0.0225	0.0430
	200	0.0185	0.0235	0.0420	0.0185	0.0235	0.0420	0.0185	0.0235	0.0420
	250	0.0245	0.0250	0.0495	0.0245	0.0250	0.0495	0.0245	0.0250	0.0495
	300	0.0235	0.0215	0.0450	0.0230	0.0230	0.0460	0.0235	0.0215	0.0450
	400	0.0260	0.0270	0.0530	0.0255	0.0270	0.0525	0.0260	0.0270	0.0530
ω_0	150	0.0200	0.0265	0.0465	0.0290	0.0215	0.0505	0.0205	0.0265	0.0470
	200	0.0200	0.0290	0.0490	0.0245	0.0255	0.0500	0.0200	0.0290	0.0490
	250	0.0270	0.0315	0.0585	0.0300	0.0290	0.0590	0.0270	0.0315	0.0585
	300	0.0200	0.0295	0.0495	0.0245	0.0265	0.0510	0.0200	0.0300	0.0500
	400	0.0240	0.0305	0.0545	0.0290	0.0270	0.0560	0.0245	0.0310	0.0555
ω_1	150	0.0215	0.0250	0.0465	0.0215	0.0250	0.0465	0.0215	0.0250	0.0465
	200	0.0240	0.0265	0.0505	0.0240	0.0265	0.0505	0.0240	0.0265	0.0505
	250	0.0330	0.0190	0.0520	0.0330	0.0190	0.0520	0.0330	0.0190	0.0520
	300	0.0235	0.0170	0.0405	0.0240	0.0165	0.0405	0.0235	0.0170	0.0405
	400	0.0275	0.0230	0.0505	0.0280	0.0230	0.0510	0.0275	0.0230	0.0505
β^*_0	150	0.0200	0.0305	0.0505	0.0255	0.0260	0.0515	0.0210	0.0315	0.0525
	200	0.0150	0.0375	0.0525	0.0185	0.0300	0.0485	0.0150	0.0380	0.0530
	250	0.0175	0.0295	0.0470	0.0235	0.0255	0.0490	0.0180	0.0305	0.0485
	300	0.0185	0.0275	0.0460	0.0230	0.0210	0.0440	0.0185	0.0275	0.0460
	400	0.0185	0.0345	0.0530	0.0220	0.0305	0.0525	0.0185	0.0345	0.0530
β^*_1	150	0.0215	0.0180	0.0395	0.0210	0.0175	0.0385	0.0210	0.0185	0.0395
	200	0.0290	0.0230	0.0520	0.0275	0.0225	0.0500	0.0295	0.0225	0.0520
	250	0.0205	0.0230	0.0435	0.0210	0.0230	0.0440	0.0205	0.0230	0.0435
	300	0.0155	0.0240	0.0395	0.0150	0.0240	0.0390	0.0155	0.0240	0.0395
	400	0.0250	0.0290	0.0540	0.0255	0.0280	0.0535	0.0255	0.0290	0.0545
ω^*_0	150	0.0195	0.0410	0.0605	0.0235	0.0325	0.0560	0.0205	0.0420	0.0625
	200	0.0175	0.0335	0.0510	0.0210	0.0255	0.0465	0.0175	0.0345	0.0520
	250	0.0230	0.0300	0.0530	0.0265	0.0230	0.0495	0.0230	0.0305	0.0535
	300	0.0200	0.0295	0.0495	0.0235	0.0245	0.0480	0.0200	0.0295	0.0495
	400	0.0250	0.0305	0.0555	0.0280	0.0270	0.0550	0.0250	0.0305	0.0555
ω^*_1	150	0.0205	0.0220	0.0425	0.0205	0.0210	0.0415	0.0205	0.0230	0.0435
	200	0.0235	0.0315	0.0550	0.0235	0.0290	0.0525	0.0235	0.0315	0.0550
	250	0.0255	0.0200	0.0455	0.0265	0.0195	0.0460	0.0260	0.0200	0.0460
	300	0.0245	0.0290	0.0535	0.0245	0.0295	0.0540	0.0245	0.0290	0.0535
	400	0.0255	0.0290	0.0545	0.0250	0.0285	0.0535	0.0255	0.0290	0.0545

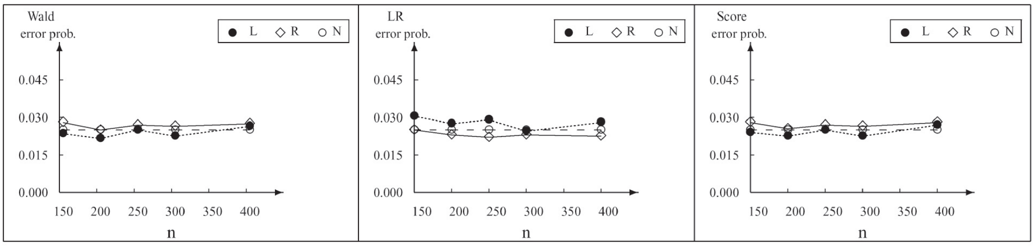

Fig. 1: Estimated error probabilities for β_0 at α= 0.05 and cp=0.1

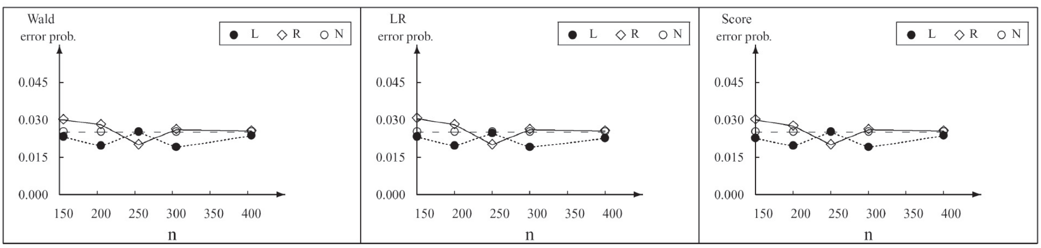

Fig. 2: Estimated error probabilities for β_1 at α= 0.05 and cp=0.1

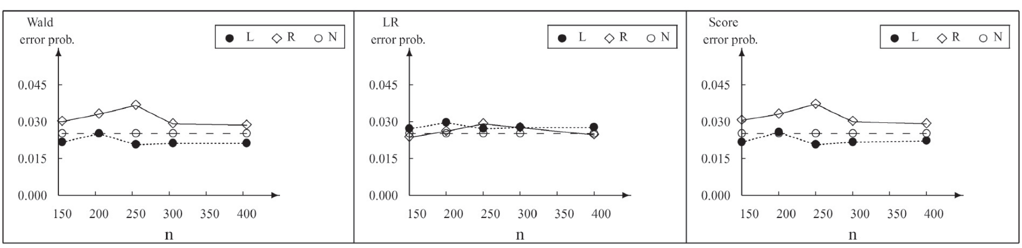

Fig. 3: Estimated error probabilities for ω_0 at α= 0.05 and cp=0.1

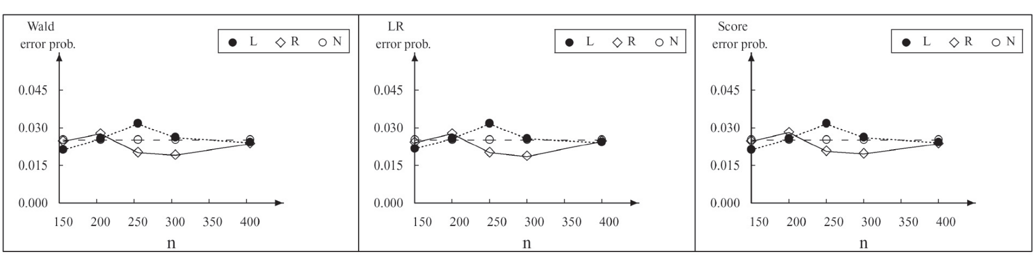

Fig. 4: Estimated error probabilities for ω_1 at α= 0.05 and cp=0.1

Thus, the Wald method can be considered when both α and the censoring proportion are high, or when a simpler and faster method is required. It is probably not advisable to consider applying the Wald intervals in other settings because it generally generates the most number of asymmetrical intervals. The score method may perform as good as the Wald, but it involves a lot of computational effort. Thus, this method should not be considered, unless as a measure of comparison. These findings are consistent with the results of Doganaksoy and Schmee (1993) who found the Wald intervals to be highly asymmetrical as compared to the LR, when dealing with censored data. However, the LR method involves a great deal of computational effort, which may not actually justify its improved performance when compared to the much more straightforward Wald method.

The discussion on the model involving time varying covariates has been restricted to two intervals, during which the covariates value remains constant. It would be possible to carry out further work to include models involving more intervals and this could be programmed relatively easily. The model can also be extended to consider the case, in which the components have the Weibull lifetime distribution.

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