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CONTROL EQUATIONS FOR THE PROPAGATION OF NONLINEAR WAVES IN VISCOUS FILM FLOWS WITH A MASS SOURCE

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The paper devotes to theoretical description and mathematical modelling the nonlinear modes of liquid film flows in the presence of possible mass sources, in the case of film condensation as a sample. The main new scientific result of the study is a theoretical description of the influence of the mass source on the characteristics of nonlinear waves arising during the flow of a viscous condensate film over a non-isothermal surface. The study was carried out within the framework of the long-wave approximation. The conditions for the existence of solutions describing the propagation of single nonlinear waves in condensate films are determined, and equations are derived for calculating the evolution of their wave characteristics. Relationships are obtained for estimating the scale of the propagation length of nonlinear waves in liquid films with variable flow rate. The conclusions of the work can be useful in design work and in developing process and apparatus control systems in the chemical and pharmaceutical industries.

Keywords: liquid films flows, mass sources, non-linear wave propagation, control parameters, govern evolution equation.

МАССА ДЕРЕККӨЗІ БАР ТҰТҚЫР ПЛЕНКАЛЫ АҒЫНДАРДА СЫЗЫҚТЫ ЕМЕС ТОЛҚЫНДАРДЫҢ ТАРАЛУЫН БАҚЫЛАУ ТЕҢДЕУЛЕРІ

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Бұл зерттеу жұмысы сұйық пленка ағындарының сызықтық емес режимдерін теориялық сипаттау және математикалық модельдеуге арналған, үлгі ретінде пленка конденсациясы жағдайында ықтимал масса көздері. Зерттеудің негізгі жаңа ғылыми нәтижесі-тұтқыр конденсат қабықшасының изотермиялық емес бетке ағуы кезінде пайда болатын сызықты емес толқындардың сипаттамаларына масса көзінің әсерінің теориялық сипаттамасы. Зерттеу ұзын толқынды жуықтау шеңберінде жүргізілді. Конденсат қабықшаларында бір сызықты емес толқындардың таралуын сипаттайтын ерітінділердің болу шарттары анықталып, олардың толқындық сипаттамаларының эволюциясын есептеу үшін теңдеулер алынады. Қатынастар ағынның өзгермелі жылдамдығы бар сұйық қабықшалардағы сызықты емес толқындардың таралу ұзындығының шкаласын бағалау үшін алынады. Жұмыстың нәтижелері химия және фармацевтика өнеркәсібіндегі аппаратураларды есептеуде және технологиялық процестерді басқаруда пайдаланылуы мүмкін.

Түйін сөздер: сұйық пленка ағындары, массалық көздер, сызықты емес толқындардың таралуы, бақылау параметрлері, эволюция теңдеуін басқару.

УПРАВЛЯЮЩИЕ УРАВНЕНИЯ ДЛЯ РАСПРОСТРАНЕНИЯ НЕЛИНЕЙНЫХ ВОЛН В ТЕЧЕНИЯХ ВЯЗКОЙ ПЛЕНКИ С ИСТОЧНИКОМ МАССЫ

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Статья посвящена теоретическому описанию и математическому моделированию нелинейных режимов пленочных течений жидкости при наличии возможных источников массы, в случае пленочной конденсации в качестве образца. Основным новым научным результатом исследования является теоретическое описание влияния источника массы на характеристики нелинейных волн, возникающих при обтекании пленки вязкого конденсата неизотермической поверхностью. Исследование проводилось в рамках длинноволнового приближения. Определены условия существования решений, описывающих распространение одиночных нелинейных волн в пленках конденсата, и выведены уравнения для расчета эволюции их волновых характеристик. Получены соотношения для оценки масштаба длины распространения нелинейных волн в пленках жидкости с переменным расходом. Выводы этой работы могут быть полезны при проектировании и разработке систем управления процессами и аппаратурой в химической промышленности.

Ключевые слова: течения пленок жидкости, источники массы, нелинейное распространение волн, управляющие параметры, уравнение эволюции управляющих волн.

Introduction. The problem of theoretical description of wave processes in condensate films, despite the well-known works [1-4], is far from exhausted, which is explained by a wide variety of manifestations of nonlinearity and dispersion effects during wave motions of films, especially in processes, the course of which is complicated by heat and mass transfer, or various phase transitions (solid phase - liquid or liquid - vapor) [2, 5, 6]. When modeling such processes, it is fundamentally important to take into account the presence of heat and mass sources, changes in the physical characteristics of the interacting media due to non-isothermality or variable concentration of individual components.

In a number of works it was established, for example, that the stationary Nusselt problem may not have a solution for non-isothermal film flows even for small Reynolds numbers. This circumstance leads to the emergence and propagation of non-linear waves of complex configuration in the dissipative system [7]. Another problem complicating the process of mathematical modeling is the complexity of the linkage of the equations that form the system [8, 9]. The main difficulty is that since in the presence of mass sources the flow rate of film flows changes in time and space, it is difficult to isolate a component constant of the solution that could then be subjected to an asymptotic analysis taking into account disturbances [10, 11].

Known works in this area [11] do not allow us to answer questions about how the intensity of the mass source affects the conditions for the occurrence of wave flows and the characteristics of the resulting nonlinear waves. At the same time, this question is of fundamental importance, since the stability of the wave-free flow regime depends significantly on the changing fluid flow rate, which leads to a changing film thickness [12, 13, 14].

The purpose of this theoretical work and its main scientific contribution is the derivation of general control equations for the flow of a viscous liquid film in the presence of a mass source. The derivation is specified using the example of the process of film condensation on a flat surface [15, 16]. The novelty of the work also lies in the fact that the developed mathematical model is subjected to asymptotic analysis [17, 18, 19], which allows us to identify the main control parameters and study the features and dependencies of the wave characteristics of the resulting nonlinear waves on the identified control parameters. The results obtained as a result of the theoretical studies carried out can find wide application in the calculation of technological processes, design of corresponding schemes and installations, as well as in methods and schemes for controlling technological film processes operating in modes of variable flow rate and the presence of mass sources.

Materials and methods. The main research method in this work was theoretical research, construction of mathematical models and their asymptotic analysis.

Previously, the method of integral relations was used to obtain a basic system of equations for film thickness and flow rate in flows with mass sources, in particular during film condensation [12].

The general form of the evolutionary equation in this case can be represented in the following form [12, 20]

$$\frac{\partial j}{\partial t} + \frac{\partial}{\partial x}\left(\frac{f_2}{f_1^2} \cdot \frac{j^2}{h}\right) + \frac{j}{f_1 h^2}\left(f_3 \nu_w - Ih\right) = g_{ef} h + \frac{\sigma h}{\rho} \cdot \frac{dK_S}{dx} \tag{1}$$

It is important to note that in the studied problem there is a parameter I, which is a control parameter characterizing the intensity of the mass source in the system.

The intensity of the condensation process decreases with increasing thickness of the condensate film. Therefore, at a sufficiently large distance from the initial point, the intensity of the mass source can be considered small [21]. At the same time, since we are talking about films of small thickness compared to the characteristic length of surface waves, we can introduce a small parameter $\varepsilon = Ij/\hat{j}_0$, into the model, where is the average velocity of the condensate flow in the unperturbed film in the area under consideration. With the help of such a small parameter, it becomes possible to scale the model by introducing slow variables $t = et$ and $z = ex$, as well as a fast phase variable $h = q(z,t)/e$.

The following system of evolutionary equations reads

$$\varepsilon\left(\frac{\partial j}{\partial \eta} + \frac{1}{\varepsilon}\frac{\partial j}{\partial \eta} \cdot \frac{\partial \theta}{\partial \tau}\right) + \varepsilon A\left[\frac{\partial}{\partial z}\left(\frac{j^2}{h}\right) + \frac{1}{\varepsilon}\frac{\partial}{\partial \eta}\left(\frac{j^2}{h}\right) \cdot \frac{\partial \theta}{\partial z}\right] + (B + \varepsilon B_1) \cdot \frac{j}{h^2} - g_{ef} h =$$
$$= \varepsilon^3 K_1 h \left[\frac{\partial^3 h}{\partial z^3} + \frac{3}{\varepsilon}\frac{\partial^3 h}{\partial z^2 \partial \eta} \cdot \frac{\partial \theta}{\partial z} + \frac{3}{\varepsilon^2}\frac{\partial^3 h}{\partial z \partial \eta^2} \cdot \left(\frac{\partial \theta}{\partial z}\right)^2 + \frac{3}{\varepsilon}\frac{\partial^2 h}{\partial z\, \partial \eta}\frac{\partial^2 \theta}{\partial z^2} + \frac{3}{\varepsilon^2}\frac{\partial^2 h}{\partial \eta^2}\frac{\partial \theta}{\partial z}\frac{\partial^2 \theta}{\partial z^2} + \frac{1}{\varepsilon^3}\frac{\partial^3 h}{\partial \eta^3}\left(\frac{\partial \theta}{\partial z}\right)^3 + \frac{1}{\varepsilon}\frac{\partial h}{\partial \eta}\frac{\partial^3 \theta}{\partial z^3}\right]$$

$$\varepsilon\left[\frac{\partial h}{\partial \tau} + \frac{1}{\varepsilon}\frac{\partial h}{\partial \eta} \cdot \frac{\partial \theta}{\partial \tau}\right] + \varepsilon\left[\frac{\partial j}{\partial z} + \frac{1}{\varepsilon}\frac{\partial j}{\partial \eta} \cdot \frac{\partial \theta}{\partial z}\right] = \varepsilon \cdot \frac{\hat{j}_0}{h} \tag{2}$$

A solution to the basic system can be presented in the form of expansions in powers of a small parameter

$$j = \sum_{i=0}^{N} \varepsilon^i J_i(\tau, z, \eta)\Big|_{\eta = \frac{\theta}{\varepsilon}} + \varepsilon^{N+1} R_{1N}(\tau, z, \eta, \varepsilon) \tag{3}$$

$$h = \sum_{i=0}^{N} \varepsilon^i H_i(\tau, z, \eta)\Big|_{\eta = \frac{\theta}{\varepsilon}} + \varepsilon^{N+1} R_{2N}(\tau, z, \eta, \varepsilon) \tag{4}$$

The zeroth order of the system reads

$$\varepsilon^0 \Rightarrow \begin{cases} \dfrac{\partial\theta}{\partial\tau}\cdot\dfrac{\partial J_0}{\partial\eta} + A\dfrac{\partial\theta}{\partial z}\cdot\dfrac{\partial}{\partial\eta}\left(\dfrac{J_0^2}{H_0}\right) + B\cdot\dfrac{J_0}{H_0^2} = g_{ef}H_0 + K_1\left(\dfrac{\partial\theta}{\partial z}\right)^3 H_0\cdot\dfrac{\partial^3 H_0}{\partial\eta^3}, \\ \dfrac{\partial\theta}{\partial\tau}\cdot\dfrac{\partial H_0}{\partial\eta} + \dfrac{\partial\theta}{\partial z}\cdot\dfrac{\partial J_0}{\partial\eta} = 0 \end{cases} \tag{5}$$

The firth order reads:

$$\varepsilon^1 \Rightarrow \begin{cases} \dfrac{\partial\theta}{\partial\tau}\cdot\dfrac{\partial J_1}{\partial\eta} + A\dfrac{\partial\theta}{\partial z}\cdot\dfrac{\partial}{\partial\eta}\left(\dfrac{2J_0}{H_0} - J_1\right) - A\dfrac{\partial\theta}{\partial z}\cdot\dfrac{\partial}{\partial\eta}\left(\dfrac{J_0^2}{H_0^2} - H_1\right) \\ \quad + B\cdot\dfrac{J_1}{H_0^2} - 2B\cdot\dfrac{J_0}{H_0^3}H_1 - g_{ef}H_1 - K_1H_0\cdot\dfrac{\partial^3 H_1}{\partial\eta^3}\left(\dfrac{\partial\theta}{\partial z}\right)^3 + K_1H_1\left(\dfrac{\partial\theta}{\partial z}\right)^3\cdot\dfrac{\partial^3 H_0}{\partial\eta^3} \\ \quad = -A\cdot\dfrac{\partial}{\partial z}\left(\dfrac{J_0^2}{H_0}\right) - B_1\cdot\dfrac{J_0}{H_0^2} + 3K_1H_0\cdot\dfrac{\partial^3 H_0}{\partial z\,\partial\eta^2}\left(\dfrac{\partial\theta}{\partial z}\right)^2 \\ \quad + 3K_1H_0\cdot\dfrac{\partial^2 H_0}{\partial\eta^2}\cdot\dfrac{\partial\theta}{\partial z}\cdot\dfrac{\partial^2\theta}{\partial z^2}, \\ \dfrac{\partial\theta}{\partial\tau}\cdot\dfrac{\partial H_1}{\partial\eta} + \dfrac{\partial\theta}{\partial z}\cdot\dfrac{\partial J_1}{\partial\eta} = \dfrac{\langle j_0\rangle}{H_0} - \dfrac{\partial H_0}{\partial\tau} - \dfrac{\partial J_0}{\partial z} \end{cases} \tag{6}$$

Next, we sequentially obtain recurrence relations for the following orders of expansion. Note that all systems, except the first, are linear with respect to the sought functions and are decoupled.

Results and discussion. Proceeding to a more detailed analysis, it is important to note that, unlike [22, 23], the systems can be decoupled without the additional assumption of a constant phase velocity. It follows from relations (5)

$$J_0 = -\frac{\dfrac{\partial\theta}{\partial\tau}}{\dfrac{\partial\theta}{\partial z}}H_0 + \Psi(\tau, z) \tag{7}$$

From the physical meaning of the problem under consideration, $Y(t,z) = 0$, which leads to the following equation for H_0

$$\frac{\left(\dfrac{\partial\theta}{\partial\tau}\right)^2}{\dfrac{\partial\theta}{\partial z}}(A-1)\cdot\frac{\partial H_0}{\partial\eta} - K_1\left(\frac{\partial\theta}{\partial z}\right)^3 H_0\cdot\frac{\partial^3 H_0}{\partial\eta^3} = g_{ef}H_0 + \frac{\dfrac{\partial\theta}{\partial\tau}}{\dfrac{\partial\theta}{\partial z}}\cdot\frac{B}{H_0} \tag{8}$$

The systems for subsequent approximations can be similarly decoupled. Moreover, there is a quasi-linear relationship between the flow rates and thicknesses as functions of fast and slow variables.

$$J_i = -\frac{\dfrac{\partial\theta}{\partial\tau}}{\dfrac{\partial\theta}{\partial z}}H_i + F\left(H_{i-1}, J_{i-1}\,;\,\tau, z\right) \tag{9}$$

The analysis of the model allows us to conclude that the surface pressure gradient along the film flow, caused by the variable curvature of the surface, contributes to the dispersion of waves only in the zero order, since waves of higher orders evolve under the influence of the effects of variability of physical characteristics. Literature analysis shows that the value of the coefficient K_1 is different, namely: ε of or ε^2, for different liquids depending on the type of liquid [24]. Therefore, in this paper, it is proposed to introduce a correction H_{02} into the solution of equation (8) to evaluate the effect of surface tension, taking into account the effect of surface tension: $H = H_{01} + H_{02}$. In this case, the inequality $H_{02} << H_{01}$ is valid.

As a result, we obtain the following equation for H_{01}

$$\frac{\left(\frac{\partial\theta}{\partial\tau}\right)^2}{\frac{\partial\theta}{\partial z}}(A-1)\cdot\frac{\partial H_{01}}{\partial\eta} = \frac{gH_{01}^2 + B\cdot\frac{\partial\theta/\partial\tau}{\partial\theta/\partial z}}{H_{01}} \tag{10}$$

The resulting equation has an obvious traveling wave solution

$$H_{01} = \sqrt{\left(H_{02}^2(\eta_0) + S_1\right)\exp\left(2gS_2(\eta-\eta_0)\right) - S_1} \tag{11}$$

Here

$$S_1 = \frac{B\left(\frac{\partial\theta}{\partial\tau}\right)}{g\left(\frac{\partial\theta}{\partial z}\right)}, \quad S_2 = \frac{\left(\frac{\partial\theta}{\partial z}\right)}{\left(\frac{\partial\theta}{\partial\tau}\right)^2(A-1)} \tag{12}$$

Discarding terms of the second and higher orders of smallness with respect to the correction, H_{02} we obtain the equation

$$\frac{\left(\frac{\partial\theta}{\partial\tau}\right)^2}{\frac{\partial\theta}{\partial z}}(A-1)\cdot\frac{\partial H_{01}}{\partial\eta} = H_{02}\left[g - \frac{B\left(\frac{\partial\theta}{\partial\tau}\right)/\left(\frac{\partial\theta}{\partial z}\right)}{H_{01}^2}\right] + K\left(\frac{\partial\theta}{\partial z}\right)^3 H_{01}\cdot\frac{\partial^3 H_{01}}{\partial\eta^3} + K\left(\frac{\partial\theta}{\partial z}\right)^3 H_{01}\cdot\frac{\partial^3 H_{02}}{\partial\eta^3} \tag{13}$$

Let' s supposed the first term is of higher order of smallness in the right-hand side, that leads to homogeneous equation. As it is shown in [12] the Wronskian of a homogeneous equation of such type is zero, because of the absence of second derivative $\frac{\partial H_{02}}{\partial\eta^2}$ [12]. From this it follows that resulting equation does not contain monotonically increasing solutions. So, for the possibility of the appearance of oscillating solutions in the form of a distortion of the wave profile of the ripple type, the following condition must be satisfied

$$(A-1)\cdot\frac{\partial\theta}{\partial z} < 0 \tag{14}$$

The last inequality relates the integral characteristics of the velocity profile in the film, depending on the varying temperature across the film thickness, and the wave number of the carrier wave. If inequality (14) is not satisfied, then we can expect that a small disturbance of the carrier wave profile will be smoothed out by capillary forces [12].

From inequality (14) it follows the condition $A < 1$.

Since the wave number and frequency in a film of variable flow rate change with time, at least two terms in their expansion in a Taylor series should be saved.

$$\theta(z,\tau,\varepsilon) = \beta(\tau,\varepsilon)\left(z+\varphi(\tau,\varepsilon)\right) + \beta_1(\tau,\varepsilon)\left(z+\varphi(\tau,\varepsilon)\right)^2 \tag{15}$$

Thus, the following evolution equation reads

$$\beta\left(\frac{\partial\varphi}{\partial\tau}\right)^2(A-1)\cdot\frac{\partial H_0}{\partial\eta}-K_1\beta^3H_0\cdot\frac{\partial^3H_0}{\partial\eta^3}=g_{ef}H_0+\frac{\partial\varphi}{\partial\tau}\cdot\frac{B}{H_0} \tag{16}$$

In order for building mathematical models for the evolution of wave disturbances of the condensate film profile, the methods of the century-old perturbation theory should be used [12].

With the help of supposition of the stationary flow regime nearby the stability boundary, the process can be considered as quasi-stationary.

Under the condition of the slowness of the functions j_0 and h_0, describing stationary solutions, it should be valid a certain relation to $j_1 = L(h_1)$ be valid, where $j_1 << j_0$ and $h_1 << h_0$ are disturbances of the stationary solution of the film condensation problem, and is also a slow function. Unlike [24], to describe the evolution of the wave packet in the weakly nonlinear approximation, we leave the second-order terms.

As a result, the following relation is correct

$$\frac{\partial j_1}{\partial t}+\alpha_1\frac{\partial j_1}{\partial x}+\alpha_2\frac{\partial h_1}{\partial x}+\alpha_3\frac{\partial^3h_1}{\partial x^3}+\alpha_4j_1+\alpha_5h_1=\beta_1j_1\frac{\partial j_1}{\partial x}+\beta_2h_1\frac{\partial h_1}{\partial x}+\beta_3j_1^2+\beta_4h_1^2 \tag{17}$$

$$\frac{\partial h_1}{\partial t}+\frac{\partial j_1}{\partial x}=zh_1+z_2h_1^2 \tag{18}$$

The coefficients of resulting system (17), (18) of equations play the role of control parameters:

$$\alpha_1=\frac{2f_2}{f_1^2}\cdot\frac{j_0}{h_0};\quad \alpha_2=-\frac{f_2}{f_1^2}\cdot\frac{j_0^2}{h_0^2};\quad \alpha_3=-\frac{\sigma}{\rho}h_0;\quad \alpha_4=\frac{2f_2}{f_1^2}\cdot\frac{\partial}{\partial x}\left(\frac{j_0}{h_0}\right)+\frac{1}{h_0}\left(\frac{\nu_wf_3}{f_1}-\frac{\lambda\Delta T}{r\rho f_1}\right)$$

$$\alpha_5=-\frac{f_2}{f_1^2}\cdot\frac{\partial}{\partial x}\left(\frac{j_0^2}{h_0^2}\right)-\left(\frac{\nu_wf_3}{f_1}-\frac{\lambda\Delta T}{r\rho f_1}\right)\cdot\frac{2j_0}{h_0^3}-g-\frac{\sigma}{\rho}\cdot\frac{\partial^3h_0}{\partial x^3} \tag{19}$$

$$\beta_1=\frac{2f_2}{f_1^2}\cdot\frac{1}{h_0};\quad \beta_2=\frac{2f_2}{f_1^2}\cdot\frac{j_0^2}{h_0^3};\quad \beta_3=-\frac{f_2}{f_1^2h_0^2}\cdot\frac{\partial h_0}{\partial x};\quad \beta_4=\frac{f_2}{f_1^2}\cdot\frac{\partial}{\partial x}\left(\frac{j_0^2}{h_0^3}\right)+\left(\frac{\nu_wf_3}{f_1}-\frac{\lambda\Delta T}{r\rho f_1}\right)\cdot\frac{3j_0}{h_0^4}; \tag{20}$$

$$z_1=-\frac{\lambda\Delta T}{r\rho h_0^2};\quad z_2=\frac{\lambda\Delta T}{r\rho h_0^3} \tag{21}$$

Let us introduce new stretched variables $X=\varepsilon x,\quad T=\varepsilon t$ and $\eta=\frac{\theta(X,T)}{\varepsilon}$ a fast variable .

Then we can look for a solution to the system (17), (18) in the form

$$h_1=H\exp(\eta),\quad j_1=J\exp(\eta) \tag{22}$$

Then, dividing the terms of the equations by powers of the small parameter and discarding rapidly oscillating components of small amplitude of the type $H^2\exp(2\eta)$ and $J^2\exp(2\eta)$, we arrive at an approximate linear system for the amplitudes

$$J\left[\frac{\partial\theta}{\partial T}+\alpha_1(X)\frac{\partial\theta}{\partial X}+\alpha_4(X)\right]+H\left[\alpha_2(X)\frac{\partial\theta}{\partial X}+\alpha_3(X)\left(\frac{\partial\theta}{\partial X}\right)^3+\alpha_5(X)\right]=0 \tag{23}$$

$$J\frac{\partial\theta}{\partial X}+H\left(\frac{\partial\theta}{\partial T}-z_1(X)\right)=0. \tag{24}$$

For the solvability of system (23), (24), the dispersion relation must be satisfied

$$\begin{bmatrix} \frac{\partial\theta}{\partial T}+\alpha_1\frac{\partial\theta}{\partial X}+\alpha_4 \\ \alpha_2\frac{\partial\theta}{\partial X}+\alpha_3\left(\frac{\partial\theta}{\partial X}\right)^3+\alpha_5 \\ \frac{\partial\theta}{\partial X} \\ \frac{\partial\theta}{\partial T}-z_1 \end{bmatrix}=0 \tag{25}$$

From this it follows the desired representation for $J=L(X,T)H$, where

$$L(X,T)=\frac{\frac{\partial\theta}{\partial T-z_1}}{\frac{\partial\theta}{\partial X}} \tag{26}$$

Considering $L(X,T)$ that is a function of slow variables, and substituting the last relation into the original system, we arrive at a zero-order equation for the function that is the film thickness simulation:

$$\frac{\partial h_1}{\partial t}+\left(\alpha_1+\frac{\alpha_2}{L}\right)\frac{\partial h_1}{\partial X}+\alpha_3\frac{\partial^3 h_1}{\partial X^3}+\left(\alpha_4+\frac{\alpha_5}{L}\right)h_1=\left(\beta_1 L+\frac{\beta_2}{L}\right)h_1\frac{\partial h_1}{\partial X}+\left(\beta_3 L+\frac{\beta_4}{L}\right)h_1^2 \tag{27}$$

In the resulting equation, it is convenient to switch to a moving coordinate system:

$$t;\quad \xi=t-\int\frac{dX}{\alpha_1+\frac{\alpha_2}{L}} \tag{28}$$

As a result, the following govern equation has been derived

$$\frac{\partial h_1}{\partial t}+\frac{\beta_1 L+\frac{\beta_2}{L}}{\alpha_1+\frac{\alpha_2}{L}}h_1\frac{\partial h_1}{\partial\xi}-\frac{\alpha_3}{\alpha_1+\frac{\alpha_2}{L}}\frac{\partial^3 h_1}{\partial\xi^3}=-\left(\alpha_4+\frac{\alpha_5}{L}\right)h_1+\left(\beta_3 L+\frac{\beta_4}{L}\right)h_1^2 \tag{29}$$

The resulting equation is close in structure to the Korteweg-de-Vries equation [12, 25] with a nonlinear perturbation of the right-hand side and slowly changing coefficients. The presence of such a perturbation leads to the fact that the dispersion relation of the last equation (29) contains a nonzero imaginary part, and an undamped wave solution can exist only on the neutral line and in the region of increasing amplitudes.

Conclusion. A mathematical model of propagation of long-wave nonlinear surface waves with small amplitude in flowing condensate films has been developed. When deriving the flow equations, the dependence of the liquid viscosity on the temperature of the supporting surface has been taken into account. Asymptotic analysis of the model has made it possible to estimate the degree of influence of variable viscosity and disturbances of the mass source intensity on the wave characteristics. The paper presents the general structure of the perturbed wave equation, which can be used for solving problems of propagation of nonlinear waves in condensate films under various boundary conditions.

References

1. Katsiavria A., Papageorgiou D. T. Nonlinear waves in viscous multilayer shear flows in the presence of interfacial slip//Wave Motion.-2022.-Vol.114.103018. DOI 10.1016/j.wavemoti.2022.103018

2. Mukhopadhyay S., Mukhopadhyay A. Waves and instabilities of viscoelastic fluid film flowing down an inclined wavy bottom//Physical Review E.- 2020-Vol.102(2): 023117. DOI 10.1103/ Physl RevE.102.023117

3. Ogrosky H. R. Linear stability and nonlinear dynamics in a long-wave model of film flows inside a tube in the presence of surfactant//Journal of Fluid Mechanics.-2021.- Vol.908.- P.1-16. DOI 10.1017/jfm.2020.878

4. Zakaria K., Sirwah M. A. Nonlinear dynamics of a liquid film flow over a solid substrate in the presence of external shear stress and electric field//The European Physical Journal Plus-2022.-Vol.137(9):1087. DOI 10.1140/epjp/s13360-022-03249-7

5. Beloglazkin A. N., Shkadov V. Y. Nonlinear Waves in Film Viscous Liquid Flows at Arbitrary Kapitsa Numbers //Fluid Dynamics.-2021.-Vol 56(4).- P.539-551. DOI10.1134/S0015462821040029

6. Mukhopadhyay S., Mukhopadhyay A. Hydrodynamic instability and wave formation of a viscous film flowing down a slippery inclined substrate: Effect of odd-viscosity // European Journal of Mechanics-B / Fluids.-2021.-Vol.89.- P.161-170. DOI 10.1016/j.euromechflu.2021.05.013

7. Duruk S. Nonlinear dynamics of thin liquid films subjected to mixed-frequency electrical field//Physics of Fluids.-2020.Vol.32(5). P. DOI 10.1063/5.0008220

8. Lerisson G., Ledda P. G., Balestra G., Gallaire F. Instability of a thin viscous film flowing under an inclined substrate: steady patterns//Journal of Fluid Mechanics.-2020.Vol.898. DOI 10.1017/jfm.2020.396

9. Mukhopadhyay S., Cellier N., Mukhopadhyay A. Long-wave instabilities of evaporating/condensing viscous film flowing down a wavy inclined wall: Interfacial phase change effect of uniform layers//Physics of Fluids.-2022.-Vol.34(4). DOI 10.1063/5.0089068

10. Witelski T. P. Nonlinear dynamics of dewetting thin films//AIMS Mathematics.-2020.-Vol. V.5(5). -P.4229-4259. DOI 10.3934/math.2020270

11. Camassa R., Marzuola J. L., Ogrosky H. R., Swygert S. On the stability of traveling wave solutions to thin-film and long-wave models for film flows inside a tube//Physica D: Nonlinear Phenomena.–2021.-Vol.415(3): 132750. DOI:10.1016/j.physd.2020.132750

12. Brener A., Yegenova A., Botayeva S. Equations of Nonlinear Waves in Thin Film Flows with Mass Sources and Surface Activity at the Moving Boundary//WSEAS Transactions on Fluid Mechanics.-2020.-Vol.15.-P.149-163. DOI 10.37394/232013.2020.15.15

13. Kim D. J., Kim D. Low-order modelling of three-dimensional surface waves in liquid film flow on a rotating disk//Journal of Fluid Mechanics. - 2024.-Vol.985. DOI 10.1017/jfm.2024.274

14. Hu T., Fu Q., Yang L. Falling film with insoluble surfactants: effects of surface elasticity and surface viscosities//Journal of Fluid Mechanics.-2020.-Vol.889. DOI 10.1017/jfm.2020.89

15. Maksymov I. S., Pototsky A. Solitary-like wave dynamics in thin liquid films over a vibrated inclined plane//Applied Sciences.-2020.-Vol.13(3):1888.2023. DOI 10.3390/app13031888

16. Ranganathan U., Chattopadhyay G., Tiwari N. Evolution of a thin film down an incline: A new perspective // Physics of Fluids .-2020.-Vol.32(1). DOI 10.1063/1.5127815

17. Zhou G., Prosperetti A. Capillary waves on a falling film // Physical Review Fluids .-2020.-Vol.5(11): 114005. DOI 10.1103/PhysRevFluids.5.114005

18. Akylas T. R. David J. Benney: Nonlinear wave and instability processes in fluid flows//Annual Review of Fluid Mechanics.-2020.-Vol.52(1).-P.21-36.DOI 10.1146/annurev-fluid-010518-040240

19. Khusnutdinova K., Gavrilyuk S., Ostrovsky L. Nonlinear dispersive waves in fluids and solids // Wave Motion.- 2023.- Vol.118:103123. DOI 10.1016/j.wavemoti.2023.103123

20. Singh G., Tiwari N. Stability of traveling waves of a thermoviscous liquid film down the outer surface of a cylinder//Physical Review Fluids.-2024.-Vol.9(2): 024002. DOI 10.1103/PhysRevFluids.9.024002

21. Sojahrood A. J., Haghi H., Karshafian R., Kolios M. C. Classification of the major nonlinear regimes of oscillations, oscillation properties, and mechanisms of wave energy dissipation in the nonlinear oscillations of coated and uncoated bubbles//Physics of Fluids.-2021.-V.33(1):16105 DOI 10.1063/5.0032766

22. Helal M. A., Badawi S. E., Mahmoud W. Wave propagation over a beach within a nonlinear theory//Inf. Sci. Lett.-2022.-V.11(05).-P.1741-1755. DOI 10.18576/isl/110531

23. Dong X., Chen W. Numerical analysis of wave effect on steam–air condensation on a vertical surface // Annals of Nuclear Energy.-2022.-Vol.168.108872. DOI 10.1016/j.anucene.2021.108872

24. Choi Y., Son G., Lee G. Numerical simulation of wavy film condensation in a vertical channel with non-condensable gas//International Journal of Heat and Mass Transfer.-2022.- Vol.149:119173. DOI 10.1016/j.ijheatmasstransfer.2019.119173

25. Iqbal M., Faridi W. A., Algethamie R., Alomari F. A., Murad M. A. S., Alsubaie N. E., Seadawy A. R. Extraction of newly soliton wave structure to the nonlinear damped Korteweg–de Vries dynamical equation through a computational technique//Optical and Quantum Electronics.-2024.-Vol.56(7):1189. DOI 10.1007/s11082-024-06880-z

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