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Performance Enhancement of Trinomial Option Pricing Model

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Abstract

Option pricing is used in the greater part of commonly traded instruments in the financial market that predicts the future stock price. The option's fair price can be found either analytically or by applying suitable numerical techniques. Black-Scholes (B-S) seminal approach is one of them, but it has less accuracy in pricing all types of options. Therefore, a lattice model, namely a trinomial model, was developed to overcome these valuation problems for American or European option pricing. However, Trinomial model is not capable of providing maximum accuracy for option pricing as the system of equations used in the model involves a non-linear growth of the stock price. It is, therefore, important to develop a new system of equations to enhance the performance of trinomial framework in contemplation of American or European options. This study aims to improve the performance of the trinomial model for American option by developing a new system of equations with a linear growth rate of the stock price in different states of the regime so that all the data can be stored in the amalgamation of a trinomial tree. The performance was assessed by comparing the trinomial model to the binomial and Black-Scholes (B-S) pricing for the American option. The results demonstrated that the trinomial option pricing gives higher accuracy than Black-Scholes (B-S) pricing and is faster than the binomial pricing.

Keywords: American Options, Binomial Model (BM), Black-Scholes model (B-S), European Options, Trinomial model (TM).

Introduction

Option pricing problems of financial derivatives are growing rapidly in the financial markets. These problems draw attention to many researchers in finding accurate solutions. However, Options are complicated derivatives in a financial market that has been traded from the ancient period. The complications in option valuation arise from its features, such as its pricing depends on an underlying asset. An intermediary negotiates the option price among the buyer, the seller, and different types of options. Therefore, participants in the financial markets need accurate option valuations. Mathematical studies in the financial industry might have been developed in recent decades, but options trading was not traded recently. It started in the ancient period and became popular as financial instruments, day by day, from the philosopher Thales (624-547 BC). Thales did the call option without using any mathematical modelling. In the sixteenth century, options were traded in Holland. But even in the seventeenth century, options trading was banned in London (1). In that same era, options trading was introduced in the USA, but stock was only presented as a financial instrument during the nineteenth century.

The option was simply called "privileges" at the beginning of options trading, where only two parties were involved in the newspaper advertisement. At that time, it was not a responding financial instrument. The option pricing formula was first addressed by a French mathematician, in 1900 (2). Brownian motion was used in his formula, which defines that price changes are distributed independently and identically, leading to the theory development of efficient markets (3). However, Bachelier's contribution was not recognized as it was unusual to apply mathematical modelling in option pricing, and it was inaccessible to study. As a result, Bachelier's theory did not get the attention of the practitioners at that time. It took sixty years to draw attention to his model. Bachelier described normal returns and investors as risk neutral.

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Boness improved the model by discounting the terminal stock price and considering the discounted price using the expected return (4). This work is the basic for the most well-known “Black-Scholes (B-S) model" although it is a widely used valuation model.

B-S model is widely used due to its binomial extension with embedded probabilities (5). The formula came with a breakthrough in practice in option pricing history. This model was established in 1973 as "The price of the option and corporate liabilities", and the Chicago Board Options Exchange was in progress to trade listed options for the first time (6). The major perception of the B-S approach is that a dynamic portfolio trading option is replicated with the returns of the option on that stock, which means the formula addresses the explicit hedging method. The B-S model is convenient for the valuation of European options only. It is unsuitable for the American option valuation because of its complex boundary conditions and early exercise for multidimensional markets. At the same time, the binomial model is convenient due to its simple mathematical calculation. The past financial crises point out that traditional models are not capable of tackling derivative pricing. This problem induces the necessity of a new financial mathematical model such as the binomial and trinomial pricing models. Only two states' pricing of the underlying stock is considered by the binomial model, which is one of the crucial disadvantages of using this model (5). Finite difference methods can be used to make such a kind of valuation, but it is quite expensive.

Several studies were carried out on option pricing using different methods and approaches (7-13). An efficient solution of the B-S equation was obtained by governing the European options (12). Few numerical examples were considered, along with their exact solutions to assess numerical accuracy. A new method based on fractional power series expansion was applied to find an analytical solution of the B-S equation for the European option pricing (10). The method was easy to apply and robust for any other problems raised in science, engineering, and technology. Trinomial model, the extension of CRR’s lattice binomial approach, differs from the binomial when the option valuation is compared for at least two-dimensional markets. It was developed by Boyle to obtain an efficient algorithm for the option valuation (14). The option value can be changed in three ways: go up, go down, or remain constant in this model. It is suitable to make pricing of the American option of multidimensional markets. The trinomial option pricing model and its solution were changed by some researchers in a more realistic way (15-18). The trinomial model was solved numerically and found faster and more accurate valuation by converging the binomial and trinomial models (19).

Markov tree model was prescribed by considering a non-IID process using first-order Markov behavior (20). A Markov chain process is followed by the interest rate, which is considered in the trinomial model (21). It was found that the trinomial option pricing is faster and shows more accuracy than the binomial option pricing. The trinomial model was again modified by governing risk-neutral probabilities (22). The trinomial model study is very important in finance as it can explain the reality of the financial markets in an exact and precise way. An extensive literature survey indicates many ongoing research studies on the option pricing model, specifically on American put due to the option uniqueness in its early exercise facility. Many studies on the trinomial model dealt with the system of equations with a non-linear rate of interest of stock price; therefore, the model cannot provide maximum accuracy for option pricing. The performance of the model can be improved further by employing the stock price growth rate as linear in the system of equations. In addition, no study is seen to measure the trinomial option pricing model's performance compared to other commonly used option pricing models. Therefore, this study aims to develop a system of equations with a linear stock price growth rate and assess its performance by comparing with the binomial and B-S pricing for the American put options. This paper is organized as follows: Section 2 describes the option pricing model. Section 3 documents the results and discusses the results and Section 4 concludes the paper.

Option Pricing

An option is a vital element in the problems of mathematical finance. An option which allows the right to buy or sell a predefined amount of underlying assets at a specific value called a strike price (K) or an exercise price, is known as a contract between a buyer and a seller (23). The buyer has no obligation to follow the contract, while the seller must maintain the contract with the predetermined price. Based on the exercise time (*T*), the option is classified into two types: American and European options. The European option is exercised on the maturity time, $t = T > 0$ only, whereas the American option is exercised before or on the maturity time $t \in [0, T]$. The buyer must pay the premiums to the seller, and the buyer gets payoff, *Y* for the trading option if the option is exercised. The payoff for European put and call options are given by $Y = max\ (0, K - S(T))$ and $Y = max\ (0, S(T)-K)$ respectively, where $S(T)$ represents the stock price on maturity time, *T*. On the other hand, the payoff for American call and put options are defined as $Y = max\ (0, S(t) - K)$ and $Y = max\ (0, K - S(t))$, respectively (24).

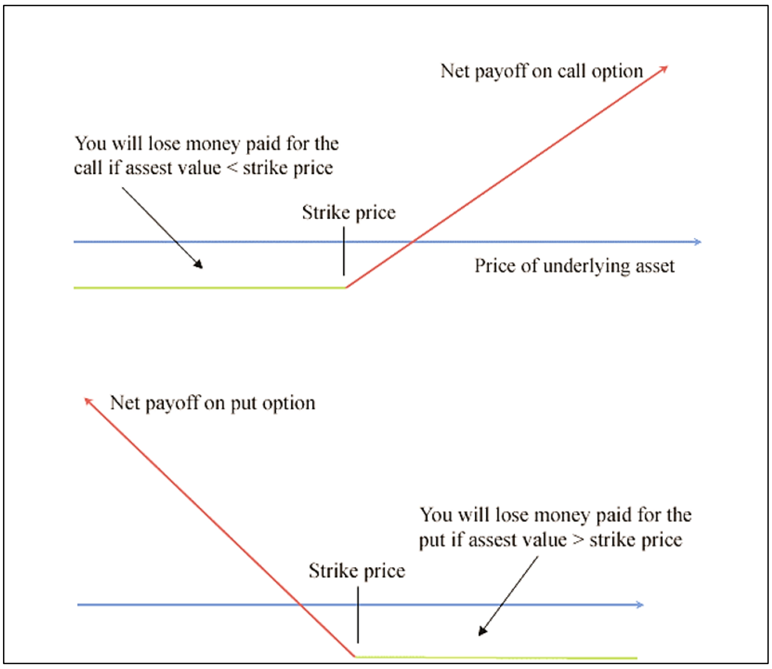

Figure 1: Payoff of Call Option and Put Option

Figure 1 depicts the payoff Call and payoff Put options. Several models were developed to trade options. Some assumptions are made for simplicity of the models such as no bid/ask spread, no transaction cost, trade that occurs instantaneously, no liquidity shortage, no borrowing limit from the money market, the same interest rate to borrow and deposit. The commonly used models on the option pricing are discussed in the following sections:

Black-Scholes (B-S) Model

The Black-Scholes model is broadly used by practitioners due to its binomial extension. As mentioned earlier, the B-S model was introduced in the trading markets in 1973. A replicating portfolio was used in this model that is composed of the risk-free and the underlying asset. The universal option price and risk-free rate are examined, highlighting the distinct risk bearers of investors who cannot affect the option price (25). This approach clarifies option pricing by use of direct parameters, apart from volatility (26). The B-S formula is essential to estimate European put and call options for a non-dividend-paying stock, especially when it comprises time-fractional derivatives. The formula is as follows:

$$C(S,T) = SN(d_1) - N(d_2)Ke^{-r(T-t)} \quad [1]$$

With

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)\right] \quad [2]$$

and

$$d_2 = d_1 - \sigma\sqrt{T-t} \quad [3]$$

where *S*, *K* are the stock and strike price, respectively, σ is the volatility, *T* and *t* are the maturities and current time correspondingly, r is the rate of interest, *N(x)* is the normal cumulative distribution function.

The Black-Scholes formula has led academic researchers to develop more mathematical modelling in the finance sector. It satisfies the terminal condition of the partial differential equation (PDE) to avoid the arbitrage opportunity. By considering a European derivative by payoff, $Y = g\,(S(T))$ at the time of maturity $T > 0$ and letting $\prod_Y^{BS}(t)$, which denotes its Black-Scholes price at time $t < T$. From here, the following is obtained.

$$\prod_Y^{BS}(t) = v\big(t, S(t)\big) \qquad [4]$$

where the price function $v\ (t, s)$ is defined as

$$v(t,s) = \left(\frac{e^{-rt}}{\sqrt{2\pi}}\right)\int_R g\left(se^{\left(r-\frac{\sigma^2 r}{2}\right)+\sigma\sqrt{\tau y}}\right)e^{\frac{y^2}{2}}\ dy, \tau = T - t \qquad [5]$$

Moreover, $v\ (t, s)$ is the solution of the terminal value problem

$$\partial_t v + rs\partial_s v + (\tfrac{1}{2}\sigma^2 s^2 \partial_s^2 v = rv, \quad s > 0, t < T, \qquad [6]$$

$$v(T,s) = g(s) \qquad [7]$$

Equation [6] is the Black-Scholes PDE.

Binomial Model

Binomial Model simplified the B-S model to value the option as the B-S derivation is mathematically complicated (5). They addressed the binomial pricing model as a new valuation method. The binomial model is formulated in a simple process where an asset can go through any of the two potential prices at any period. The stock price formulation follows the binomial path as shown in Figure 2, where S, the present stock price, rises to S_u with the probability p and falls to S_d with the probability *(1-p)* at any time. The binomial model is useful for both the European put and call options. But, for the American put options, the Black-Scholes model with its binomial extension does not give accurate solutions. These difficulties led to further research into the option pricing solution. As a result, the binomial stock price is given as follows:

$$S(t) = \begin{cases} S(t-1) \quad e^u, \text{probability } p \\ S(t-1)\ e^d, \text{probability } (1-p) \end{cases} For\ t\ \epsilon\ I = \{1,2,\ldots N\} \qquad [8]$$

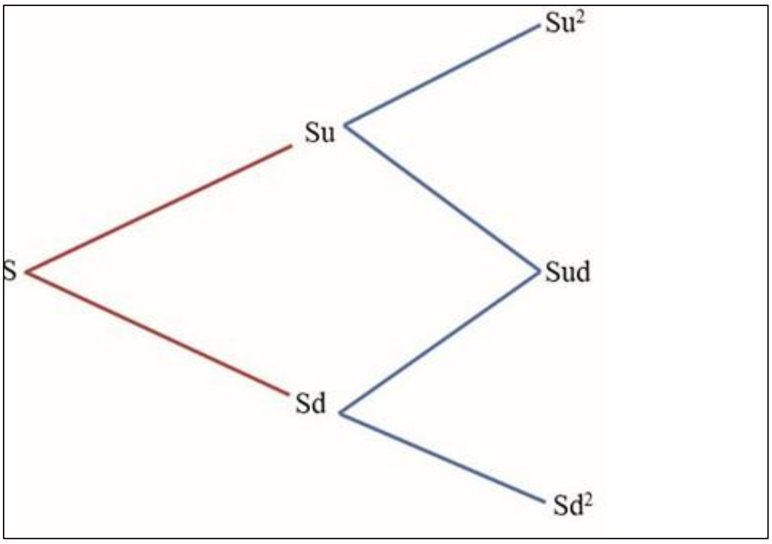

Figure 2: Binomial Price Path

Figure 2 shows the path of a binomial model, where S, the present stock price, rises to S_u with the probability p and falls to S_d with the probability *(1-p)* at any time. In simple words the stock price either goes up or goes down in each node.

Trinomial Model

To use the option pricing model more efficiently, Boyle (1986) developed the trinomial framework in 1986 (14). A highly effective numerical calculation of the option price within the pioneering Black-Scholes approach is given by the trinomial pricing structure, which is an addition of the binomial approach (27). One of the advantages of using this approach is to determine the early exercise property of the American option. The trinomial model can be formulated by recombining the trinomial tree. To make a recombining trinomial tree, a method was presented to estimate a time-homogeneous diffusion process (28). The stock price is lighting the trinomial model to pass in three various directions at every time step in Figure 3, for example $S(0) = S_0 > 0$ and the following dynamics are considered to formulate the trinomial option pricing model:

$$S(t) = \begin{cases} S(t-1)\ e^{u}, \text{probability } p_u \\ S(t-1)\, e^{m}, \text{probability } p_m = 1 - p_u - p_d, for\ t \epsilon I \\ S(t-1)\ e^{d}, \text{probability } p_d \end{cases} \qquad [9]$$

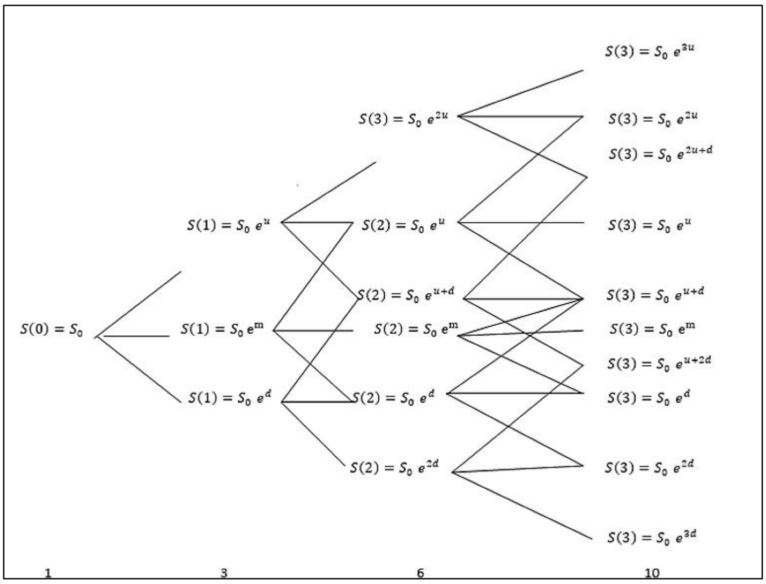

Figure 3: Trinomial Price Path When (u ≠ d)

Figure 3 represents a trinomial model where the path of stock prices has three directions. There are three possible directions for each time. The possible number of prices are $(t + 1)\ (t + 2)/2$ at time t, and in this case, we obtained a quadratic rate of growth for the trinomial tree.

In the case of the binomial model, it was linear growth. Where $u > m > d$, $p_u, p_m, p_d \epsilon (0, 1)$ and $p_u + p_m + p_d = 1$, (p_u, p_m, p_d) is called the physical probability. A system of equations for the trinomial model is introduced and discussed in this section. The admissible values have been formulated for the trinomial model by using the probability theory. There are so many portfolios in the existing trinomial option pricing model which are parameterized by arbitrary constants. The options market fluctuates continuously due to the good number of arbitrary constants. As a result, short sellers benefit within a short period for which the market becomes vulnerable. The number of nodes created for these arbitrary constants should be reduced for the betterment of the options market. The following recombination conditions can be imposed to minimize the number of nodes:

$$m = \frac{u+d}{2} \qquad [10]$$

Hence the trinomial model is restricted to the following form. Therefore, we obtained a linear growth rate like the Binomial model in Figure 4.

$$S(t) = \begin{cases} S(t-1)\ e^{u},\ \text{probability } p_u \\ S(t-1)\, e^{\frac{u+d}{2}}, \text{probability} p_m = 1 - p_u - p_d, for\ \ t \epsilon I \\ S(t-1)\ e^{d},\ \text{probability } p_d \end{cases} \qquad [11]$$

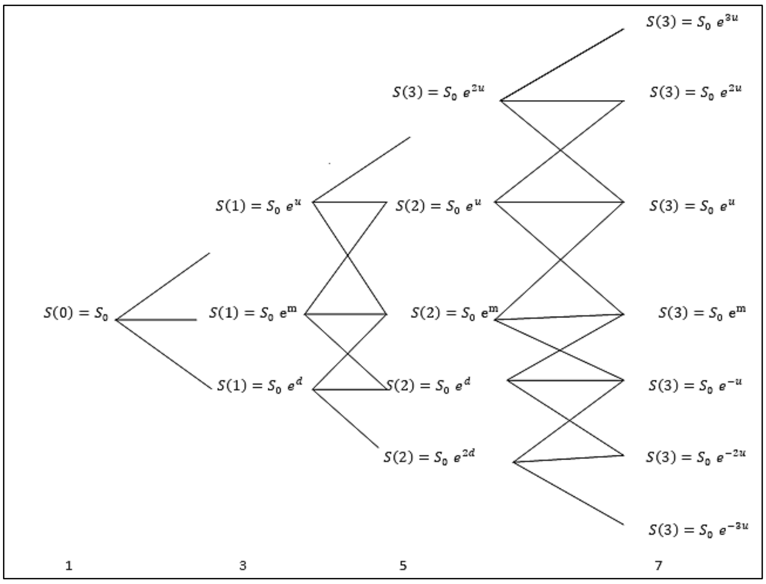

Figure 4: Stock Price in The Trinomial Tree When (u= —d)

Figure 4 draws the recombining trinomial model. Since there is a quadratic growth in trinomial models, there is arbitrage opportunity. As a result, trinomial models are restricted here and thus create a linear growth of stock price.

Therefore, the sum of the probability that follows the path *x* is

$$\sum_{x\in\{-1,0,1\}^N} P(S^x) = 1 \qquad [12]$$

Probabilistic Interpretation

When studying the trinomial model in further depth, it becomes necessary to build a probability theory formulation for the model. We can analyze the convergence of the stock price to the Geometric Brownian Motion by defining the stock price (in the trinomial model setting) as a stochastic process, as we will do in this section. The martingale condition, which we also derive here, will be used to prove convergence.

Probabilistic Formulation

Given, Ω = x = $(x_1, ...x_N)$ ϵ { -1, 0, 1} ^{N} define, S (t, x) = S_0 exp $(x_1 + x_2 + ... + x_t)$ u, t ∈ I. The vector S^x = (S (1, x), ..., S (N, x)) is called the x - path of the stock price. Given p = (p_u, p_m, p_d) such that 0<p_u, p_m, p_d<1 and $p_u + p_m + p_d$ = 1. The probability that the stock follows the path x is defined by on the sample space Ω is **P**p (ω) = **P** (S^x) = $(p_u)^{N+(x)}$ $(p_d)^{N-(x)}$ $(1- p_u - p_d)^{N_0(\omega)}$ = $(p_u)^{N+(\omega)}$ $(p_d)^{N-(\omega)}$ $(p_m)^{N_0(\omega)}$. Where $N_+(\omega)$ is the number of +1 in x, N_- (ω) the number of -1 in x and $N_0(\omega)$ =N – N-(ω) - $N_+(\omega)$ the number of 0's in x in the sample ω. The stock price of a trinomial model is defined as a stochastic process within the probability space (Ω, $\mathbf{P}_p$). This stochastic processes of $\{X_t\}_{t=1,...,N}$ can be defined as ω = $(\Upsilon_1, ..., \Upsilon_N)$ ∈Ω as X (ω)=Υ_t that is

$$X_t(\omega) = \begin{cases} 1 \ \ if \ \ \gamma_t = 1 \\ 0 \ \ if \ \ \gamma_t = 0 \\ -1 \ if \ \gamma_t = -1 \end{cases} \qquad [13]$$

The above equation represents probabilistic formulation of the trinomial model. If X_1, ..., X_N are the random variables which are independent and identically distributed. So, we have from equation [11] is

$$S(t) = S(t-1)exp\left[\left(\frac{u+d}{2}\right) + X_t\left(\frac{u-d}{2}\right)\right] \qquad [14]$$

For stock price of trinomial model at time t=1, 2, …., N is

$$S(t) = S(0)exp\left[t\left(\frac{u+d}{2}\right) + Z_t\left(\frac{u-d}{2}\right)\right], Z_t = X_1 + X_2 + \cdots + X_t, \Omega_N = R \qquad [15]$$

Assuming Z_0 = 0, that means $\{S_t\}_{t=0...N}$ is measurable for $\{Z_t\}_{t=0...N}$.

Theorem

Probability measure **P** is martingale if $p = q = (q_u, q_m, q_d)$, where (q_u, q_m, q_d) be a triple of real number satisfying.

$q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d = e^r$ [16]

$p_u + p_m + p_d = 1$ [17]

$0 < p_u, p_m, p_d < 1$ [18]

Proof of Theorem: Discounted value of an asset indicates the current value of an asset, therefore the condition on q_u, q_m and q_d we impose towards sample space Ω_N produce trinomial price is martingale. As we know from the expected value of S(t) is

$$\mathbb{E}[S(t)] = S_0 \prod_{i=1}^{t} \mathbb{E}[x_i] = S_0 \prod_{i=1}^{t} \left(q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d\right) = S_0 \left(q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d\right) \mathrm{t} \quad [19]$$

Where, S (t-1) is known from expected valve of S(t)

$$E[S(t)|S(t-1)] = E[S_0\, t \prod i = 1\, x_i | S(t-1)] = S_0 X(t-1) E[x_i] = S_0 X(t-1)\left(q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d\right)$$

Now, we can write

$$E[e^{-rt} S(t)] = S_0 e^{-rt} t \prod i = 1\ E[x_i] = S_0 e^{-rt} X(t-1)\left(q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d\right) t \quad [20]$$

Similarly, as from above, we can write,

$$E[e^{-rt} S(t)|S(t-1)] = S_0 e^{-rt} X(t-1) E[x_i] = S_0 e^{-rt} X(t-1)\left(q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d\right) \quad [21]$$

The equation satisfies the martingale condition

$E[e^{-rt} S(t)|S(t-1)] = e^{-r(t-1)} S(t-1)$ and $E[e^{-rt} S(t)] < \infty$ if and only if

$$q_u e^u + q_m e^{\frac{u+d}{2}} + q_d e^d = e^r$$

Self-Financing Portfolio

As mentioned earlier, there exist many portfolios in the trinomial model that are parameterized by arbitrary constants $q_0 \in R$. Now it demonstrates that the European option acceptable price is not defined uniquely identified in the framework. Considering V*(N)* as a self-financing portfolio, $[\{h_s(t), h_B(t)\}_{t \in I}]$ value at time *N*. Now let us assume that the price of a risk-free asset in the money market at the time is B(t) and the interest rate r ≥0. So that

$B(t) = B_0 e^{rt}$ [22]

By recalling a portfolio $[\{h_s(t), h_B(t)\}_{t \in I}]$ invested in stock and risk-free money is called self-financing if

$h_s(t) S(t-1) + h_B(t) B(t-1) = h_s(t-1) S(t-1) + h_B(t-1) B(t-1)$ [23]

It means that the value changes in the portfolio indicate only the changes in the position within the portfolio. Here we do not consider the risk in an asset. We consider risk-neutral probabilities q_u,$q_m$and q_d to adjust the probabilities. Having three variables and only two equations, the values of q_u,$q_m$and q_d are not uniquely defined. Choosing qm as an independent variable, it may solve of the above system by the form

$$q_u = \frac{e^r - e^d}{e^u - e^d} - \omega \frac{e^{\frac{d}{2}}}{e^{\frac{u}{2}} - e^{\frac{d}{2}}}, \quad q_m = \omega, \quad q_u = \frac{e^u - e^r}{e^u - e^d} - \omega \frac{e^{\frac{u}{2}}}{e^{\frac{u}{2}} - e^{\frac{d}{2}}}$$

Market parameters r, u, d and free parameter ω satisfy the above equation. The limit ω--->0 impose trinomial model to converge with martingale measure with the condition. r<u and $0 < \omega < \frac{e^u - e^r}{e^u - 1}$. Where r>0, u>0, and u=-d are not arbitrage. To compute trinomial stock price in some intervals where T>0. The interval [T,0] is classified into N divisions having length of h=T/N, where h is smaller. Fair price of the derivatives in the trinomial model is defined by self-financing hedging portfolios value. In the trinomial model, a self-financing portfolio value is not defined uniquely. That is why we can say that the trinomial model market is incomplete. Therefore, the derivative price depends on the parameter ω can be used for better calibration.

Results and Discussion

Black-Scholes (B-S) PDE or terminal value problem is considered to avoid the arbitrage opportunity. The B-S model may be integrated with trinomial option pricing, both analytically and numerically, for European options (29). Assume that $p_u = p_d = p$, where p_d and p_u represents the probabilities and the underlying price of assets going down and up. For simplification of the calculation, let $\alpha = 0$ in Geometric Brownian Motion (GBM), where α is the instantaneous means of return of the Black-Scholes. The trinomial model convergence to the B-S model at leading order in h if and only if

$$u = \sigma\sqrt{\frac{h}{2}p} \qquad [23]$$

And

$$q_0 = 1 - 2p, \quad p_u = p_d = p \in \left(0, \frac{1}{2}\right). \qquad [24]$$

Numerical Convergence

The numerical investigation for the convergence rate of trinomial option price for European Options via B-S has been investigated here. Table 1 tabulated the error values for different probabilities, *p,* and different numbers of steps, *N*. Error-values are calculated from the difference between the trinomial and the B-S option pricing values. The values used are *T = 10/252, N = 50, S₀ = 10, r = 0.01, K = 10, σ = 0.2*. The B-S price has been found by using MATLAB function *blsprice*. For *p = 0.5,* $q_0 = 0$, this assumption is valid for the binomial price.

Follows: $\frac{T-BS}{T}100\%$

Table 1: The Error Values of the Trinomial Option Price for Different *p* and *N*

N	*p=0.1*	*p=0.2*	*p=0.3*	*p=0.4*	*p=0.5*	B-S price
10	5.74561	1.91145	0.80850	0.294527	2.49535	0.1609
15	3.60727	1.26043	0.54247	0.192530	1.63087	0.1609
20	2.63560	0.94031	0.40811	0.147426	1.24162	0.1609
25	2.07842	0.74989	0.32708	0.119062	0.98215	0.1609
30	1.71628	0.62362	0.27289	0.099862	0.82625	0.1609
35	1.46180	0.53374	0.23410	0.085988	0.70260	0.1609
40	1.27312	0.46651	0.20497	0.075496	0.61911	0.1609
45	1.12762	0.41432	0.18228	0.067285	0.54692	0.1609
50	1.01199	0.37264	0.16412	0.060685	0.49505	0.1609

Table 1 shows error diminishes swift for the trinomial framework in case of *p = 0.2*, *p = 0.3*, and *p = 0.4* compared to binomial framework, where *p = 0.5*. However, when p = 0.1 error is higher than binomial framework for *p = 0.5*. This convergence error is less than the binomial model for a set of steps, *N = 50,* as shown in Figure 5. The values of *T = 10/252, N = 50, S₀ = 10, r = 0.01, K = 10, σ = 0.2* are considered for numerical convergence that gives the trinomial model to have less error with *p* over the interval [0.17, 0.49]. The error is higher in the trinomial model when $p \to 0$.

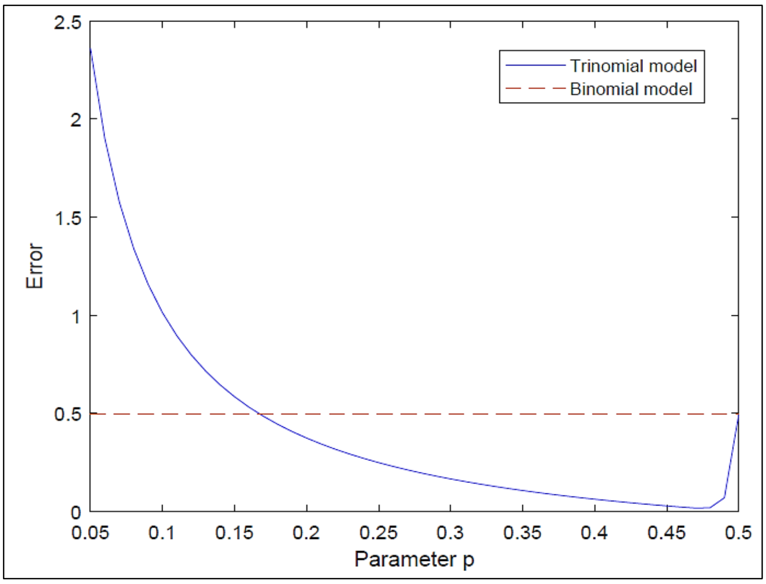

Figure 5: Error for Trinomial Option Pricing Framework with Different *p Value*

Figure 5 describes the numerical convergence error of binomial & trinomial model. The blue line represents a trinomial model whereas red dot line is for binomial. The convergence error is decreasing on and over *p=0.17* and the error is minimized at point *p=0.49*.

Comparison in Convergence for American Options

Since no exact formula is found for the B-S price of the American put, showing the convergence of the trinomial price to it is not simple. The comparison was carried out by comparing the binomial and the trinomial prices. The convergence comparison of the trinomial and binomial prices for the American put option is shown in Figure 6. The figure is drawn by using values of $T = 10/252$, $S_0 = 10$, $K = 10$, $r = 0.01$, $\sigma = 0.2$ in the trinomial model. The value, $p = 0.5$, is considered for getting the binomial price for the American put. It is seen that the trinomial pricing model converges quicker than the binomial option pricing with *P=0.4*. The trinomial model converges on $p = 0.4$, whereas the binomial framework converges on $p = 0.5$ with an identical American put option.

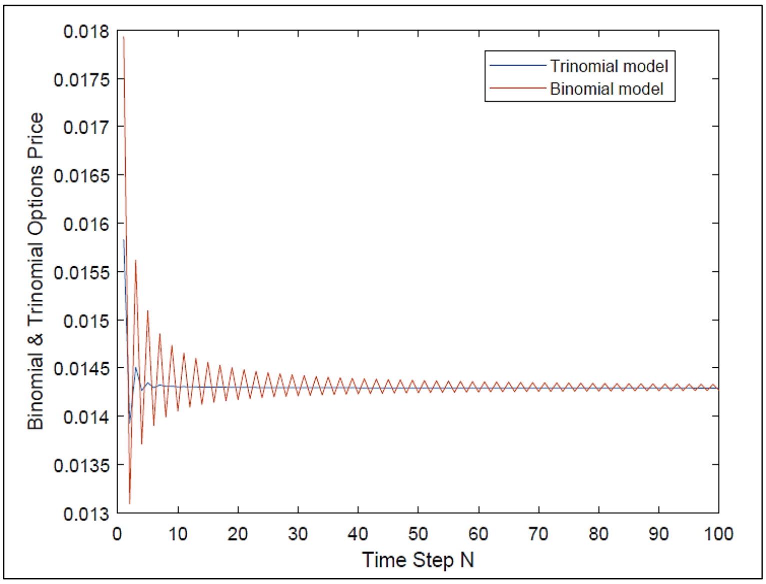

Figure 6: The Convergence of American Put ($N \to \infty$) For the Trinomial and Binomial Model

Figure 6 depicts the convergence comparison between binomial model and trinomial model. The blue line represents trinomial model where the red line represents binomial model. In the figure the blue line convergences at point *p=0.4* and the red line convergences at point *p=0.5*. This shows that trinomial model convergence faster than binomial model.

It was found that the trinomial pricing converges to the B-S pricing of perpetual American put, as shown in Figure 7. The closed B-S price of perpetual put is 3.849001795, where the values of the parameters are $S_0 = 10, K = 10, r = 0.01, \sigma = 0.2$ and $p = 0.4$, which are used in the trinomial model.

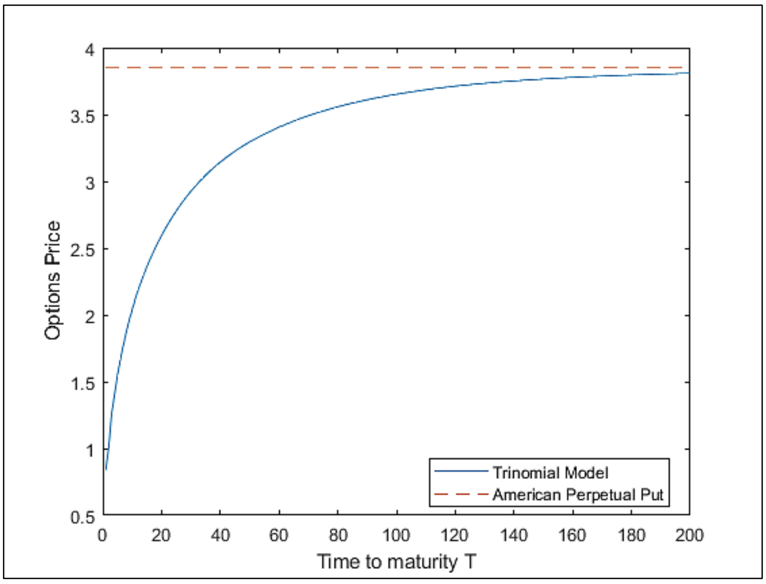

Figure 7: Trinomial and Black-Scholes Price for Perpetual American Put Convergence As $T \rightarrow \infty$

Figure 7 shows the trinomial price, which converges to the closed formula of Black-Scholes when the expiration time tends to infinity. Thus, as seen in the figure, the trinomial option pricing model associated with the new system of equations provides an accurate valuation for pricing the American put option.

Comparative Analysis

Binomial Model: A comparative table illustrates Convergence rates, error margin and execution time in relation to conventional models. It was found that the convergence between binomial pricing and B-S pricing as shown in Figure 8. The values of the parameters are $S_0 = 10, K = 10, r = 0.01, \sigma = 0.2$ which are used in the binomial model. We got Black-Scholes Price: 0.1609, CRR Binomial Model Performance: Final Price (n=500): 0.1608, Final Error: 0.000079, Final Execution Time: 0.009615 seconds, Convergence Rate: 0.9994 (approximate order of convergence).

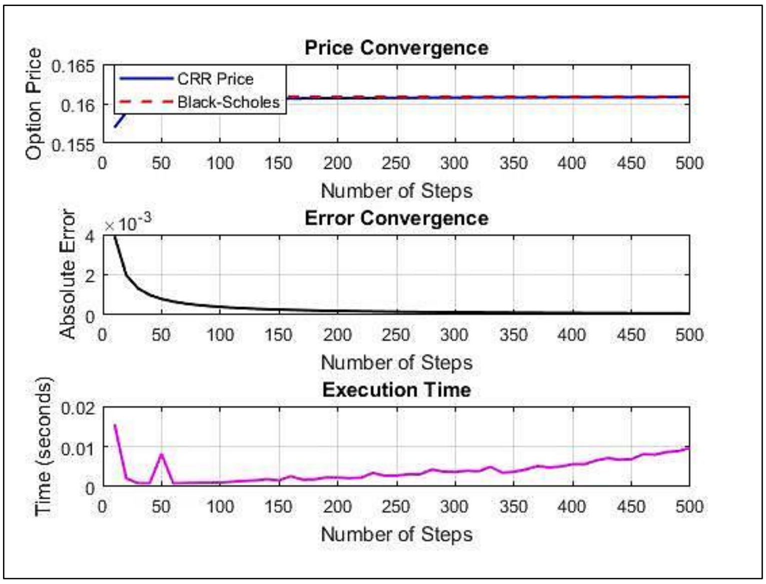

Figure 8: Comparative Figure Illustrating Convergence Rates, Error Margin and Execution Time Between Binomial and Black-Scholes Model

This figure shows the binomial convergence parameters. The parameters are price convergence, error convergence and execution time.

Table 2: Comparison Convergence Rates, Error Margin and Execution Time for Trinomial Model

Steps	Price	Error	Time(ms)	Conv Rate
10	0.069646	0.001829	8.03	NaN
13	0.070079	0.001396	2.25	1.03
17	0.070413	0.001062	0.74	1.02
23	0.070694	0.000781	0.67	1.01
31	0.070897	0.000578	32.57	1.01
40	0.071028	0.000447	0.39	1.01
53	0.071138	0.000337	0.49	1.01
71	0.071224	0.000251	0.53	1.00
94	0.071285	0.000190	10.59	1.00
124	0.071331	0.000144	0.98	1.00
164	0.071366	0.000109	7.25	1.00
216	0.071393	0.000082	1.55	1.00
286	0.071413	0.000062	133.91	1.00
378	0.071428	0.000047	5.72	1.00
500	0.071439	0.000036	7.55	1.00

Trinomial Model:
In Table 2, the values of the parameters are $S_0 = 10$, $K = 10$, $r = 0.01$, $\sigma = 0.2$ which are used in the trinomial model. Option Type: CALL, Black-Scholes Price: 0.071475. Trinomial model Performance: Final Price (n=500): 0.071439, Final Error: 0.000036, Final Execution Time: 7.55 (ms), Convergence Rate: 1.004 (approximate order of convergence). From table 2 it has shown that trinomial model takes 0.00755 seconds for convergence, but binomial model takes 0.009615 seconds to converge. So, the numerical comparison shows that trinomial models converge faster than binomial models.

Stability

From the numerical convergence, it is seen that the trinomial model converges with the B-S when 'h' approaches zero. It is not possible to get accurate results by using the recurrence formula. The rate of convergence is considered as an error value. The trinomial model will be stable if these errors remain bounded when the step size tends to zero. For reasonable stability, it is indeed to restrict some inequalities. Suppose e is the difference between the trinomial and the B-S price at the time $t = 0$. By letting $e_{i,j} = \alpha^i e^{j\sqrt{-1}\omega}$, e will then remain bounded if the following inequalities hold: $(1/\alpha) \le 1$ and $1 - 4p\sin^2(\omega/2) \le 1$. Moreover, these inequalities will remain the same if $p \le 1/2$.

Conclusion

A system of equations was developed for the trinomial option pricing model to increase its efficacy. The newly developed system of equations involved a good number of nodes that were parameterized by arbitrary constants. To eliminate the arbitrary constants, the nodes were reduced by linearizing the system of equations. Since these nodes are not identical, the use of decreasing nodes becomes confusing. Therefore, future research can be conducted for this non-identical node in the trinomial model. To measure the performance of trinomial option pricing, a numerical investigation was made for the American put option to compare with other option pricing, namely Black-Scholes and binomial pricing. The convergence test demonstrated that the trinomial pricing model shows more accuracy for the American put option. It was also found that the trinomial pricing is faster than the binomial for the same stock. Therefore, the trinomial model performs better than any other model, and it is more convenient when the option value remains stable. Since institutional and individual investors use option pricing for hedging their risk, the trinomial model would be the best option for them due to its accuracy and fast pricing. As a result, investors can trade huge stocks for option trading, although the profit from options trading is negligible. The findings recommend using the trinomial option pricing for equity in troubled firms, natural resource companies, stockholders, and the people involved in the financial markets.

Abbreviations

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Author Contributions

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Conflict of Interest

The authors declare no conflict of interest.

Declaration of Artificial Intelligence (AI) Assistance

The authors declare no use of artificial intelligence (AI) for the write-up of the manuscript.

Ethics Approval

Not applicable.

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References

1. Timothy J. The Financial Revolution of the Late Seventeenth Century. Ethics in Quantitative Finance. 2017. DOI: https://doi.org/10.1007/978-3-319-61039-9
2. Robert CM. Influence of mathematical models in finance on practice: past, present and future. Philosophical Transactions of the Royal Society A. 1994; 347: 451-463. DOI: https://doi.org/10.1098/rsta.1994.0055.
3. Paul HC. The Random Character of Stock Market Prices. The MIT Press. 1969; WEB: https://mitpress.mit.edu/9780262530040/the-random-character-of-stock-market-prices/
4. A James B. Elements of a Theory of Stock-Option Value. Journal of Political Economy. 1964; 72(2). DOI: https://doi.org/10.1086/258885
5. John CC, Stephen AR, Mark R. Option pricing: A simplified approach. Journal of Financial Economics. 1979; 7(3): 229-263. DOI: https://doi.org/10.1016/0304-405X(79)90015-1
6. Fischer B, Myron S. The Pricing of Options and Corporate Liabilities. Journal of Political Economy. 1973; 81(3): 637-654. http://www.jstor.org/stable/1831029
7. Iñigo A, Beatriz S, Daniel S, Carlos V. PDE models for American options with counterparty risk and two stochastic factors: Mathematical analysis and numerical solution. Computer & Mathematics with Applications. 2020; 79(5): 1525-1542. DOI: https://doi.org/10.1016/j.camwa.2019.09.014
8. Edeki SO, Jena RM, Chakraverty S, Baleanu D. Coupled transform method for time-space fractional Black-Scholes option pricing model. Alexandria Engineering Journal. 2020 Oct 1;59(5):3239-46. DOI: https://doi.org/10.1016/j.aej.2020.08.031
9. Rajarama MJ, S. Chakraverty. A new iterative method based solution for fractional Black–Scholes option pricing equations (BSOPE). SN Applied Sciences. 2019; DOI: https://doi.org/10.1007/s42452-018-0106-8
10. Qijie J, Zhuoyao X, Yue L, Sheng M, Qianyou Z. Differential dynamic decision-making model for multi-stage investment of scenic area. 2020; 59(4): 2819-2826. DOI: https://doi.org/10.1016/j.aej.2020.06.028
11. Grzegorz K, Marcin M, Łukasz P. A weighted finite difference method for subdiffusive Black–Scholes model. Computers & Mathematics with Applications. 2020; 80(5): 653-670. DOI: https://doi.org/10.1016/j.camwa.2020.04.029
12. Pradip R. A high accuracy numerical method and its convergence for time-fractional Black-Scholes equation governing European options. Applied Numerical Mathematics. 2020; 50: 472-493. DOI: https://doi.org/10.1016/j.apnum.2019.11.004
13. Mahboubeh S, Longsheng C. Probabilistic approach for optimal portfolio selection using a hybrid Monte Carlo simulation and Markowitz model. 2020; 29(5): 3381-3393. DOI: https://doi.org/10.1016/j.aej.2020.05.006
14. Phelim PB. Option valuation using a tree-jump process. International Options Journal. 1986; 3: 7-12. https://cir.nii.ac.jp/crid/1572261550510527744
15. Phelim PB. A Lattice Framework for Option Pricing with Two State Variables. Journal of Financial and Quantitative Analysis. 1988; 23(1). DOI: https://doi.org/10.2307/2331019.
16. Phelim PB PP, Evnine J, Gibbs S. Numerical evaluation of multivariate contingent claims. The Review of Financial Studies. 1989; 2(2): 241-250. DOI: https://doi.org/10.1093/rfs/2.2.241
17. Phelim PB, Sok. HBumping Up Against the Barrier with the Binomial Method. The Journal of Derivatives. 1994; 1(4): 6-14. DOI: https://doi.org/10.3905/jod.1994.407891
18. Jianye L, Zuxin L, Dongkun L, Ruolei L. Study on the valuation method for overseas oil and gas extraction based on the modified trinomial tree option pricing model. Mathematical Problems in Engineering. 2020; 5: 1-15. DOI: https://doi.org/10.1155/2020/4803909
19. Xiaoping H, Jiafeng G, Tao D, Lihua C, Jie C. Pricing options based on trinomial markov tree. Discrete Dynamics in Nature and Society. 2014; 3: 1-7. DOI: https://doi.org/10.1155/2014/624360
20. Bhat HS, Kumar N. Option pricing under a normal mixture distribution derived from the Markov tree746 model. European Journal of Operational Research. 2012; 223(3): 764-764. DOI: https://doi.org/10.1016/j.ejor.2012.07.003
21. AitSahlia F, Lai TL. Valuation of discrete barrier and hindsight options. Journal of Financial Engineering. 1997; 6(2): 169-177.

https://scholar.google.com/scholar?cluster=6633353762311488825&hl=en&oi=scholarr
22. Yuen FL, Yang H. Pricing Asian options and equity-indexed annuities with regime switching by the trinomial tree method. North American Actuarial Journal. 2010; 14(2): 256-272. https://researchportal.hw.ac.uk/en/publications/pricing-asian-options-and-equity-indexed-annuities-with-regime-sw
23. Vinayak M, Panesar HA, Saulo QdS, Thulasiram RK, Thulasiraman P, Appadoo SS. Analyzing Financial Smart Contracts for Blockchain. 2018 IEEE International Conference on Internet of Things (iThings) and IEEE Green Computing and Communications (GreenCom) and IEEE Cyber, Physical and Social Computing (CPSCom) and IEEE Smart Data (SmartData). 2018. DOI:https://doi.org/10.1109/Cybermatics_2018.2018.00284
24. Wein-kai W. Building recombining trinomial trees for time-homogeneous diffusion processes. Journal of Computational and Applied Mathematics. 2020; 364: 112351. DOI: https://doi.org/10.1016/j.cam.2019.112351
25. Yu-Fen C, Ming-Hua H, Chenghsien T. Valuation and analysis on complex equity indexed annuities. Pacific-Basin Finance Journal. 2019; 57: 101175. DOI: https://doi.org/10.1016/j.pacfin.2019.101175
26. Fahed M, Tharam D, Elizabeth C. Computational intelligence applications to option pricing, volatility forecasting and value at risk. Studies in Computational Intelligence. 2017; 697. DOI: https://doi.org/10.1007/978-3-319-51668-4
27. Young SK, Stoyan S, Svetlozar R, Frank JF. Enhancing binomial and trinomial equity option pricing models. Finance Research Letters. 2019; 28: 185-190. DOI: https://doi.org/10.1016/j.frl.2018.04.022
28. Arlie OP, Xiaoying D. An introduction to mathematical finance with applications. Springer Undergraduate Texts in Mathematics and Technology (SUMAT). 2016; XVII: 483. WEB: http://ndl.ethernet.edu.et/bitstream/123456789/53301/1/Arlie%20O.%20Petters.pdf
29. John H. Options, futures, and other derivatives securities. University of Toronto. Prentice Hall. Englewood Cliffs, New Jersey 07632. WEB: https://personal.lse.ac.uk/ROBERT49/teaching/ph232/pdf/Hull-Ch1.pdf

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