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# Modelling of Vehicle Longitudinal Dynamics for Speed Control

Syed Idros<sup>1</sup>, Muhammad Abdullah<sup>1,\*</sup>, Siti Fauziah Toha<sup>2</sup>, Mohamad Syazarudin Md Said<sup>3</sup>

<sup>1</sup> Department of Mechanical and Aerospace Engineering, International Islamic University Malaysia, Jalan Gombak, 53100, Kuala Lumpur, Malaysia

<sup>2</sup> Department of Mechatronics Engineering, International Islamic University Malaysia, Jalan Gombak, 53100, Kuala Lumpur, Malaysia

<sup>3</sup> Department of Chemical and Environmental Engineering, Faculty of Engineering, Universiti Putra Malaysia, 43400, Selangor, Malaysia

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### ABSTRACT

Longitudinal dynamics control is one of the essential tasks for an autonomous vehicle, where it deals with speed regulation to ensure smooth and safe operations. To design a good controller, a simple yet reliable mathematical model is needed so that it can be used as a plant and to tune the controller. Although there are many types of mathematical models available in the literature, finding the right one for control application is essential. The model cannot be too complex and can be too simple. Thus, the main objective of this work is to derive a simple yet reliable vehicle longitudinal model so that it can be used as a simulation plant in MATLAB Simulink to test or tune various types of control algorithm's performance. The model consists of three main parts which are the vehicle body dynamics, simplified power train dynamic, and braking dynamic. To validate the reliability of the model, standard urban drive cycles will be used as a reference speed and a hierarchical PID control structure with inverse plant model is used to control the pedal inputs replacing the driver in simulation environment. Results show that the controller managed to track the drive cycle with an acceptable pedal pressing response which is between 40% throttle press and 20% brake press that in line with the normal operation of a vehicle. Although only simulation result is presented, the model can be used as a good starting point for further development and testing of different types of control algorithms for future work.

## 1. Introduction

Vehicle longitudinal dynamics refer to the motion and behaviour of a vehicle when it travels in the longitudinal axis. The motion includes factors such as acceleration, deceleration, speed, position, as well as the forces that are acting on the vehicle. There is a wide range of applications in the automotive industry that utilise vehicle longitudinal dynamics such as design and optimisation of vehicle performance, evaluation of vehicle safety, development of vehicle control system, and simulation analysis [1-3]. In general, there are various modelling approaches and levels of complexity in the model, which depend on the type of application. For design and optimisation work, a high-

\* Muhammad Abdullah.

E-mail address: mohd\_abdl@iiu.edu.my

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fidelity mathematical model is usually used to analyse the vehicle's performance for improving its fuel efficiency and safety requirements [4]. Nevertheless, this type of model will consume high computation time due to the usage of complex equations to retain the accuracy of the result. For control applications, a simple model is usually enough to ensure fast calculation and decision-making processes, especially if model-based control were to be implemented.

Based on the literature review, there are two ways to obtain a mathematical model which is via system identification and derivation using the first principal law. System identification is a process of building mathematical models based on observed data. The goal is to identify the underlying dynamics of a system, which may be difficult to measure directly. The process of system identification involves collecting data from the real system using sensors and then using mathematical techniques such as least squares regression, maximum likelihood estimation, and Bayesian methods to estimate the parameters of a model that best describes the observed data. Although there are many successful implementations to develop the vehicle longitudinal dynamics as reported by Jin *et al.*, [5], Bernardo *et al.*, [6] and Syahira *et al.*, [7], it should be noted that this technique can be quite challenging, where it requires careful consideration of the limitations and uncertainties of the data, as well as the choice of appropriate modelling techniques. Besides, the obtained model can only be used for a specific vehicle that has been trained by their data and it is quite difficult to generalise the model for different types of vehicles.

The second approach or the most common approach is by using the first principal modelling, which builds a mathematical model based on the fundamental physical laws. There are many examples of this model specifically for vehicle longitudinal dynamics that can be found in literature such as in the work of Cole *et al.*, [2], Gillespie [8], and Rajamani [9]. Nevertheless, the first principles modelling approach can be time-consuming and challenging, requiring a deep understanding of the physical principles and properties of the system being modelled. Besides, the determination of individual parameters such as masses, coefficients and others can be quite tedious. However, it provides a rigorous and accurate description of the system's behaviour that can be used to design and optimise control systems for a wide range of applications. Besides, if ones would like to develop a general model, this will be the best approach as one only needs to change the parameters according to the vehicle of interest.

In general, vehicle longitudinal dynamics has three major components, namely: vehicle body dynamics, powertrain dynamics and brake dynamics. Although many works have covered the derivation of these systems, finding a suitable one may be challenging. This is because different authors provide different kinds of simplification. For example, in the work of Amer *et al.*, [10], the vehicle body model includes the effect of bending while other's work such as Fauzi *et al.*, [11] neglected it. Some authors also improved the fidelity by including the dynamics of road slope [12] or by considering a full dynamic braking system which included a tire model that covers the effect of deflection and coefficient of friction [13]. As in powertrain dynamics, several authors have considered a simplified power loss dynamic relationship between the mechanical device ranging from the torque converter to the final drive [14]. While others just assumed a simple equation to sum all the power lost based on a look-up table [15]. Similar concepts can be found in describing engine dynamics where some used a complete engine equation such as [16], and others by using a lookup table based on the engine dyno chart [17]. Clearly, one needs to use a suitable strategy for one's application. As in this work, the focus is more on a model that can be used to develop a controller and thus a simplified, yet reliable model is needed that will only focus on longitudinal motion by neglecting the motion in other axes.

The application of mathematical model in automotive control system is very crucial. For instance, researchers have extensively considered the use of Model Predictive Control (MPC) due to its ability

to handle multiple constraints and predict future vehicle states. In the longitudinal dynamics, hybrid MPC approaches have been utilised for their efficiency in managing intelligent vehicle speeds and accelerations while considering external constraints and uncertainties that showcases significant improvements in handling dynamic traffic scenarios [18]. Similarly, for tackling the combined lateral and longitudinal dynamics, robust control methods like the super-twisting sliding mode controller have been applied. This method offers high robustness against model uncertainties and disturbances, crucial for maintaining path stability and preventing collisions [19]. Moreover, the integration of longitudinal collision avoidance with lateral stability has led to advanced adaptive control systems. These systems synergize multiple control strategies to simultaneously ensure collision prevention and vehicle stability [20]. In general, all these control methods require a reliable mathematical model to derive the control law or tune the required parameters.

Based on the previous discussion, it should be note that the main contribution of this paper is to develop a simple and reliable mathematical model for vehicle longitudinal dynamics that can be used a simulation plant to test and tune various control methods. The model of interest for the Internal Combustion Engine (ICE) based vehicle is derived based on the available parameters in MATLAB extension file [21], which is quite good for testing and validating a simple speed and distance control for Adaptive Cruise Control (ACC) development. A few modifications will be made to enhance its reliability to fit a real application. The paper is organized as follows: Section 2 discusses the methodology that covers the modelling work of each component along with its simplification and modification. Section 3 provides the hierarchical PID control architecture for simple speed control to track a standard drive cycle for validation purposes. Section 4 analyses and discusses the simulation results and Section 5 provides the conclusions.

## 2. Vehicle Longitudinal Dynamics Model

The model for vehicle longitudinal dynamics consists of three main parts as shown in Figure 1: vehicle body dynamics, powertrain dynamics, and braking dynamics. The inputs to the model are the percentage of throttle pressing  $x_t$  and percentage of brake pressing  $x_b$ . The output will be the vehicle speed  $v$  and the parameters of  $F_t$  and  $F_b$  both denote the states that consist of tractive force  $F_t$  and braking force  $F_b$ , respectively. The detailed derivation for each part is given in the following subsections, where the parameters and equations are based on these references [9,21,22].

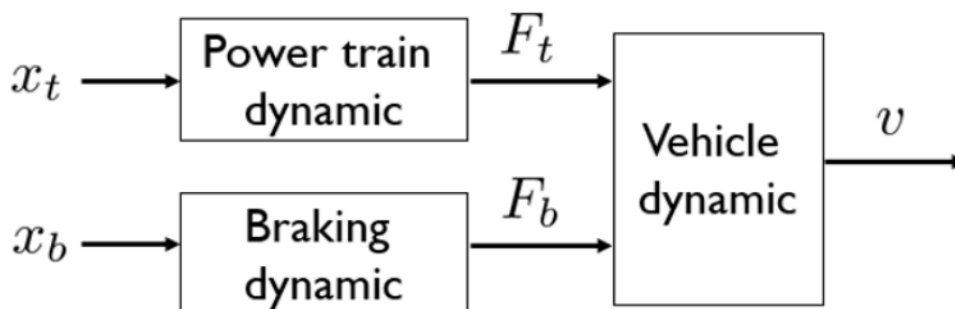
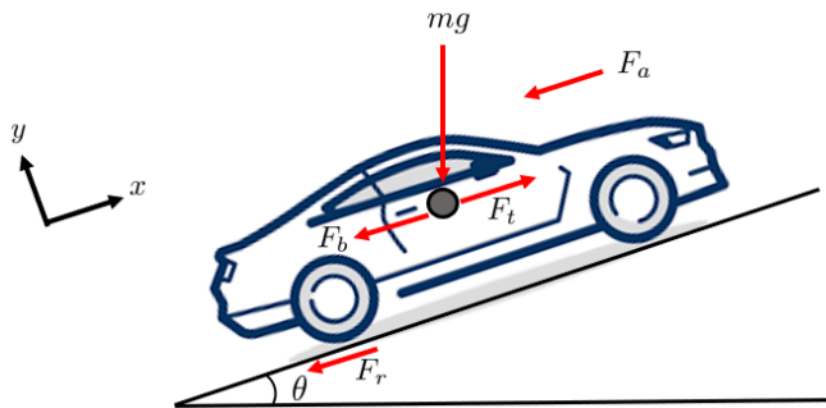


Fig. 1. Block diagram for longitudinal vehicle dynamic

### 2.1 Vehicle Body Dynamics

The vehicle body dynamic consists of the summation of forces that are acting on the car as shown in Figure 2. There are five main forces namely: total tractive force  $F_t$ , total braking force  $F_b$ , weight  $mg$ , total rolling resistance force  $F_r$ , and aerodynamic force  $F_a$ .



**Fig. 2.** Free body diagram for vehicle body [19]

Assuming the respective forces that developed in the tires can be summed directly, the equation of motion can be derived based on Newton's second law:

$$m \frac{dv}{dt} = F_t - F_b - F_r - F_a - mg \sin \theta \quad (1)$$

The inputs to this system are the total tractive force that is generated from the powertrain dynamics and the total braking force that is developed by braking dynamics. These inputs will be generated by their own equations which will be discussed in the later subsection. As for the aerodynamic force and rolling resistance force, it can be expressed as:

$$F_a = 0.5\rho AC_d(v + v_w)^2 \quad (2)$$

$$F_r = C_r mg \cos \theta \quad (3)$$

The description and value for parameters in Eq. (2) and Eq. (3) that are used in this work are given in Table 1. It should be noted that the rolling resistance force depends on the normal force that is generated by the car. To simplify the modelling process, only the normal force that is related to the weight is considered although more detailed calculations that involved the summation of the moment can be used for higher fidelity response [9].

**Table 1**

Parameters for vehicle body dynamics [21]

| Parameters                    | Value                   |
|-------------------------------|-------------------------|
| Mass of vehicle, $m$          | 1535 kg                 |
| Gravitational constant, $g$   | 9.81 m/s <sup>2</sup>   |
| Slope angle, $\theta$         | 0°                      |
| Rolling coefficient, $C_r$    | 0.015                   |
| Air density, $\rho$           | 1.202 kg/m <sup>3</sup> |
| Front cross-section area, $A$ | 1.88 m <sup>2</sup>     |
| Drag coefficient, $C_d$       | 0.31                    |
| Wind gust, $v_w$              | 0 m/s <sup>2</sup>      |

It should be mentioned that the parameters of slope angle and wind gust depend on the instantaneous environmental conditions. Currently, these parameters are fixed according to the values in the table, where it is assumed that the vehicle is traveling on a flat road without any wind gust. However, it can be changed to assess the robustness of a controller for future work.

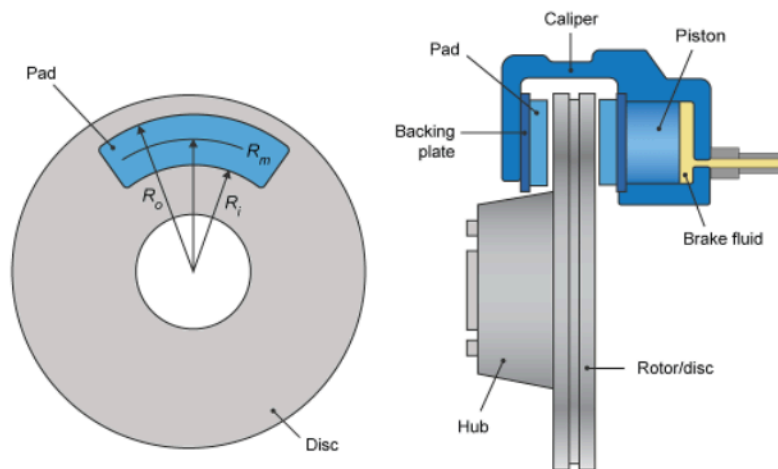
**Remark 1:** Technically, negative velocity means that the vehicle is moving in the reverse direction. To avoid the model from giving negative velocity, a minor modification on the set of conditions needs to be implemented based on the current value of brake pressing and velocity:

- i. If the brake is activated and velocity is negative then the net tractive force ( $F_t - F_b$ ) is set to zero, which represents the vehicle is in a complete stop.
- ii. Similarly, the rolling resistant force is set to zero if the current velocity is negative. This is to indicate that when the vehicle is not moving, the coefficient of kinetic friction should be zero.

## 2.2 Brake Dynamics

For simplicity, the brake dynamic can be considered to have a linear relationship between the input brake pressing  $x_b$  and output brake force  $F_b$ . The equation for brake can be represented as in Eq. (4) by referring to the schematic diagram in Figure 3.

$$F_b = \frac{\mu_k P \pi D_b^2 R_m N}{4 R_w} \quad (4)$$



**Fig. 3.** Schematic of disk brake system [18]

where  $\mu_k$  is the disk pad coefficient of kinetic friction,  $P$  is the applied brake pressure,  $D_b$  is the brake actuator bore diameter,  $R_m$  is the mean radius of brake pad force on the brake rotor,  $N$  is the number of brake pad assembly  $R_w$  is the wheel radius. Based on Eq. (4), it can be noted that all the parameters other than  $P$  are constant. Empirically,  $P$  can be directly mapped to the percentage of pedal pressing  $x_b$ , thus the equation can be reduced to:

$$F_b = k_b x_b \quad (5)$$

Here, the  $k_b$  is assumed as a brake constant that consists of all the parameters other than  $P$  in Eq. (4). The brake constant can be obtained by using a brake testing machine. However, in this work for simplicity, the constant  $k_b$  value is assumed arbitrarily to be 100 just to get a direct map between pedal pressing and brake force.

### 2.3 Powertrain Dynamics

The powertrain dynamics consist of many interconnecting subsystems such as engine torque map, gear scheduling logic, engine speed estimator, and driveline. Figure 4 shows the overall block diagram for powertrain dynamics where the input is throttle pressing  $x_t$  while the output is the traction force  $F_t$ . Note that the velocity  $v$  in the figure is taken from the output from the vehicle body dynamic model in Section 2.1.

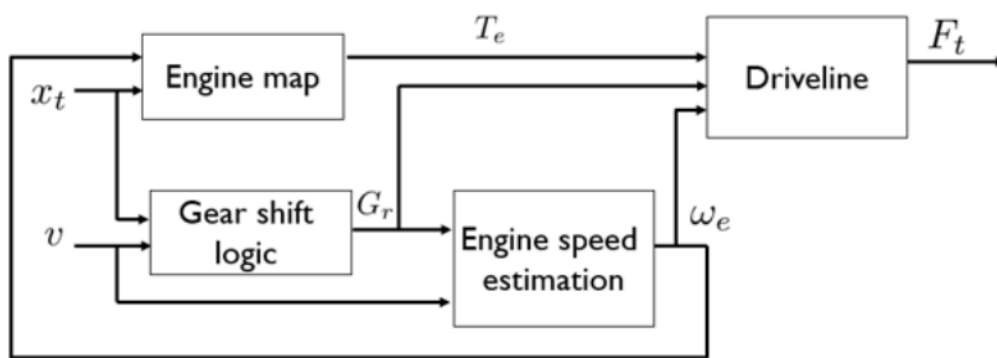


Fig. 4. Block diagram for powertrain component

#### 2.3.1 Engine torque map

The engine torque is estimated based on the current engine speed,  $\omega_e$  and its Brake Mean Effective Pressure ( $BMEP$ ) using the relationship below [18]:

$$T_{BMEP} = \frac{BMEP(V_{disp})}{4\pi} \quad (6)$$

where  $V_{disp} = 0.0053$  l is the engine vehicle's stroke displacement measured in litres. Note that the  $BMEP$  can be obtained from a correlation between the  $BMEP$  and the  $\omega_e$  through experiment data as shown in Figure 5 by a simple lookup table [9,21].



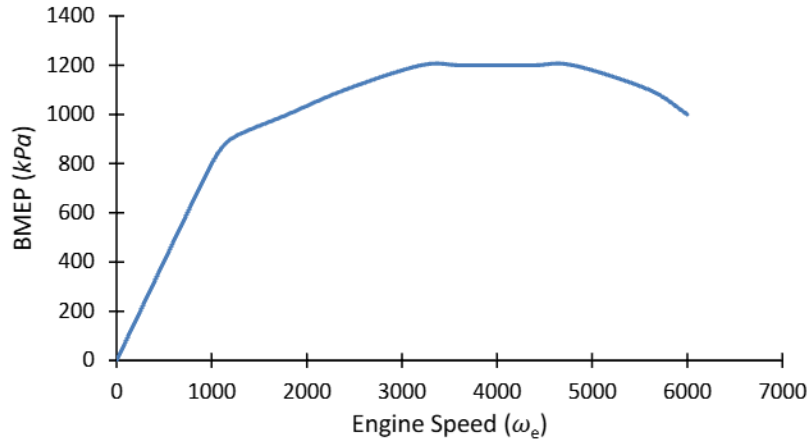


Fig. 5. Typical BMEP vs engine speed curve

It is important that the  $T_{BMEP}$  obtained from Eq. (6) does not exceed the specified maximum engine torque calculated based on the maximum engine power, given by [21]:

$$T_{e,lim} = \frac{P_{e,max}}{\omega_e} \quad (7)$$

Where in this case,  $P_{e,max} = 280000$  W is assumed as the maximum engine power. This value is usually specified in the vehicle's technical specification. Thus, the maximum engine torque  $T_e$  at a given pedal position,  $x_t$ , is given by:

$$T_{e,lim} = \frac{P_{e,max}}{\omega_e} \quad (8)$$

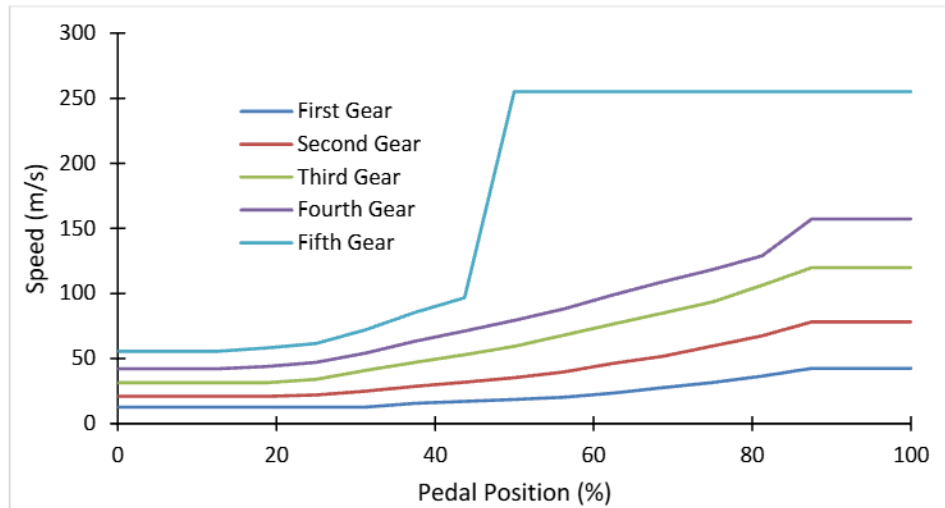
where  $T_{e,max}$  is set by taking the minimum value between the two as formulated in Eq. (9).

$$T_{e,max} = \min(T_{BMEP}, T_{e,lim}) \quad (9)$$

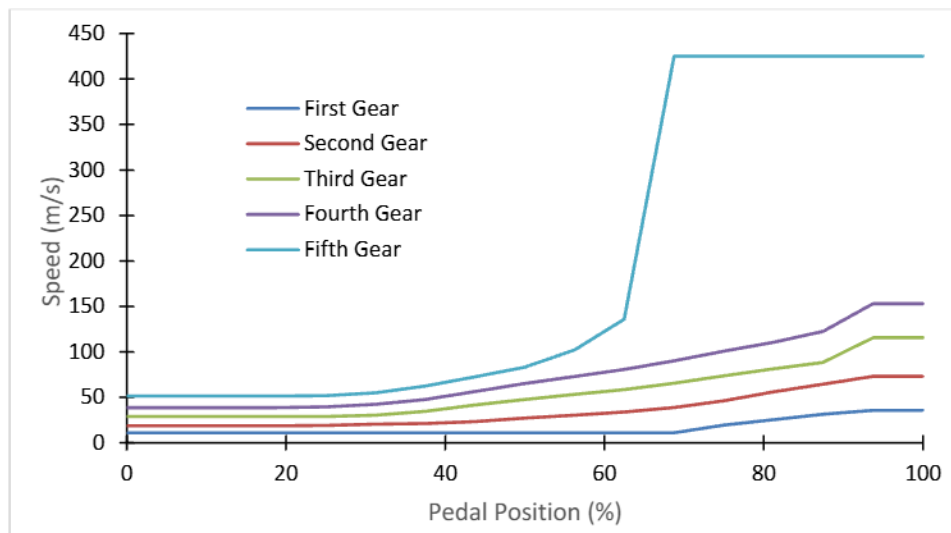
**Remark 2:** When the vehicle is stationary or not moving i.e.,  $v = 0$  m/s, the engine torque should be  $T_e = 0$  since  $x_t = 0$ . This is because although the engine is still running, the torque is not transferred to the drive line. Thus, it is important to include this rule in the Simulink model.

### 2.3.2 Gear shift logic

In ICE vehicles, a gear ratio needs to be changed to provide optimum torque to the driveline. This is because the engine can only provide maximum torque at a specific engine speed and hence a suitable gear ratio is needed for different driving speeds. Since the real gear logic shift is quite difficult to obtain from a manufacturer, a default logic from the MATLAB example is used in this work [21]. Assuming a car has 6 different gears including the final drive, the upshift and downshift logics as shown in Figures 6 and 7, respectively can be used. These logics are based on the current measurement of speed and throttle pressing. Again, a simple lookup table can be used to obtain a suitable value based on these parameters.



**Fig. 6.** Upshift gear schedule



**Fig. 7.** Downshift gear schedule

For increasing vehicle speeds (typically a result of vehicle acceleration), an upshift gear schedule (shown in Figure 6) was followed. Conversely, decreasing engine speeds will observe a downshift gear schedule, as shown in Figure 7. Table 2 provides the values for each gear including the final drive ratio.

**Table 2**  
Value for respective gear ratio [21]

| Parameters  | Value |
|-------------|-------|
| Gear 1      | 4.47  |
| Gear 2      | 2.47  |
| Gear 3      | 1.47  |
| Gear 4      | 1     |
| Gear 5      | 0.8   |
| Gear 6      | 0.65  |
| Final Drive | 3.4   |



### 2.3.3 Engine speed estimation

In actual operation, the engine speed can be directly measured from a sensor in a vehicle. However, when developing a mathematical model, this parameter needs to be estimated. Thus, the standard equation below can be used [9]:

$$\omega_e = \frac{vG_r}{R_w} \quad (10)$$

where  $G_r$  is the overall gear ratio determined from Section 2.3.2 and  $R_w = 0.288$  m is the wheel radius. The engine speed also should be constrained to a minimum value of 0.001 rad/s since by default it will still move even during idling.

### 2.3.4 Driveline

To propel a vehicle, the torque generated from the engine needs to be transferred to the driveline. Commonly, transferred torque will be susceptible to mechanical losses due to heat, friction, and other factors. Mathematically, these losses can be estimated as [21]:

$$T_l = c_0 + \frac{c_1}{200} T_{d,i} + \frac{c_2}{2000} (\omega_e - 200) \quad (11)$$

where  $c_0=8$ ,  $c_1=10$ , and  $c_2=4$  are constants obtained from curve fitting from experimental data [18], and  $T_{d,i}$  is the engine torque at a specific gear, given by:

$$T_{d,i} = T_e G_r \quad (12)$$

Therefore, the net output torque can be calculated to be:

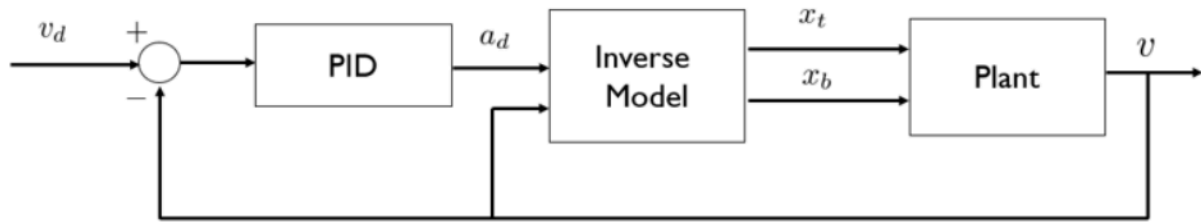
$$T_{d,o} = T_{d,i} - T_l \quad (13)$$

Finally, the traction force that will be fed to the vehicle dynamic block in Section 2.1 can also be estimated using Eq. (14). The maximum limit for this value is set to 5000 N [21].

$$F_T = \frac{T_{d,o}}{R_w} \quad (14)$$

## 3. Model Validation with Hierarchical PID Structure

In this work, the mathematical model is validated with the standard drive cycle to see whether it can track the reference velocity with an acceptable level of pedal pressing. For tracking the drive cycle in a simulation environment, a Proportional Integral Derivative (PID) controller is used to represent a driver. Usually, a PID controller can be tuned directly based on the mathematical model of a system. Nevertheless, as the model in Section 2 is nonlinear and has many interconnected subsystems, a hierarchical control structure can be utilized for ease of tuning and implementation as shown in Figure 8 [9,19].



**Fig. 8.** Hierarchical PID control structure

For this structure, the PID controller is tuned based on a simple kinetic vehicle model rather than the longitudinal model. Based on the error between the desired velocity  $v_d$  and actual velocity  $v$ , the PID controller will provide the desired acceleration to achieve the target. This signal is then passed to the inverse model where a reverse calculation based on the previous equations is used to calculate a suitable throttle and brake pedals pressing inputs to actuate the plant. A detailed explanation of each block is provided in the next subsections.

### 3.1 Tuning of PID Control

As discussed before, the PID control is tuned based on a simple kinematic relationship between the output velocity and input acceleration of a vehicle. Ideally, the demanded velocity should be equal to the output velocity, however in reality there will be some delay due to the power transfer from the engine to the wheel. According to Rajamani [9], a small delay of 0.5 s is often observed. Thus, the relationship between desired velocity and output velocity can be presented as a first-order transfer function:

$$v(s) = \frac{1}{0.5s + 1} v_d(s) \quad (15)$$

For ease of calculation in the inverse model, the relationship between the desired velocity  $v_d$  and output velocity  $v$ , can be converted to the desired acceleration  $a_d$  by a simple integrator  $1/s$ . Thus, the kinematic model in Eq. (15) can also be presented as:

$$v(s) = \frac{1}{s(0.5s + 1)} a_d(s) \quad (16)$$

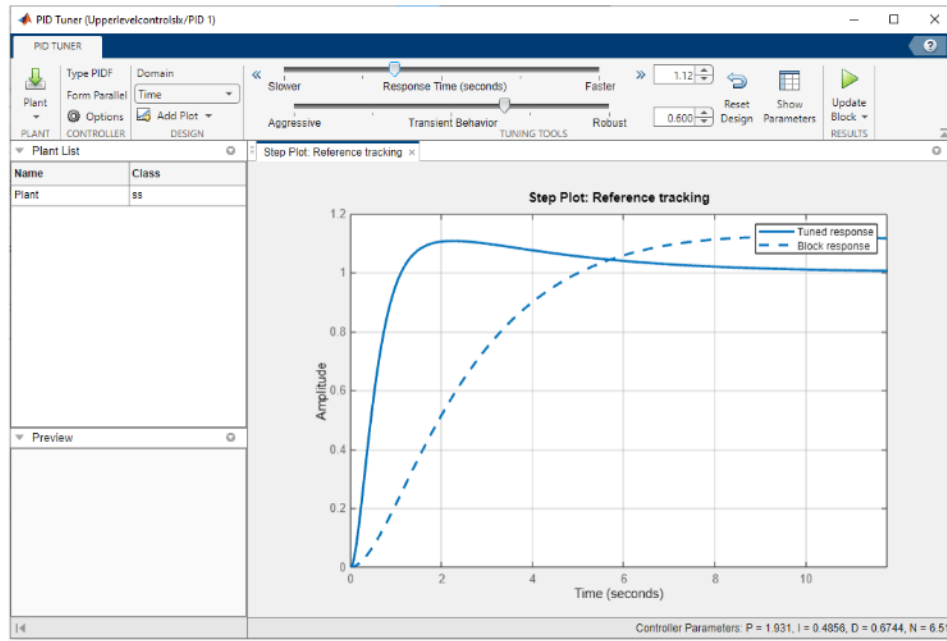


Fig. 9. Tuning apps for PID controller

With the transfer function in Eq. (16), tuning the PID controller should be straightforward. Various methods can be used such as Zigler Nichols, Cohen Coon, and others. Yet in this work, a PID tuner toolbox in MATLAB is utilised. This feature can provide a user with suitable gain values for  $k_p$ ,  $k_i$ , and  $k_d$  by visual inspection of their step response via adjusting the speed of convergence and robustness properties as shown in Figure 9 (refer to the two sliders). As the slider is moved, the step response also will change. Once suitable response and PID gains value are obtained, it can be used to control the plant by generating the acceleration signal.

### 3.2 Inverse Model

Based on this hierarchical structure, the PID controller only provides a desired acceleration and thus this signal needs to be converted to the actual inputs which are throttle and brake pressing. From Eq. (1) the net tractive force  $F_{net}$  can be calculated from the desired acceleration  $a_d$  and thus:

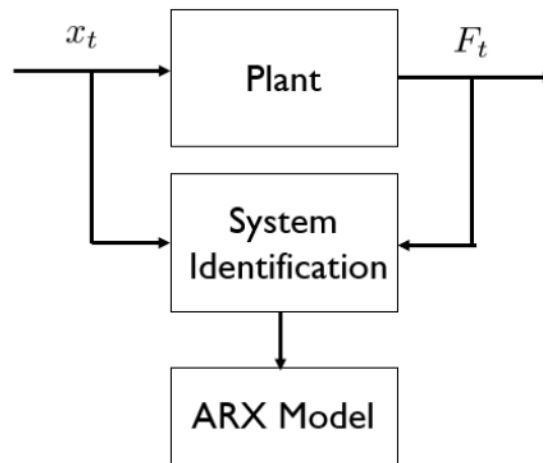
$$F_{net} = ma_d + F_r + F_a + mg \sin \theta \quad (17)$$

If the net force value is negative, then braking input should be applied by reversing Eq. (5) to get the  $x_b$  value. However, if net force is equal to zero, a minimum brake pressing equal to 5% is applied to overcome the default traction force that will come from a vehicle when no throttle pedal is applied to keep a vehicle from moving.

As when the net force is positive, the throttle pedal needs to be applied. Nevertheless, as can be seen in Section 2.3, the powertrain dynamic cannot be simply inverse since it has many interconnecting subsystems and logic rules. This issue has the potential to generate an algebraic loop error in the Simulink environment. To overcome this problem, a system identification model can be used to represent the powertrain dynamic in a simple discrete Auto-Regressive with eXogenous variables (ARX) model which has a form of:

$$y(k+1) = b_1u(k) + b_2u(k-1) - a_1y(k) - a_2y(k-1) \quad (18)$$

where  $k$  is the current sample,  $y$  is the output traction force and  $u$  is the input throttle pressing. The parameters  $a$  and  $b$  in the model will be identified based on a set of training data consisting of input and output with a sample time of 0.1 s. Figure 10 shows the overall system identification process.



**Fig. 10.** System identification process for powertrain dynamics

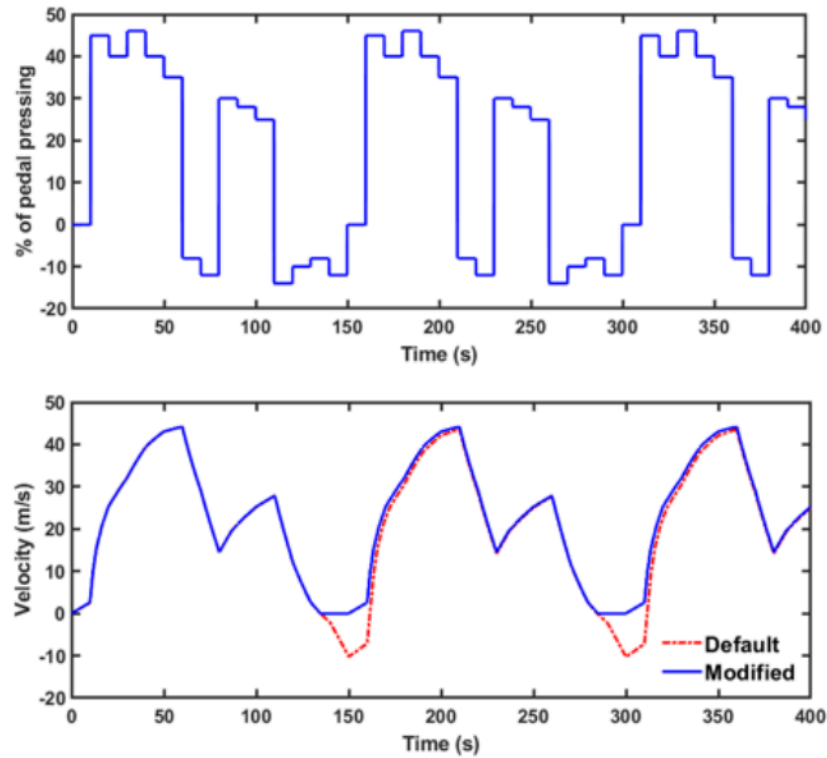
The input throttle pressing can be estimated by reversing Eq. (18) to find the  $u(k)$  and substituting the future output  $y(k + 1)$  with the positive traction force that comes from Eq. (17). Note that the past input  $u(k - 1)$  and past output  $y(k - 1)$  can be taken from stored memory in the microprocessor.

## 4. Results

This section presents the simulation results which consist of the open loop response of the vehicle longitudinal dynamic model, upper-level PID control tuning performance in tracking the step response, system identification model accuracy in representing power train dynamics for inverse calculation and the overall model validation performance in tracking the drive cycle.

### 4.1 Open-loop Response

The model developed in Section 2 is assumed to be a plant that represents a real car. An input signal of the throttle and brake pedal is generated by using *Repeating Sequence Stair* function in MATLAB Simulink to mimic the actual pedal pressing by varying the value between 100 to -100 percent. Figure 11 shows the open-loop response of the plant. It should be noted that the positive input represents the percentage of throttle pressing, while the negative input is for the brake pressing. It is assumed that while driving, both pedals will not be pressed simultaneously and hence both inputs can be assumed to be a single signal.



**Fig. 11.** Open-loop response of the plant model

The results show that the velocity output gives a response for each change in pedal pressing. The modifications that have been made in *Remark 1* (blue line) provide a logical response. As can be observed in the duration between 120 to 150 seconds, the velocity is zero when the brake is pressed. When no pedals are pressed, the vehicle resumes cruising with a minimum velocity that mimics the behavior of a real car. Compared to the model without *Remark 1* modification (red dash-dotted line), the velocity goes to a negative value since the braking force is producing negative tractive force and acceleration should the modification is not implemented, which is not logical if compared to the real situation.

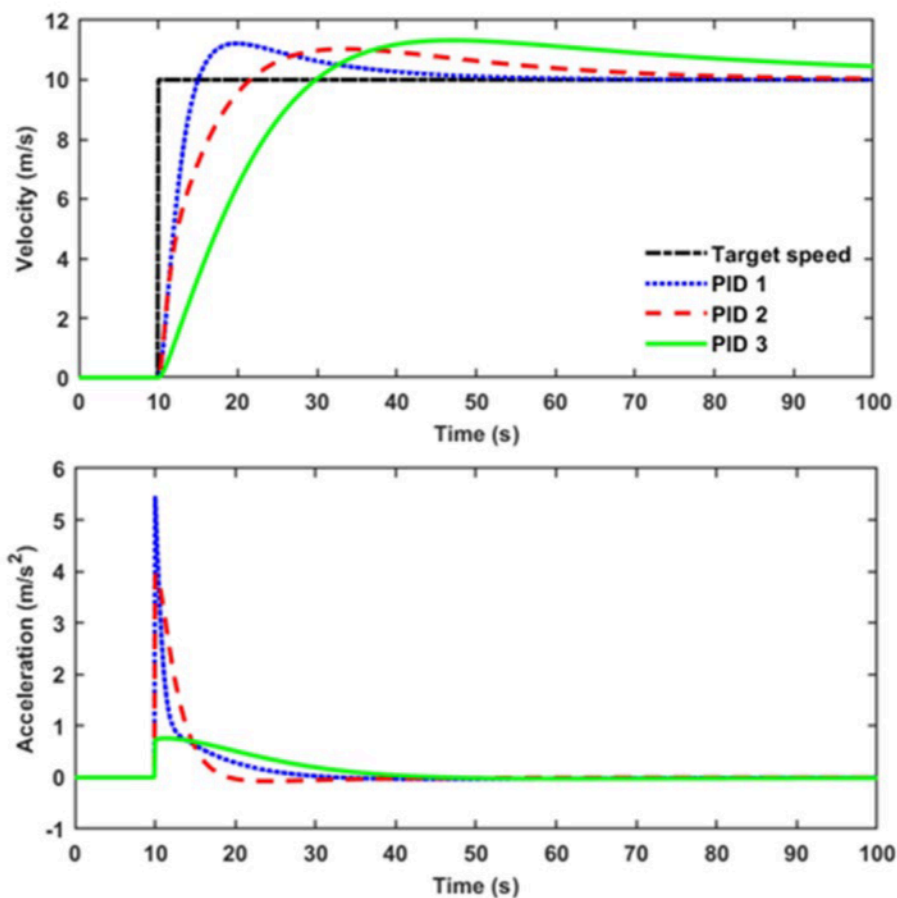
#### 4.2 Upper-level Control Performance

Once the plant has provided a logical response, the next task is to tune the PID controller based on the kinetic model in Eq. (16). In this task, 3 sets of PID gains are tuned for comparison analysis of the step response and the control effort of the pedal pressing, where the values are given in Table 3. During the tuning process, these gains are tuned for a balance performance between tracking the desired setpoint and robustness to uncertainties by using the MATLAB PID tuner app. This aspect is important when using a hierarchal control structure as the PID is not tuned based on the full kinetic model.

**Table 3**  
Gain for different PID controllers

| Gain   | PID 1 | PID 2   | PID 3  |
|--------|-------|---------|--------|
| $k_p$  | 0.39  | 0.214   | 0.1    |
| $k_i$  | 0.027 | 0.00083 | 0.0019 |
| $k_d$  | 0     | 0.271   | -0.16  |
| Filter | 100   | 1.23    | 0.169  |

Figure 12 shows the response of the three PID controllers. As can be observed, the PID 1 (blue dotted line) is tuned to reach the setpoint faster compared to the other two. Although this requirement can be achieved, often a user forgets to check the control effort that is needed where it demands around  $6 \text{ m/s}^2$  acceleration to reach the target, which for a certain car is not feasible. Observing the PID 3 (green solid line), this controller is tuned for a slower convergence, yet after 100 s, the controller still not converging to the setpoint. PID 2 (red dashed line) shows an optimum performance in this case where it managed to converge around 60 s with minimum overshoot at a reasonable and comfortable control effort. Thus, the gain values of the PID 2 controller will be used to track the drive cycle. Table 4 shows the qualitative time response performance comparison of the three PID.



**Fig. 12.** Response of different PID controllers with the control effort

**Table 4**

Time response performance for different PID controllers

| Time Response           | PID 1 | PID 2 | PID 3 |
|-------------------------|-------|-------|-------|
| Percentage of overshoot | 12.3  | 1.59  | 13.5  |
| Settling time (s)       | 33.2  | 17.4  | 124   |
| Rise time (s)           | 3.39  | 10.6  | 14.2  |

#### 4.3 System Identification for Powertrain Dynamics

The control input generated from the PID in the previous subsection only gives a reference acceleration signal. To get the actual input the inverse model needs to be utilized. For brake pressing



the calculation is straightforward but for throttle pressing the system-identified model is used. To identify this model, a set of input (throttle pressing) and output data (traction force) is generated as can be observed in Figure 13 by using *Repeating Sequence Stair* varied between 0 to 100% pressing. This is to ensure that the training input is covering most of the important dynamics.

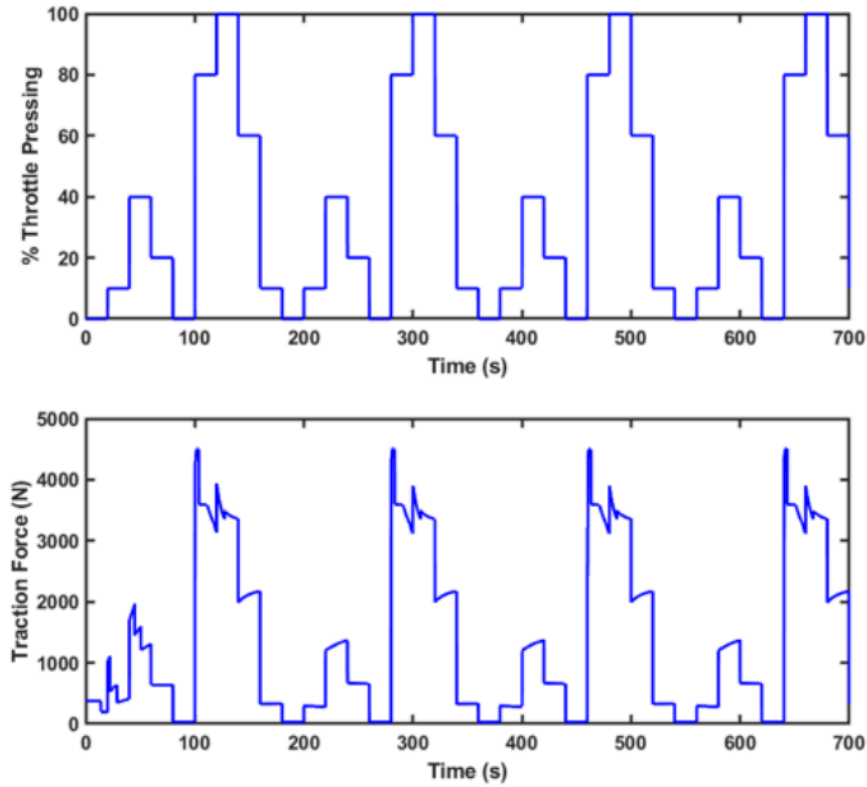
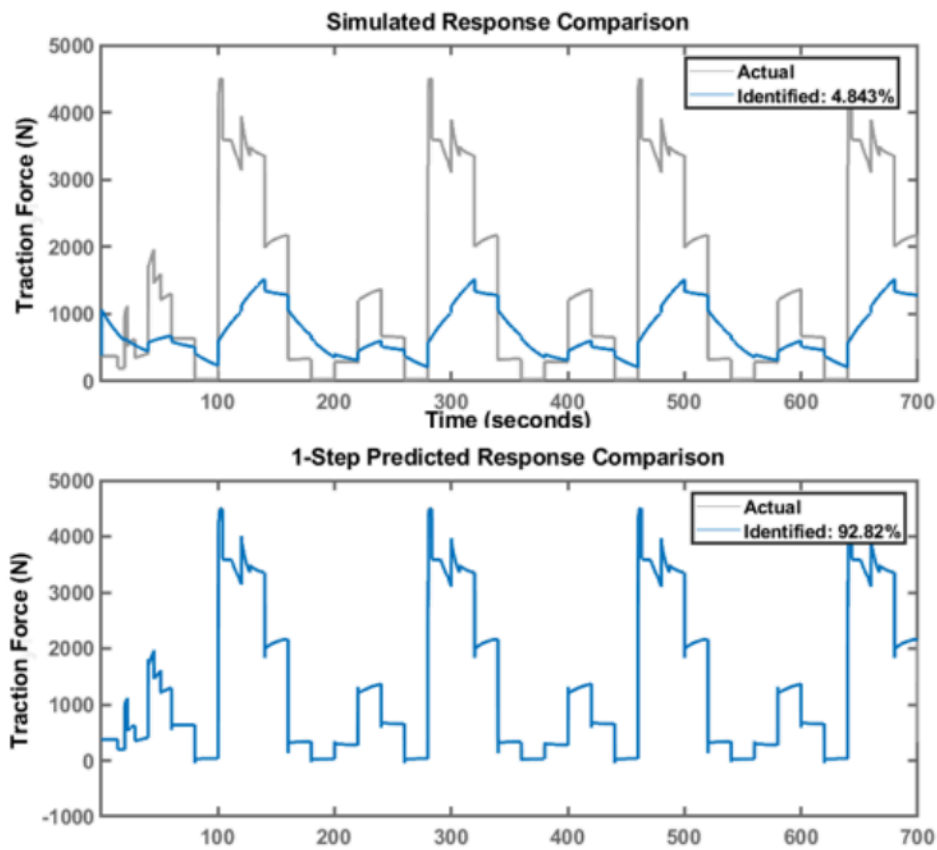


Fig. 13. Input and output profile for training data

Next, the parameters of the ARX model are identified as given in Eq. (19) and used to represent the system. As can be observed in Figure 14, the model is unable to produce a similar response as compared to the actual data. The percentage of fit is just around 4.84 %, which is quite low. Nevertheless, it is well known that the main advantage of the ARX structure is for predicting future output based on the current input and output. The percentage of fit for 1 step ahead prediction based on the current data is quite high which is around 92.82% (refer to Figure 14). Since the inverse model is responsible for calculating the current input based on the future desired target, this model is quite suitable to be used since it has the capability to predict.

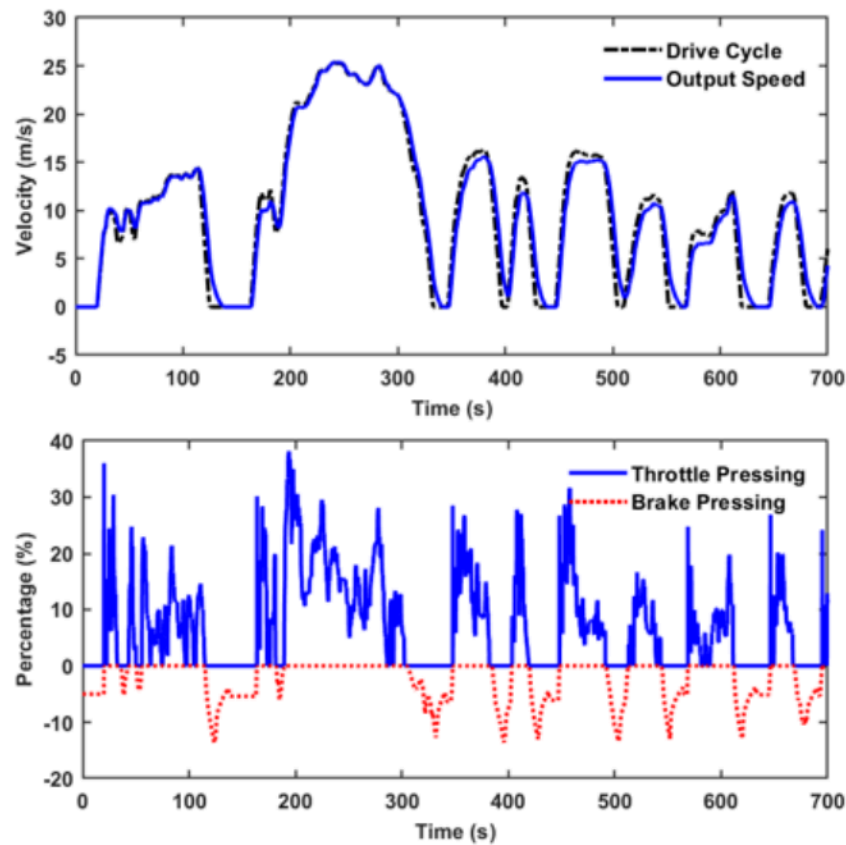
$$y(k + 1) = 4.544u(k) + 4.484u(k - 1) - 0.9783y(k) - 0.01872y(k - 1) \quad (19)$$



**Fig. 14.** Simulated and 1 step ahead prediction response of the identified model

#### 4.4 Tracking Drive Cycle using Hierarchical Control Structure

In this part, the hierarchical control structure is employed to track a standard urban drive cycle to assess its capability to track real data speed. A drive cycle is a predefined pattern or sequence of vehicle operating conditions that represent typical driving behaviour. The drive cycle is often developed for various purposes such as vehicle testing, emissions certification, fuel efficiency evaluation, and development of control strategies [23]. By using the same PID 2 gain as in Section 4.2, Figure 15 shows that the proposed control structure managed to track the US urban drive cycle [21] with acceptable delay. When the car needs to be at zero m/s, the controller holds it by pressing the brake to prevent the vehicle from moving. As the desired speed is increasing, the throttle pedal is pressed. Conversely, as the desired speed is decreasing, a brake pedal is pressed. Besides, the input signal for throttle pressing and brake pressing do not conflict with each other and is within an acceptable range for daily usage between -20% to 40%.



**Fig. 15.** Drive Cycle tracking response using hierarchical PIC controller

Nevertheless, it also should be noted that the PID controller itself has several weaknesses where in this case the controller is only designed to track a speed without considering the operational constraints such as maximum and minimum acceleration and safe distancing. For future work, an advanced controller such as model-based control can be implemented by using the same structure and vehicle plant to develop and improve the existing Cruise Control system, where the conflicting performance such as fast response, driving comfort, safe distancing and fuel efficiency can be optimised.

## 5. Conclusions

In summary, this work has provided a detailed explanation and derivation of a simple vehicle longitudinal dynamic model that is suitable for control applications. The model only focuses on the longitudinal motion without considering the motion in other axes. Besides, a PID controller has been tuned and employed by using a hierarchical structure where it produces a reference acceleration signal for the lower-level system to track via the inverse model relationship. The system identification method is also utilized to represent the powertrain dynamics to simplify the reverse calculation of the throttle pedal pressing. The results show that the proposed system managed to track a standard urban drive cycle with acceptable throttle and brake pressing with minimum delay in tracking the set point and without conflict. For future work, the same plant and set-up can be used to design and tune other advanced controllers such as model-based controllers for designing an ACC system that can track the speed while maintaining a safe following distance.

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