Innovative Approaches Using ICT to Support Teaching and Learning Advanced Mathematics

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Abstract

The UK has seen a worrying decline in the number of students taking up Higher Education courses in both Mathematics and ICT. The opportunity exists within an A-level mathematics course for teachers and students to use technology in an imaginative way to enhance the learning experience and to encourage students to consider further study in either discipline.

Students taking A-level are taught in a variety of institutions. An A-level mathematics class in an 11-18 school could be a small cohesive group with similar background, prior learning experiences and familiarity with the use of ICT in mathematics. A class in a Further Education college is likely to be much larger and to include students from a wide range of previous institutions whose experiences both with regards to teaching and learning styles and to their use of ICT could vary widely.

Catering for the individual teacher and student in these diverse circumstances can be a big deterrent to the use of technology, so in the ICTAM (Interactive Companion to Advanced Mathematics) project [3] the authors have put together a unique collection of software which has been designed to offer something to every A-level mathematics teacher and student, whatever their background institution or technological expertise.

The paper considers a range of activities from ICTAM which includes:-

- Activities based on files for a wide range of powerful mathematics software.
- Its own graphing software Java applet, TIM, which teachers can adapt.
- Dynamic activities based on real world problems and images
- Ideas for different teaching approaches.
- Supporting instructions and video demonstrations for student/teacher self-help.
- Extension challenges for able students.

Introduction

In his report [4] Sir Gareth Roberts notes that:
Although, relative to many other countries, the UK has a large and growing number of young people studying science and engineering, this overall growth has masked a decline in the numbers studying the physical sciences, engineering and mathematics.

The report draws attention to the drop during the 1990s of nearly 10 per cent in the numbers taking A-level mathematics in England. At the same time, the report also notes that:
The demand for graduates and postgraduates in these strongly mathematically oriented subjects has grown significantly over the past decade, not only in science and engineering areas, but also in the financial services and ICT sectors.

In addition to the supply problem, the report identifies concerns expressed by employers about the mismatch between skills acquired during formal education and those required in the workplace.

In referring to [4], Professor Adrian Smith notes [5] that:
In addition, many of the generic problems identified across science and engineering in 'SET for Success' manifest themselves most acutely in the area of mathematics. For example: there has long been deep concern about the supply of appropriately qualified mathematics teachers in secondary schools and colleges.
There has also been considerable concern about many young people's perception of mathematics as being "boring and irrelevant" and "too difficult, compared with other subjects".

The arrangements for Advanced level teaching in the United Kingdom vary widely between local education authorities. A substantial proportion of post 16 students are now taught in colleges of Further Education. These large institutions, which cater for almost the entire post-16 population in some areas, have to work within very tight budgetary restraints. As a result, classes for A-level students in FE colleges tend to be considerably larger than those in traditional school sixth forms. Access to technology is limited both by funding and by a lack of specialised accommodation. Effective use of technology is further compounded by the varied backgrounds of the students, who may come from a wide range of previous institutions – and countries.

In their Interactive Companion to Advanced Mathematics (ICTAM) the authors have set out to produce a comprehensive product that addresses all of these concerns. Firstly, they have aimed to produce a package that will not only support students' A-level study but will also stimulate their interest in further study, and will encourage them to see the use of technology as an integral part of a mathematics course. Secondly, they provide comprehensive support for teachers, thus enabling them to have the confidence to make greater use of technology in their A-level teaching.

Motivating Students - Dynamic Geometry and the Use of Real World Images

The use of dynamic models and photographic images can bring mathematics to life and make it relevant to the real world, as well as giving it greater appeal for visual and kinaesthetic learners. ICTAM tries wherever possible to take advantage of the power of software such as *The Geometer's Sketchpad* and *Cabri Geometry II Plus* to incorporate photographic images. This feature is also incorporated in its own java applet called TIM – *Tool for Interactive Mathematics*.

TIM (Tool for Interactive Mathematics)

ICTAM also has its own specially designated Java applet: TIM. A particular feature of TIM is the ability to import images into a 'click for data' and `fit a function' application. The user can click points along the image - in this case a roller coaster and experiment by inserting functions to try to fit their graphs to the points shown.

Pre-Written Dynamic Geometry Files

In this example an image, Figure 1, of a rear screen of a car has been incorporated into a pre-written dynamic geometry program. The image can be animated to illustrate the motion of a wiper blade, and the angles and areas swept out by the blade are displayed. This is one example of many pre-written programs for teachers to use as demonstration or for students to use independently.

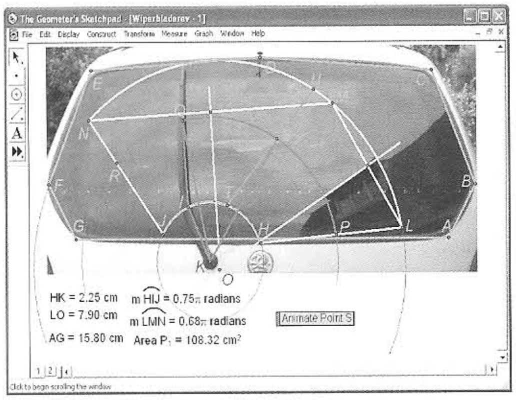

Figure 1: Rear windscreen wiper

Self-help Files

In addition to pre-written files, allowance has been made for students or teachers who may wish to use the software themselves, by the incorporation of written instructions and video demonstrations of how to do this.

In the example shown in Figure 2, instructions are given for fitting an arc of a circle to an image and using this to estimate the radius and angle of the arc.

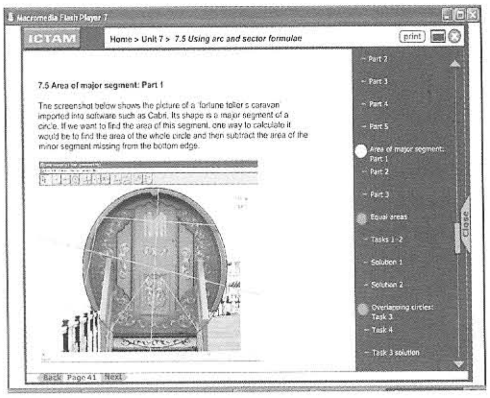

Figure 2: Fitting an arc to a circle

One of the challenges in the graphs unit of ICTAM is to find a suitable cubic function to fit part of a roller coaster image Figure 3. Not only does this relate graphs of mathematical functions to the real world, but it also provides a stimulating intellectual challenge, and an opportunity for discussion of successful strategies. Detailed instructions for using TIM are included in both written and video form.

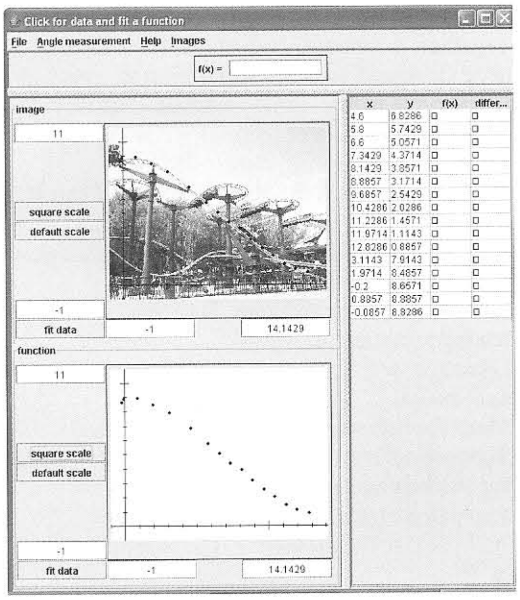

Figure 3: Output from TIM

Supporting Students and Teachers in the Use of a Wide Range of Software

The resources available to teachers and individual students vary widely. Ideally students and teachers should be able to choose which software they think is most appropriate for a particular application. To facilitate a variety of approaches, and as an aid to those whose resources are limited, alternative files are offered wherever possible e.g. for *The Geometer's Sketchpad*, *Cabri Geometry II Plus*, *MS Excel*, *TI InterActive!*, as well as instructions for graphical calculators. To encourage both students and teachers to see the use of these as an integral part of mathematics, they are integrated throughout the product, ranging at one extreme from major pre-written files, to the other with instructions for how to use a graphical calculator or *TI InterActive!* to check solutions to examination questions.

Facilitating the Teaching of Difficult Topics

The use of dynamic imagery can help with the visualisation of difficult concepts. One such example is in the understanding of how circular functions are generated. ICTAM includes animated files showing how all the major trig function graphs are generated in both degrees and radians – Figure 4. Another file, Figure 5, is an animated tangent which moves around a curve demonstrating stationary points and points of inflexion in a very effective way. These ready-made visual aids are much appreciated by teachers. The teachers' notes and detailed contents list are designed so that teachers can easily access files like this and use them as stand-alone activities.

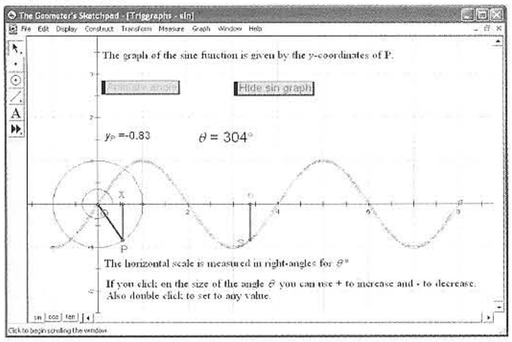

Figure 4: Graphing trigonometric functions

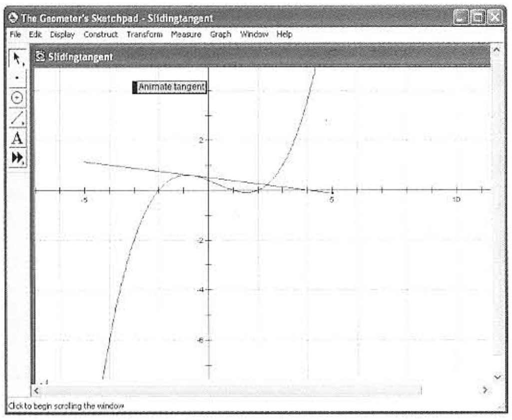

Figure 5: A sliding tangent line

Supporting an Investigative Approach to Teaching Mathematics

Between 1988 and 1995 the Raising Achievement in Mathematics (RAMP) project directed by Prof. Afzal Ahmed and based at the Mathematics Centre (now part of the University of Chichester) operated an innovative A-level course in which students were assessed on a portfolio of work [2]. A key feature of the course was the use of 'Rich mathematical activities' as starting points for stimulating student investigation. In [1] such an activity is described as having the following characteristics:-

It is accessible to all students at the start: It allows for further challenges and extension: It invites decision making, speculating, hypothesis making, proving, explaining, reflecting, and interpreting.

These are all qualities which would not only enhance student motivation and participation but would enable students to develop skills that would be useful in higher education or the workplace.

The principle of using this type of rich activity to stimulate learning still remains valid. Despite its apparent text-book style format, each unit of ICTAM contains an activity which could be used as a starting point, and further details of this approach are included in the teachers' booklet. One such activity is finding the area cleaned by a car's screen wipers, mentioned earlier.

A slightly different approach is used in the Coordinate geometry unit, where, assuming no previous knowledge of coordinate geometry, students are given the following problem.

Fit a circular patio or pond around 3 fixed objects (Figure 6)

Figure 6: A circular pond

How can you find the centre and radius of the circle which passes through the points with coordinates (4,10), (11,9) *and* (7,1). *What is the equation of this circle? Generalise. What steps, calculations and formulas do you need in order to be able to do this?*

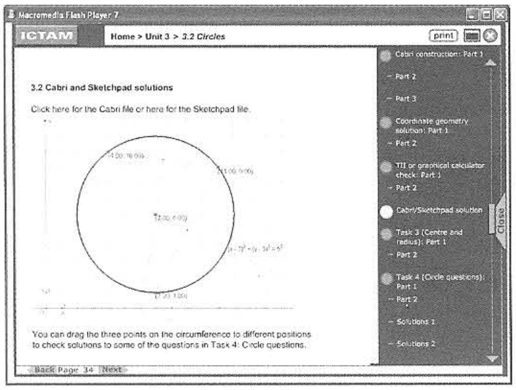

Figure 7: Student help with properties of circles

This problem encourages students to develop their own strategies and formulas in coordinate geometry in a more meaningful way. Activities to help them and a variety of supporting software are included plus video demonstrations for using dynamic geometry software. Generalising the equation for any 3 points and creating a dynamic geometry file to go with this, would provide a challenging extension. There is also an extension activity of fitting an ellipse.

Design a pond and calculate its surface area is an example mentioned in the teachers' booklet (Figure 8).

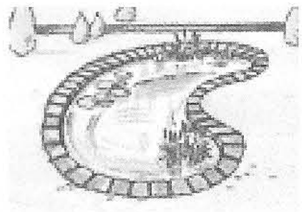

Figure 8: A garden pond

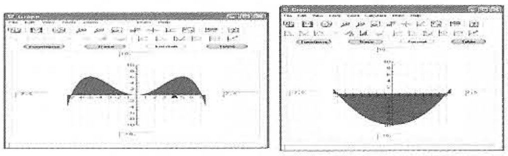

Figure 9: Approaches to modelling the pond

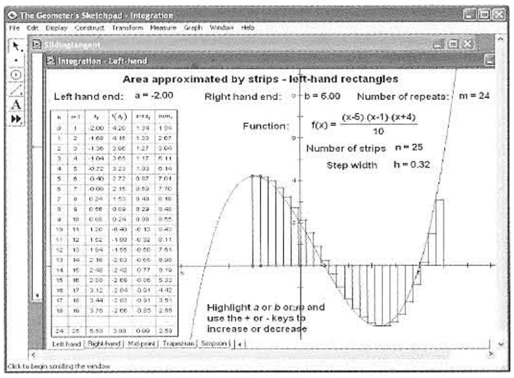

Figure 10: Student investigation with integrals

Using the materials in the Integration unit, students could be encouraged to experiment with a variety of choices for the curves using graphical calculators or graphing software; to consider further properties of functions and their graphs in order for them to pass through specific points; to examine whether functions can be integrated by simple methods or whether they need to find alternatives, such as using approximate methods or researching more complex techniques. Software activities using dynamic geometry could be adapted by students, who could also use the video demonstrations of graphical calculators and graphing software. There are lots of challenges for more able students e.g. to consider smooth joins and to obtain a particular value for the area. Further ideas can be found in [7].

Recognising the Importance of the Role of the Teacher

Gray in [6] makes the following prophetic statement about teaching 11-16 pupils using textbooks, which now applies equally to A-level teaching where publishers compete with each other to match a textbook to every examination syllabus.

The translation of the explicit requirements of the National Curriculum in mathematics into action within the classroom may increasingly make teachers turn to textbooks. If history is anything to go by writers and publishers will certainly respond to the need. It is overstating the value of a textbook to believe it can take on the responsibility for teaching. The ability to make more than a passing comment to the qualitatively different ways in which children do mathematics is beyond them: only teachers can respond to this.

The authors of ICTAM were constrained to work in a text book like format, but because the entire product is contained on a single CD with detailed indexing and ease of navigation from one unit or activity to another, there is considerable scope for both students and teachers to use the materials in a more imaginative way. Whilst the CD is designed to allow students to work on as individuals, the role of teachers in facilitating more imaginative use of the materials is a key one.

Guidance for a variety of approaches, with comprehensive details of all the activities and demonstrations, learning objectives and links to examination syllabuses is included in the teachers' booklet. The role of the teacher as facilitator is fundamentally important. If ICTAM had been designed to be used in the same way as a text book, then there would be no perceived need for further professional development for those using it. The effective use of ICTAM is greatly enhanced, both for students and teachers alike, if teachers have opportunities to discuss and share ideas.

Conclusion

ICTAM contains a variety of student activities catering for a wide range of abilities and prior experiences, both mathematical and technological. A comprehensive variety of different software and approaches is included with supporting materials for teachers. The authors believe that the use of visual and dynamic imagery, together with the supporting materials, will encourage students to develop their expertise and interest in both mathematics and the use of ICT, and to see the two as mutually supportive. They also believe that the product will offer teachers much of the support they need to plan how to embed the use of ICT into the A-level mathematics curriculum.

References

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