



UNIVERSITI PUTRA MALAYSIA

**GOODNESS-OF-FIT TEST FOR STANDARD LOGISTICS
DISTRIBUTION WITH OUTLIERS**

**LIM FONG PENG
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**R_{MAD}^2 GOODNESS-OF-FIT TEST
FOR STANDARD LOGISTICS DISTRIBUTION
WITH OUTLIERS**

LIM FONG PENG

**MASTER OF SCIENCE
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**R_{MAD}^2 GOODNESS-OF-FIT TEST
FOR STANDARD LOGISTICS DISTRIBUTION
WITH OUTLIERS**

By

LIM FONG PENG

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia
in fulfilment of the requirement for the degree of the Master of Science

R_{MAD}^2 **GOODNESS-OF-FIT TEST**
FOR STANDARD LOGISTICS DISTRIBUTION
WITH OUTLIERS

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LIM FONG PENG

March 2010

Chairman: Noor Akma Ibrahim, PhD

Faculty: Science

Alternative to the least square coefficient of determination (R_{OLS}^2), the coefficient of determination based on median absolute deviation, R_{MAD}^2 , is an attractive consideration in the construction of goodness-of-fit test based on regression and correlation, due to its robustness.

This study presents the observations made from the resulting plots and descriptive measures obtained from contaminated standard logistic distribution. Contamination is introduced to investigate perseverance of robustness property of R_{MAD}^2 for samples from the standard logistic distribution. The sampling distribution of R_{MAD}^2 is simulated for various sample sizes ($n = 20, 40, 100$), percentage of contamination (5%, 15%, 25%) and distribution of the contaminants (logistic (2, 0.2), logistic (0, 0.2), logistic (2, 1) and normal (3, 0.2) contaminants). The symmetry of the sampling distribution of R_{MAD}^2 is observed and followed by the investigation of the



confidence intervals of R_{MAD}^2 in the presence of outliers. The study of confidence interval estimates for the mean and standard deviation of R_{MAD}^2 was conducted using the bootstrap (BCa) method.

Tables of critical values for samples from the standard logistic distribution using $Z_{MAD} = 1 - R_{MAD}^2$ and $Z_{OLS} = 1 - R_{OLS}^2$ are constructed. The tables obtained then are used in the power study on the goodness-of-fit tests using test statistic for alternative distributions and contaminated alternative distributions. For lognormal, exponential and standard logistic alternatives, Z_{MAD}^* and Z_{OLS}^* are simulated for various sample sizes ($n = 10, 20, 30, 50, 100$), percentage of contamination (5%, 15%, 25%, 40%) and distribution of the contaminants (logistic (2, 0.2), logistic (0, 0.2), logistic (2, 1) and normal (3, 0.2) contaminants) for different percentiles ($\alpha = 0.01, 0.025, 0.05, 0.1$), respectively. The results indicated that the test statistic Z_{MAD} is able to discriminate the sample that comes from alternative distributions as the test statistic Z_{OLS} .



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

R_{MAD}^2 **UJIAN KEBAGUSAN PENYUAIAN**
BAGI TABURAN LOGISTIK PIAWAI
DENGAN PENCEMAR

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Alternatif bagi pekali penentuan kuasa dua terkecil (R_{OLS}^2), iaitu pekali penentuan yang berdasarkan median lencongan mutlak, R_{MAD}^2 , merupakan satu pertimbangan menarik dalam pembinaan ujian kebagusan penyuaian berdasarkan regresi dan korelasi, disebabkan keteguhannya.

Kajian ini memerihalkan pemerhatian yang dibuat daripada plot yang diperolehi dan ukuran pemerihalalan daripada taburan logistik piawai tercemar. Pencemaran diperkenalkan untuk menyelidiki sifat keteguhan R_{MAD}^2 bagi sampel daripada taburan logistik piawai. Taburan pensampelan R_{MAD}^2 disimulasikan bagi pelbagai saiz sampel ($n = 20, 40, 100$), peratusan pencemaran (5%, 15%, 25%) dan taburan pencemar (pencemar logistik (2, 0.2), logistik (0, 0.2), logistik (2, 1) dan normal (3, 0.2)). Sifat simetri bagi taburan pensampelan R_{MAD}^2 diperhatikan dan ini diikuti dengan kajian selang keyakinan bagi R_{MAD}^2 dengan kehadiran titik terpencil.



Jadual nilai kritis bagi sampel daripada taburan logistik piawai yang menggunakan $Z_{MAD} = 1 - R_{MAD}^2$ dan $Z_{OLS} = 1 - R_{OLS}^2$ masing-masing dibina. Jadual-jadual yang diperolehi itu kemudian digunakan dalam kajian kuasa pada ujian kebagusan penyuaian yang menggunakan statistik ujian bagi taburan alternatif dan taburan alternatif tercemar. Bagi alternatif lognormal, eksponen dan logistik piawai, Z_{MAD}^* dan Z_{OLS}^* masing-masing disimulasikan bagi pelbagai saiz sampel ($n = 10, 20, 30, 50, 100$), peratusan pencemaran (5%, 15%, 25%, 40%) dan taburan pencemar (pencemar logistik (2, 0.2), logistik (0, 0.2), logistik (2, 1) dan normal (3, 0.2)) bagi persentil yang berbeza (0.01, 0.025, 0.05, 0.1). Keputusan yang diperolehi menunjukkan bahawa statistik ujian Z_{MAD} berupaya untuk mendiskriminasi sampel yang datang daripada taburan alternatif seperti statistik ujian Z_{OLS} .

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I certify that a Thesis Examination Committee has met on 3 March 2010 to conduct the final examination of Lim Fong Peng on her thesis entitled “ R_{MAD}^2 Goodness-of-fit Test for Standard Logistic Distribution with Outliers” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

—

Lim Fong Peng

Date: 30 January 2010



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CHAPTER 1

INTRODUCTION

1.1 Background

Goodness-of-fit techniques are methods of examining how well a sample of data agrees with a given distribution. In the formal framework of hypothesis testing for goodness-of-fit techniques, the hypothesis set is as below,

H_0 : the data follow a specified distribution.

H_1 : the data do not follow the specified distribution.

Then, it is usually hoped to accept the null hypothesis and proceed with other analyses (D' Agostino and Stephens, 1986).

In addition to formal hypothesis testing procedures, goodness-of-fit techniques also include less formal method, in particular, graphical techniques. In these methods, graphs are drawn so that deviation from the hypothesized distribution is reflected in certain features of the graph. Graphical techniques are useful because of their ease and informality.

Regression and correlation based test is a graphical technique, related to probability plots, in which the order statistics $X_{(i)}$ (on the vertical axis) are plotted against a suitable function of i , T_i , (on the horizontal axis). A straight line is then fitted to the



points, and goodness-of-fit tests are based on the correlation coefficient between X and T .

In this study, a regression and correlation based test is conducted for samples from standard logistic distribution. The statistics of interest are the coefficient of determination from regression obtained by using ordinary least square (OLS) and median absolute deviation (MAD).

The coefficient of determination from ordinary least square regression, R_{OLS}^2 , is the ratio of the variability of the modeled values, which is obtained by ordinary least square regression, to the variability of the original data values. Or it can be defined as the square of the correlation coefficient, r_{OLS} , between the original and modeled data values.

Meanwhile, the median absolute deviation for the correlation coefficient (r_{MAD}) has been defined by Gideon (2007). This definition is discussed in Chapter 3. He discovered that r_{MAD} sometimes may go beyond +1 or -1, that is $|r_{MAD}| \leq 1$ is not always true. R_{MAD}^2 is an attractive alternative to the coefficient of determination from ordinary least square regression, R_{OLS}^2 , due to its robustness. Mean absolute deviation (MAD) about the median is a very robust scale estimator (Hampel, 1974). The MAD is clearly not influenced very much by the presence of a large outlier, and as such provides a good robust alternative to the sample standard deviation (Maronna, 2006).

However, the properties of the sampling distribution of R_{MAD}^2 have not been well

researched. Hence, a simulation study is conducted in this study to obtain graphical and numerical description on the sampling distribution of R_{MAD}^2 , focusing on samples from the standard logistic distribution. The sampling distribution of R_{MAD}^2 above can be used to construct tables of the critical values and to conduct a power study on the goodness-of-fit tests.

In addition, checking for the bound of R_{MAD}^2 for samples from standard logistic distribution is also presented before conducting a power study on the goodness-of-fit tests. It is an advantage if $|r_{MAD}|$ is always less than 1, so that the critical values, $Z_{MAD} = 1 - R_{MAD}$, can be calculated in conducting the power study. Hence, the bootstrapping can be a good “tool” to investigate the bound of R_{MAD}^2 whether $|r_{MAD}|$ is always less than 1 or not. The bootstrapping method is often used as an alternative for inference based on parametric assumptions when those assumptions are in doubt, or where parametric inference is impossible or requires very complicated formulas for the calculation of standard errors. The goal of bootstrap confidence interval theory is to construct dependable confidence limits for a parameter of interest from the bootstrap distribution of $\hat{\theta}$. For more details, refer to DiCiccio and Efron (1996).

There are several bootstrap techniques for constructing confidence intervals, such as bootstrap- t , percentile, BC_a (*bias-corrected and accelerated*) and ABC (*approximate bootstrap confidence intervals*) methods. However, BC_a method is preferred and widely used in statistical study since it is an improved version of the percentile