Heat Transfer Optimization in Hybrid Nanofluid Flow over a Convectively Heated Stretching/Shrinking Sheet Considering Different Nanoparticle Shapes

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Abstract

A sophisticated statistical tool called the response surface methodology (RSM) can be utilized to find the optimal settings for efficient heat performance of hybrid nanofluids with various flow conditions. The present study examines the steady flow of Ag-MgO/H_2O hybrid nanofluid over a permeable stretching/shrinking sheet. The effects of chemical reaction, activation energy, magnetic field, various nanoparticle shapes, zero mass flux, velocity slip, and convective boundary condition are incorporated into the flow problem. Similarity variables are then used to transform the governing partial differential equations and boundary conditions into non-linear ordinary differential equations. These equations are solved numerically using the bvp4c solver in MATLAB, and optimization is conducted using the RSM in Minitab. Hybrid nanofluid with platelet- and spherical-shaped nanoparticles is found to have the lowest local skin friction coefficient in the stretching and shrinking cases, respectively. Meanwhile, the augmentation of the suction parameter (S) and Biot number (Bi) improves the local Nusselt number. Through the RSM, the local Nusselt number is estimated to be optimized at $Bi = 0.7$ and $S = 5.2$. With the desirability of 100%, the local Nusselt numbers for the stretching and shrinking cases are approximated to be maximized at 0.68501 and 0.68493, respectively.

Keywords: Convective boundary condition, Hybrid nanofluid, Nanoparticles shapes, RSM, Stretching/shrinking sheet.

1. Introduction

A statistical method called the response surface methodology (RSM) is an efficient tool for optimization. It is based on the design of experiment (DoE), which minimizes the number of needed runs using the central composite design (CCD) [1]. The RSM, introduced by Box and Wilson [2], is utilized in determining the interactive effects of parameters in the presence of various desired criteria and decides the significant parameters for a response. According to Han et al. [3], the RSM has the benefits of lowering experimental cost, decreasing the variability around a target when bringing the performance value to the target value, and the optimal conditions found in the simulation can be reproduced in actual applications. Therefore, the RSM has been employed in various fluid flow problems to determine the relationship between controlling parameters and responses (e.g., heat and mass transfer rates) and find the best condition to optimize the responses.

The prior work by Yahaya et al. [4] had discussed the effects of nanoparticle shapes on hybrid nanofluid flow over a stretching/shrinking sheet in the presence of slip, magnetic field, and chemical reaction. The present study will extend this work with convective boundary condition. In addition, the heat transfer optimization for the current flow problem will be examined through RSM. New boundary conditions will be introduced into the mathematical formulation of the flow problem. Similarity transformation will then reduce the governing equations and boundary conditions into non-linear ordinary differential equations. Then, numerical computation will be carried out using the bvp4c solver in MATLAB. The results will be recorded, analyzed, and discussed, focusing on identifying the optimal conditions for achieving maximum heat transfer rate.

2. Mathematical Formulation

The considered flow problem is described mathematically using governing partial differential equations and boundary conditions. Similar continuity, momentum, energy, and concentration equations as those used by Yahaya et al. [4] are also employed in the current study. However, the present study incorporates convective boundary conditions, expressed as:

$$-k_{HN}\frac{\partial T}{\partial y} = h_f(T_f - T), \tag{1}$$

where k_{HN} is the thermal conductivity of the hybrid nanofluid, h_f is the heat transfer coefficient, T_f is the convective surface temperature, and T is the temperature of the hybrid nanofluid.

Similarity transformation is then conducted to reduce the governing equations and boundary conditions into the following non-linear ordinary differential equations:

$$\frac{1}{\rho_{HN}/\rho_{BF}}\left[\frac{\mu_{HN}}{\mu_{BF}}f''' - \frac{\sigma_{HN}}{\sigma_{BF}}Mf'\right] + ff'' - f'^2 = 0, \tag{2}$$

$$\frac{k_{HN}/k_{BF}}{(\rho C_p)_{HN}/(\rho C_p)_{BF}}\theta'' + Prf\theta' + PrNb\theta'\phi' + PrNt\theta'^2 = 0, \quad (3)$$

$$\frac{1}{Sc}\phi'' + f\phi' + \frac{Nt}{ScNb}\theta'' - \xi\phi(1+\delta\theta)^n \exp\left(-\frac{E}{1+\delta\theta}\right) = 0 \quad , \quad (4)$$

$$f'(0) = \lambda + A_1 f''(0),\ f(0) = S,\ \theta'(0) = -\frac{k_{BF}}{k_{HN}} Bi[1 - \theta(0)],\ Nb\phi'(0) + Nt\theta'(0) = 0, \quad (5)$$

$$f'(\infty) \to 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0. \quad (6)$$

Here, the prime denotes differentiation with respect to η, f, θ, and ϕ are functions related to velocity, temperature, and concentration, respectively, M is the magnetic parameter, Pr is the Prandtl number, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, Sc is the Schmidt number, ξ is the dimensionless reaction rate, δ is the temperature difference parameter, E is the dimensionless activation energy, A_1 is the velocity slip parameter, S is the suction parameter, and Bi is the Biot number. Meanwhile, ρ, μ, σ, and ρC_p are the density, dynamic viscosity, electrical conductivity, and heat capacity, respectively. The correlations and values for these thermophysical properties are given by Yahaya et al. [4]. The difference in nanoparticle shapes is incorporated through the correlations of these thermophysical properties. Additionally, the subscripts HN and BF represent the hybrid nanofluid and base fluid, respectively. The hybrid nanofluid comprises Ag and MgO, while the base fluid is water.

Then, the local skin friction coefficient $\left(Re_x^{1/2} C_{fx}\right)$ and Nusselt number $\left(Re_x^{-1/2} Nu_x\right)$ are given by:

$$Re_x^{1/2} C_{fx} = \frac{\mu_{HN}}{\mu_{BF}} f''(0), \quad Re_x^{-1/2} Nu_x = -\frac{k_{HN}}{k_{BF}}\theta'(0), \quad (7)$$

with Re_x as the local Reynolds number.

Equations (2) to (6) and the physical quantities of interest (7) are computed using the bvp4c solver in MATLAB.

3. Response Surface Methodology (RSM)

The RSM is conducted according to the following steps:

1. Identify the independent variables and their respective ranges.
2. Determine the experimental design.
3. Predict and evaluate the model equation.
4. Determine the optimum combination of independent variables that maximize the response.

In this study, the Biot number and suction parameter are the independent variables, while the local Nusselt number that is related to the heat transfer rate at the surface is the response. The relationship between the independent variables and response can be written as the following second-order equation [5]:

$$\text{Response} = a_0 + a_1\text{A} + a_2\text{B} + a_{11}\text{AA} + a_{22}\text{BB} + a_{12}\text{AB}, \quad (8)$$

with the coefficients to be determined through the RSM in Minitab software. For simplification, the independent variables Bi and S are represented as A and B, respectively. These variables are assigned with specific range of actual values, which are then coded into levels of high (1), medium (0), and low (-1).

4. Results and Discussion

The numerical results of Eqs. (2) to (6) are tabulated in Table 1. It can be observed that the platelet-shaped and spherical-shaped nanoparticles produce the lowest local skin friction coefficient for the stretching ($\lambda = 1$) and shrinking ($\lambda = -1$) cases, respectively. However, the variation in the nanoparticle shape does not have a significant impact on the local Nusselt number. This observation differs from the findings of Yahaya et al. [4], where variations in nanoparticle shape significantly affected the local Nusselt number. The discrepancy may be attributed to the inclusion of the convective boundary condition in this study, which was not considered by Yahaya et al. [4]. Nevertheless, it can be seen that the increase in the convective-related parameter, called the Biot number, provides significant improvement towards the local Nusselt number. Similar behavior is observed with the increase in the suction parameter.

Table 1. Values of $Re_x^{1/2} C_{fx}$ and $Re_x^{-1/2} Nu_x$ for different nanoparticle shapes and Biot number when $\phi_{Ag} = \phi_{MgO} = 0.015, Pr = 6.2, Nt = Nb = 0.55, Sc = M = E = \delta = 1, \xi = 2, n = 0.5, A_1 = 0.3$

λ	Nanoparticle shapes	Bi	S	$Re_x^{1/2} C_{fx}$	$Re_x^{-1/2} Nu_x$
1	Platelet	0.3	4.2	-3.49610	0.29656
		0.5	3.2	-3.10717	0.48766
			4.2	-3.49610	0.49051
			5.2	-3.85314	0.49230
		0.7	4.2	-3.49610	0.68151
	Cylindrical	0.3		-3.19666	0.29656
		0.5		-3.19666	0.49051
		0.7		-3.19666	0.68150
	Blade	0.3		-2.64409	0.29656
		0.5		-2.64409	0.49050
		0.7		-2.64409	0.68149
	Spherical	0.3		-2.13784	0.29656
		0.5		-2.13784	0.49049
		0.7		-2.13784	0.68147
	Brick	0.3		-2.57871	0.29656
		0.5		-2.57871	0.49050
		0.7		-2.57871	0.68148
-1	Platelet	0.3	4.2	3.24146	0.29652
		0.5		3.24146	0.49040
		0.7		3.24146	0.68129
	Cylindrical	0.3		3.01964	0.29652
		0.5		3.01964	0.49040
		0.7		3.01964	0.68130
	Blade	0.3		2.56378	0.29653
		0.5		2.56378	0.49041
		0.7		2.56378	0.68131
	Spherical	0.3		2.10514	0.29653
		0.5	3.2	1.87277	0.48744
			4.2	2.10514	0.49042
			5.2	2.28254	0.49225
		0.7	4.2	2.10514	0.68134
	Brick	0.3		2.50644	0.29652
		0.5		2.50644	0.49041
		0.7		2.50644	0.68132

Next, a statistical analysis is carried out to analyze the relationship between the Biot number and suction parameter with the local Nusselt number, as well as to determine their optimal settings for maximizing the heat transfer rate. A face-centered central composite design is utilized to fit the second-order equation (8). A total of 13 runs are conducted for this experimental design. Then, an analysis of variance (ANOVA) is performed using Minitab software. The results, shown in Tables 2 and 3, indicate that all terms have p-values less than 0.05. Thus, these terms are significant in correlation (8).

Table 2. ANOVA for stretching case

Source	DF	Adj SS	Adj MS	F-value	P-value
Model	5	0.222	0.044	802653.71	0.00
Linear	2	0.222	0.111	2006419.13	0.00
A	1	0.222	0.222	4012132.04	0.00
B	1	0.000	0.000	706.22	0.00
Square	2	0.000	0.000	95.21	0.00
AA	1	0.000	0.000	114.17	0.00
BB	1	0.000	0.000	16.03	0.01
2-Way Interaction	1	0.000	0.000	239.83	0.00
AB	1	0.000	0.000	239.83	0.00
Error	7	0.000	0.000		
Lack-of-Fit	3	0.000	0.000	*	*
Pure Error	4	0.000	0.000		
Total	12	0.222			

Table 3. ANOVA for shrinking case

Source	DF	Adj SS	Adj MS	F-value	P-value
Model	5	0.221	0.044	693865.81	0.00
Linear	2	0.221	0.111	1734466.46	0.00
A	1	0.221	0.221	3468274.61	0.00
B	1	0.000	0.000	658.30	0.00
Square	2	0.000	0.000	86.47	0.00
AA	1	0.000	0.000	100.58	0.00
BB	1	0.000	0.000	16.35	0.00
2-Way Interaction	1	0.000	0.000	223.21	0.00
AB	1	0.000	0.000	223.21	0.00
Error	7	0.000	0.000		
Lack-of-Fit	3	0.000	0.000	*	*
Pure Error	4	0.000	0.000		
Total	12	0.221			

Then, the interactive impact between the independent variables is analyzed through the contour plots in Figs. 1 and 2. It is observed that the Biot number (A) has a greater impact than the suction parameter (B) in enhancing the local Nusselt number (Response). Furthermore, the optimization results in Tables 4 and 5 denote that the local Nusselt number can be maximized at 0.68501 (stretching case) and 0.68493 (shrinking case) when both independent variables are at their high levels.

Fig. 1. Contour plot for stretching case

Fig. 2. Contour plot for shrinking case

Table 4. Optimization for stretching case

Solution	A	B	Response fit	Composite desirability
1	1	1	0.68501	1.00000

Table 5. Optimization for shrinking case

Solution	A	B	Response fit	Composite desirability
1	1	1	0.68493	1.00000

5. Conclusion

The heat transfer performance of hybrid nanofluid flow over a convectively heated permeable stretching/shrinking sheet can be optimized by incorporating high levels of Biot number and suction parameter.

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