

Research Article

Usman Afzal, Khalid Masood, Azhar Ali Zafar, Rizwan A. Farade, Nehad Ali Shah and Jae Dong Chung*

Comparative RSM-based study of nano, hybrid, and ternary nanofluid heat transfer with Cattaneo–Christov flux in porous media

<https://doi.org/10.1515/ntrev-2025-0261>

Received September 11, 2025; accepted November 29, 2025;

published online January 13, 2026

Abstract: This study conducts a comparative analysis of nano, hybrid, and ternary nanofluids in an Oldroyd-B fluid model within a Darcy–Forchheimer porous medium, focusing on heat transfer efficiency. It examines the effects of thermal radiation, heat source/sink, viscous dissipation, relaxation, and retardation time under different thermo-physical conditions. Nanofluids are widely used in medicine, electronic cooling, heat exchangers, and renewable energy systems. However, the comparative performance of nano, hybrid, and ternary nanofluids in an Oldroyd-B framework remains unexplored. This study fills this gap by evaluating TiO_2 , Fe_3O_4 , and CoFe_3O_4 nanoparticles and optimizing their thermal efficiency using numerical and statistical modeling. The governing nonlinear partial differential equations are transformed into ordinary differential equations using similarity variables and solved with the BVP4C method. Response surface methodology (RSM) is applied for parameter optimization, while multilinear regression analysis provides further insights. Graphical and tabulated results show that nano nanofluids achieve the highest heat transfer, followed by hybrid, while ternary nanofluids perform the lowest due to increased

resistance. Thermal radiation improves heat transfer, whereas the Forchheimer number reduces it. The obtained findings can guide the selection of nanofluids for high-performance heat exchangers, magnetohydrodynamic (MHD) cooling systems, solar-thermal collectors, nuclear and industrial cooling, and biomedical thermal regulation technologies. The integration of RSM further provides a practical optimization framework for engineering design and performance improvement in energy-efficient systems. These findings offer valuable insights into optimizing nanofluid-based thermal systems, aiding advancements in heat exchangers, MHD cooling, and energy-efficient technologies.

Keywords: response surface methodology (RSM); ternary hybrid nanofluid; thermal radiation; magnetohydrodynamics (MHD); heat source/sink

Nomenclature

r and z	radial and axial coordinates
v and w	components of velocity
B_0	magnetic field strength
δ	film thickness
β_0 and β_1	relaxation and retardation times
β_2	temperature relaxation time
q_r	radiative heat flux
$F = \frac{c^*}{\sqrt{K}}$	inertial coefficient
K	porous permeability factor
c^*	drag component
T_{ref}	reference temperature
T_b	temperature at the outer cylindrical boundary
ξ	similarity variable
Q_0	heat generation/absorption parameter

Subscript

f	basic fluid
thnf	ternary nanofluid

*Corresponding author: **Jae Dong Chung**, Department of Mechanical Engineering, Sejong University, Seoul 05006, Republic of Korea, E-mail: jdchung@sejong.ac.kr

Usman Afzal and Nehad Ali Shah, Department of Mechanical Engineering, Sejong University, Seoul, 05006, Republic of Korea, E-mail: usmanafzal9884@gmail.com (U. Afzal), nehadali199@yahoo.com (N.A. Shah). <https://orcid.org/0000-0002-1949-5643> (N.A. Shah)

Khalid Masood, Department of Mathematics and Statistics, College of Science, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh, 11623, Saudi Arabia, E-mail: kmaali@imamu.edu.sa

Azhar Ali Zafar, Department of Mathematics, Government College University, Lahore, 54000, Pakistan, E-mail: azharalizafar@gcu.edu.pk

Rizwan A. Farade, Department of Electrical and Electronic Engineering, Faculty of Engineering, Advanced Lighting, ALPER, University Putra Malaysia, 43400, Serdang, Malaysia, E-mail: rijavan@upm.edu.my

s_1, s_2 and s_3	nano solid nanoparticles (c_p) _{thnf} , k_{thnf} heat capacity and heat conductance of ternary hybrid nano fluid ρ_{thnf} , μ_{thnf} , σ_{thnf} density, viscosity, electric conductivity of ternary hybrid nano fluid
B_t	thermal relaxation time parameter
Q	heat source or sink
S	unsteadiness parameter
Re	Reynolds number
Fr	local inertia parameter
Ra	radiation parameter
M	magnetic field parameter
Pr	Prandtl number
θ	dimensionless function of temperature
χ	dimensionless function of velocity φ_1, φ_2 nanomaterials volume fraction De_d, De_x Deborah number related to relaxation and retardation times
hnf	hybrid nanofluid
w	wall
np	nanoparticles

1 Introduction

Fluids were categorized as Newtonian and non-Newtonian, where Newtonian fluids exhibited a linear relationship between shear stress and shear rate, while non-Newtonian fluids displayed complex rheological behaviors [1]. Non-Newtonian fluids, such as polymer solutions, mud, and blood, demonstrated varying viscosity patterns, making their study essential for industrial and scientific applications. The Oldroyd-B fluid, a well-researched viscoelastic model, was widely studied due to its ability to exhibit both elastic and viscous responses under deformation [2]. This model incorporated stress relaxation time and a convective derivative, making it applicable in polymer processing, the food industry, and biological systems [3]. Researchers examined various aspects of Oldroyd-B fluids. Some studies analyzed cylindrical particles containing Oldroyd-B-fluid-based nanofluids in a circular cross-section pipe, while others derived shear stress and velocity field solutions for helical flow around an infinite circular cylinder [4, 5]. Additional investigations explored irregular flow using fractional derivatives and shearing flow classes in viscoelastic fluids [6, 7]. Researchers also studied Oldroyd-B fluid flow under slip boundary conditions and its continuous secondary flow in a rectangular curved conduit powered by a direct electric field [8, 9]. Nanotechnology advanced fluid engineering through nanofluids, enhancing thermal conductivity and heat transfer. Ternary hybrid nanofluids have found applications in energy, cooling, aerospace, pharmaceuticals, and biosensors, influencing various industries (see Figure 1) [10].

Researchers expanded fluid engineering by developing ternary hybrid fluids, which combined distinct fluid phases

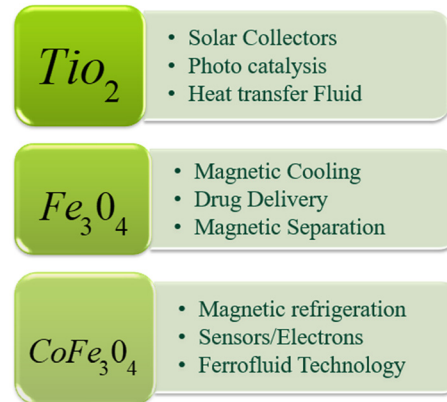


Figure 1: Applications of ternary hybrid nanofluids.

with complementary rheological properties. By adjusting viscosity, density, and surface tension, these fluids were tailored for specific applications. They were utilized in microfluidic devices for precise fluid control and in industrial processes for improved mixing and dispersion, marking a significant advancement in fluid engineering and optimization [11]. The best configuration and performance of a ternary hybrid nanofluid were examined using a contemporary machine learning prediction method, with a focus on symmetry analysis for thermal and momentum boundary layers, impacting heat transfer across various industries [12]. Scholars examined thermal performance of a Casson-type hybrid nanofluid over a porous stretching plate under Cattaneo–Christov (CC) heat conduction [13]. Researchers explored the influence of different nanoparticle shapes (Fe_3O_4 and TiO_2) suspended in transformer oil, providing relevant insights into ternary hybrid nanosuspensions despite the difference in base fluid [14].

They investigated the entropy generation in ternary hybrid nanofluid flow with non-Fourier heat flux [15]. The authors presented the entropy optimization in magnetized ternary hybrid nanofluid flow with viscous dissipation [16]. Scholars studied heat source and radiation effects on hybrid nanofluid flow with activation energy [17]. The hydrothermal integrity of a ternary hybrid nanosuspension ($TiO_2 + CoFe_3O_4 +$ water) over a rotating disk was analyzed, considering the effect of orientation [18]. The heat transport behavior of a ternary hybrid nanofluid under the influence of a magnetic dipole was also investigated [19]. In addition, an investigation was conducted on the bidimensional nonlinear convective flow of a ternary hybrid nanofluid past a nonlinear extending surface, yielding further insight into heat transfer features [20]. The heat transfer efficiency of a ternary hybrid nanofluid ($TiO_2 + Fe_3O_4 + CoFe_3O_4/H_2O$) over a rotating disk was also explored [21].

Solar energy is the beaming heat and light emitted by the Sun. It serves as a crucial source of renewable energy, harnessed through various technologies. These technologies fall into two main categories: passive solar (which captures and distributes solar energy without active mechanical systems) and active solar (which involves converting solar energy into usable power), Figure 2 summarizes the applications of ternary hybrid nanofluids in solar energy systems, while Figure 3 presents a schematic of a parabolic trough collector employed in solar thermal applications.

Solar systems convert sunlight to electricity via photovoltaic cells. Solar thermal systems produce heat for water and steam. Solar architecture maximizes natural light exposure and heat retention in buildings. Proper orientation, materials, and ventilation enhance energy efficiency. Researchers explored the utilization of hybrid nanofluids in solar thermal and photovoltaic/thermal (PV/T) systems, demonstrating how nanoparticles modified fluid properties to enhance solar energy performance [22]. The effectiveness of nano-thermal energy storage and hybrid nano-coolants was evaluated, considering factors such as nanoparticle stability, base fluid types, and thermal properties for efficient solar energy applications [23]. The ecological and

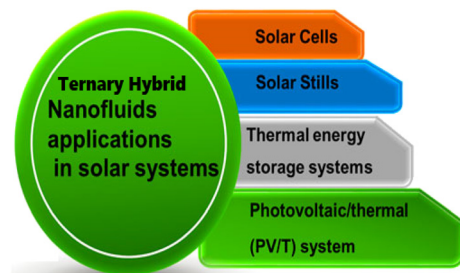


Figure 2: Ternary hybrid nanofluids' applications in solar systems.

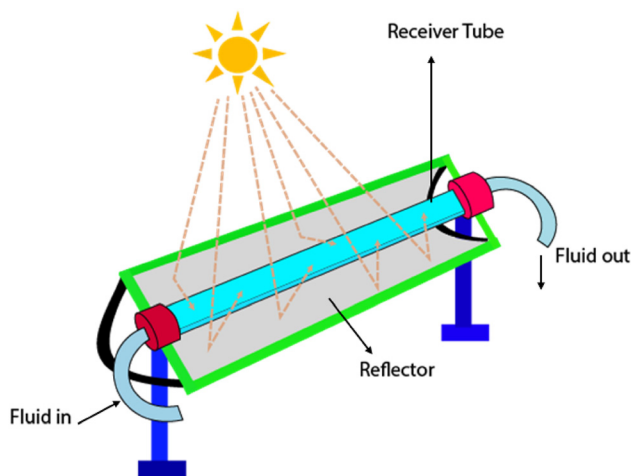


Figure 3: Diagram showing a solar collector with a parabolic trough.

economic optimization of hybrid energy systems, including solar components, was analyzed using non-dominated sorting genetic algorithms to assess various scenarios [24]. The function of stagnation point flow using a Riga plate in solar optimization was investigated, showing how hybrid nanofluids improved thermal performance and ensured uniform heat distribution in solar panels [25]. The solar-to-thermal energy conversion process was optimized by adjusting nanoparticle volume fraction, nanofluid temperature, and solar concentration [26]. They investigated the heat transfer behavior of micropolar tangent hyperbolic hybrid nanofluid flow for enhanced thermal performance [27]. The authors examined radiation and chemical reaction impacts on magnetohydrodynamic (MHD) Casson nanofluid flow supported by artificial intelligence modeling [28]. Researchers also examined enhanced heat transfer using nanofluids in different collector geometries to improve solar energy absorption [29]. Additionally, studies specifically analyzed PV/T systems, which combined solar electricity generation with thermal energy collection, focusing on key components such as PV modules, absorbers, and cooling fluids [30–32].

Response surface methodology (RSM) is a statistical tool used in fluid dynamics to optimize and analyze the effects of multiple parameters on heat transfer, flow behavior, and thermophysical properties [33]. It helps in developing predictive models for nanofluid performance, boundary layer characteristics, and drag reduction. By employing design of experiments (DOE) and regression analysis, RSM enhances efficiency in solving complex fluid flow problems [34].

After a thorough review of existing research, it was observed that the interaction of ternary hybrid nanoparticle shapes, radiative heat transfer, CC heat flux, and energy sinks or sources on unsteady thermal transmission in an Oldroyd-B mixture nanofluid confined in a Darcy–Forchheimer porous medium had not been comprehensively explored. This study addresses this gap by investigating the impact of nano, hybrid, and ternary nanofluids, incorporating TiO_2 , Fe_3O_4 , and CoFe_2O_4 nanoparticles, on heat transfer and fluid flow behavior. Unlike conventional flow through a cylinder, this study proposes a concentric cylinder configuration to optimize heat transfer, particularly analyzing differences in heat distribution near the lower and upper cylindrical surfaces across the three nanofluid types.

Although various studies have investigated heat transfer using either nano or hybrid nanofluids, the literature lacks a unified comparative analysis of nano, hybrid, and ternary nanofluids within an Oldroyd-B viscoelastic framework subjected to a Darcy–Forchheimer porous medium, CC heat flux, and MHD effects. Additionally, the influence of these nanofluid types in a concentric cylindrical geometry

and the associated variations in heat distribution between different cylindrical regions have not been adequately addressed. These limitations in existing studies highlight the need for a comprehensive comparative investigation.

This study simultaneously compares nano, hybrid, and ternary nanofluids under the same thermophysical and geometric conditions, offering new insight into the role of nanoparticle composition on heat transfer performance. A concentric cylinder configuration is used to enhance heat transfer evaluation, and RSM is integrated with BVP4C numerical simulation to statistically optimize governing parameters, providing predictive capabilities beyond traditional numerical approaches. To the best of our knowledge, such a combined assessment has not been reported previously. The results, including the comparative impact of key parameters on velocity, temperature profiles, and heat transfer rates for nano, hybrid, and ternary nanofluids, are presented through detailed graphical and tabulated analyses.

This research seeks to address the following fundamental questions:

- How does nano, hybrid, and ternary nanofluids compare with respect to heat transfer performance and temperature distribution?
- What is the influence of ternary hybrid nanoparticles on the Nusselt number, and how does it differ from hybrid and nano nanofluids?
- How does heat transfer behavior vary near the lower and upper surfaces of the concentric cylinder configuration for each nanofluid type?
- How do variations in governing physical parameters such as the Reynolds number, the Forchheimer number, and thermal radiation influence the comparative performance of nano, hybrid, and ternary nanofluids?

- What are the potential practical implications of the comparative heat transfer behavior of these nanofluids in industrial systems such as MHD cooling, energy storage, and biomedical thermal regulation?

Thus, this work provides a comparative evaluation of nano, hybrid, and ternary nanofluids, offering valuable insights into their thermal efficiency and potential applications in heat exchanger designs, MHD cooling systems, and nanofluid-based energy technologies.

2 Mathematical formulation

The Darcy–Forchheimer porous medium is utilized as a framework to examine the behavior of an unsteady, two-dimensional (2D), incompressible flow of an Oldroyd-B ternary nano liquid. This liquid consists of a base fluid comprising water, TiO_2 , Fe_3O_4 , and CoFe_2O_4 flowing along an elongated tube with flow confinement beyond $r > 0$. The radial direction is represented by the r -axis, while the z -axis denotes the cylinder's axis, sketched in Figure 4. While the inner cylinder remains stationary, the outer cylinder experiences variation. A radial magnetic field of strength B_0 is applied, with assumed negligible impact on the stemming magnetic field. The time-varying cylinder's stretching velocity is expressed as $w = 2cz/(1 - \alpha_1 t)$, where $c > 0$ and $\alpha_1 > 0$ are constants. The study accounts for influences such as CC thermal flux, radiative heat, and energy generation/sink effects.

The problem under discussion is governed by the following equations [35–38]:

$$(rv)_r + (rw)_z = 0 \quad (1)$$

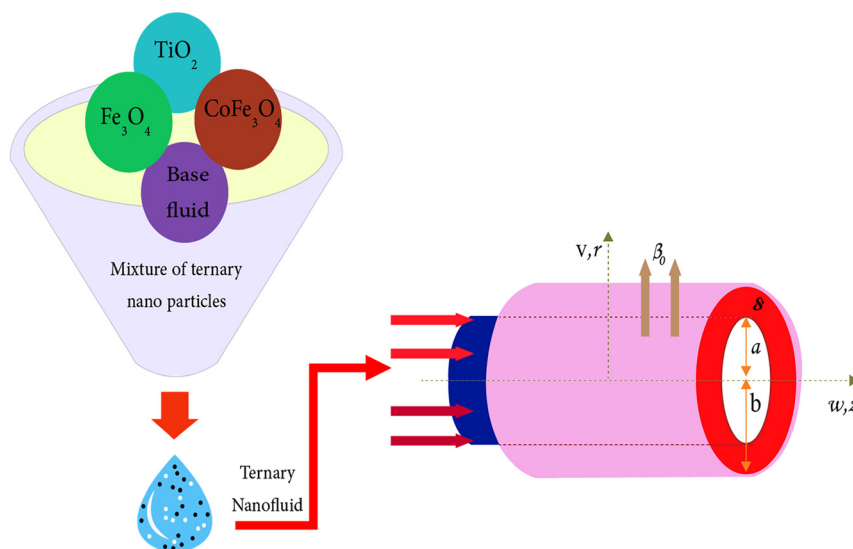


Figure 4: Flow diagram of the model.

$$\begin{aligned}
& \rho_{\text{thnf}} (w_t + v w_r + w w_z + \beta_0 (w_{tt} + w^2 w_{zz} + v^2 w_{rr} + 2v w w_{rz} + 2w w_{tz} + 2v w_{tr})) = \\
& = \left. \begin{aligned} & \mu_{\text{thnf}} \left(w_{rr} + \frac{1}{r} w_r \right) + \mu_{\text{thnf}} \beta_1 \left(\begin{aligned} & (w_{rr})_t + \frac{1}{r} w_{tr} + \frac{v}{r^2} w_r - \frac{1}{r} w_r w_z - \frac{2}{r} v_r w_r + \frac{w}{r} w_{rz} \\ & - w_r w_{rz} - 2w_r v_{rr} + \frac{v}{r} w_{rr} - w_z w_{rr} + w (w_{rr})_z + v (w_{rr})_r \end{aligned} \right) - \end{aligned} \right\} \\
& - \rho_{\text{thnf}} F w^2 - \sigma_{\text{thnf}} B_0^2 (w + \beta_0 (w_t + v w_r))
\end{aligned} \tag{2}$$

$$\begin{aligned}
& (\rho C_p)_{\text{thnf}} \left(T_t + w T_z + v T_r + \beta_2 \left(\begin{aligned} & v w_r T_z + w w_z T_z + v v_r T_r + w T_r v_z + 2v w T_{rz} \\ & + T_{tt} + w_t T_z + 2w T_{tz} + v_t T_r + 2v T_{tr} + v^2 T_{rr} + w^2 T_{zz} \end{aligned} \right) \right) = \\
& = k_{\text{thnf}} \left(T_{rr} + \frac{1}{r} T_r \right) - \frac{1}{r} (r q_r^*)_r + Q_0 (T - T_b) \}
\end{aligned} \tag{3}$$

With boundary conditions

$$\left. \begin{aligned} r = a : w = w_w = \frac{2cz}{(1 - \alpha_1 t)}, v = v_w = \frac{-ca}{\sqrt{1 - \alpha_1 t}}, T = T_w = T_b - \frac{T_{\text{rf}}}{(1 - \alpha_1 t)^{\frac{3}{2}}} \left(\frac{cz^2}{vf} \right), \\ r = b : w_r = w_{rr} = \delta_r = T_r = 0, \delta_z = v \end{aligned} \right\} \tag{4}$$

The expression for thermal radiation flux using Rosseland's approximation is [39]

$$q_r^* = -\frac{4\sigma_0}{3k_0} (T^4)_r; \quad T^4 \cong 4T_b^3 T - 3T_b^4, \tag{5}$$

where k_0 and σ_0 represent the mean absorption factor and the Stefan–Boltzmann constant, respectively.

The following set of dimensional variables is adopted to transform the governing equations into their dimensionless counterparts χ , ψ , and similarity variable ξ :

$$\left. \begin{aligned} v &= -ca\chi'(\xi)(1 - \alpha_1 t)^{-\frac{1}{2}}\xi^{-\frac{1}{2}}, w = 2cz\chi'(\xi)(1 - \alpha_1 t)^{-1}, \\ T &= T_b - T_{\text{rf}} \left(\frac{cz^2}{vf} \right) (1 - \alpha_1 t)^{-\frac{3}{2}} \theta(\xi), \xi = \left(\frac{r}{a} \right)^2 (1 - \alpha_1 t)^{-1}. \end{aligned} \right\} \tag{6}$$

At the outer cylinder of the radius b , the thickness $\xi = \left(\frac{b}{a} \right)^2 = h$.

The governing equations for a non-dimensional system are provided below.

$$\frac{2A_1}{\text{Re}} (\chi''' \xi + \chi'') - A_2 \left(\frac{s}{2} (\chi'' \xi + \chi') - \chi \chi'' + \chi'^2 + \text{De}_x \left(\frac{s^2}{2} (\chi''' \xi^2 + 4\chi'' \xi + 2\chi') + 2\chi^2 \chi'' + \frac{\chi^2 \chi''}{\xi} \right) - 4\chi \chi' \chi'' + 2s(\chi' \chi'' \xi + 2\chi'^2 - \chi \chi'' \xi - 2\chi \chi'') \right) + \quad (7)$$

$$\frac{A_1}{\text{Re}} \text{De}_d (2s(\chi'' \xi + 4\chi'' \xi + 2\chi'') + 4\chi'^2 \xi - 4\chi \chi'' \xi - 8\chi \chi'') - A_2 \text{Fr} \chi'^2 - A_3 M (\chi' + \text{De}_x S (\chi'' \xi + \chi')) - 2\text{De}_x \chi' \chi'' = 0$$

$$A_4 \text{Pr} \left(\chi \theta' - 2\chi \theta' - \frac{s}{2} \left(\theta' \xi + \frac{3}{2} \theta \right) + B_t \left(-\frac{s^2}{2} \left(\theta'' \xi^2 + 5\theta' \xi + \frac{15}{4} \theta \right) + S(2\chi \theta'' \xi + 5\chi \theta' - 2\chi'' \theta \xi - 8\chi' \theta - 3\chi' \theta' \xi) \right) \right) + \quad (8)$$

$$2(A_5 + \text{Ra})(\theta' \xi + \theta') + \frac{Q \text{Pr} \theta}{2} = 0.$$

Subjected to boundary conditions

$$\xi = 1: \chi = \chi' = \theta = 1, \quad (9) \quad \xi = h: \chi'' = \chi''' = \theta' = 0$$

In the preceding equations, $\text{Re} = \frac{\rho_f c a^2}{\mu_f}$ is the Reynolds number, $\text{Pr} = \frac{(\rho c_p)_f c a^2}{k_f}$ is the Prandtl number, $\text{Ra} = \frac{16\sigma^* T_b^3}{3k^* k_f}$ is the radiation parameter, $M = \frac{\sigma_f B_0^2 (1-\alpha_1 t)}{2\rho_f c}$ is the magnetic field, $B_t = \frac{\beta_2 c}{(1-\alpha_1 t)}$ is the thermal radiation time, $Q = \frac{(1-\alpha t)}{(\rho c_p)_f c}$ is the heat source/sink, $\text{De}_x = \frac{\beta_0 c}{(1-\alpha_1 t)}$ is the relaxation time, $\text{De}_d = \frac{\beta_1 c}{(1-\alpha_1 t)}$ is the retardation time, $\text{Fr} = \frac{c^* z}{\sqrt{K}}$ is the local inertia parameter, $S = \frac{a}{c}$ is the unsteadiness parameter, and

$$A_1 = \frac{\mu_{\text{thnf}}}{\mu_f}, A_2 = \frac{\rho_{\text{thnf}}}{\rho_f}, A_3 = \frac{\sigma_{\text{thnf}}}{\sigma_f}, A_4 = \frac{(\rho c_p)_{\text{thnf}}}{(\rho c_p)_f}, A_5 = \frac{k_{\text{thnf}}}{k_f}.$$

Correlation of ternary hybrid nanofluids [12].

Viscosity:

$$\mu_{\text{thnf}} = \frac{\mu_f}{(1-\phi_1)^{2.5} (1-\phi_2)^{2.5} (1-\phi_3)^{2.5}}$$

Density:

$$\rho_{\text{thnf}} = (1-\phi_1) \left[(1-\phi_2) \left[(1-\phi_3) + \phi_3 \frac{\rho_3}{\rho_f} \right] + \phi_2 \frac{\rho_2}{\rho_f} \right] + \phi_1 \frac{\rho_1}{\rho_f}$$

Heat capacity:

$$(\rho c_p)_{\text{thnf}} = (1-\phi_1) \left[(1-\phi_2) \left[(1-\phi_3) + \phi_3 \frac{(\rho c_p)_3}{(\rho c_p)_f} \right] + \phi_2 \frac{(\rho c_p)_2}{(\rho c_p)_f} \right] + \phi_1 \frac{(\rho c_p)_1}{(\rho c_p)_f}$$

Electrical conductivity:

$$\frac{\sigma_{\text{thnf}}}{\sigma_{\text{nf}}} = \frac{(1+2\phi_1)\sigma_1 + (1-2\phi_1)\sigma_{\text{thnf}}}{(1-\phi_1)\sigma_1 + (1+\phi_1)\sigma_{\text{thnf}}};$$

$$\frac{\sigma_{\text{thnf}}}{\sigma_{\text{nf}}} = \frac{(1+2\phi_2)\sigma_2 + (1-2\phi_2)\sigma_{\text{nf}}}{(1-\phi_2)\sigma_2 + (1+\phi_2)\sigma_{\text{nf}}};$$

$$\frac{\sigma_{\text{nf}}}{\sigma_f} = \frac{(1+2\phi_3)\sigma_3 + (1-2\phi_3)\sigma_f}{(1-\phi_3)\sigma_3 + (1+\phi_3)\sigma_f}$$

Thermal conductivity:

$$\frac{k_{\text{thnf}}}{k_{\text{thnf}}} = \frac{k_1 + 2k_{\text{thnf}} - 2\phi_1(k_{\text{thnf}} - k_1)}{k_1 + 2k_{\text{thnf}} + 2\phi_1(k_{\text{thnf}} - k_1)};$$

$$\frac{k_{\text{thnf}}}{k_{\text{nf}}} = \frac{k_2 + 2k_{\text{nf}} - 2\phi_2(k_{\text{nf}} - k_2)}{k_2 + 2k_{\text{nf}} + 2\phi_2(k_{\text{nf}} - k_2)};$$

$$\frac{k_{\text{nf}}}{k_f} = \frac{k_3 + 2k_f - 2\phi_3(k_f - k_3)}{k_3 + 2k_f + 2\phi_3(k_f - k_3)}$$

Case 1: (nano) Case 2: (hybrid) Case 3: (ternary).

$\text{TiO}_2 = 0.01$	$\text{TiO}_2 = 0.01$	$\text{TiO}_2 = 0.01$
$\text{Fe}_3\text{O}_4 = 0.01$	$\text{Fe}_3\text{O}_4 = 0.01$	$\text{Fe}_3\text{O}_4 = 0.01$
$\text{CoFe}_3\text{O}_4 = 0.01$	$\text{CoFe}_3\text{O}_4 = 0$	$\text{CoFe}_3\text{O}_4 = 0.01$

The Nusselt number is framed as [38]

$$(1 - \alpha_1 t)^{\frac{1}{2}} \text{Nu}_z = -2(A_5 + R_a) \psi'(1) \quad (10)$$

where R_a is the Rayleigh number.

3 Results and discussions

The system of ordinary differential equations (ODEs), as represented by Eqs. (7) and (8) under the specified conditions in Eq. (9), is solved using the BVP4C technique. To ensure computational accuracy, appropriate factors such as $\text{De}_d = 0.2$, $M = 2$, $\text{Fr} = 0.5$, $\text{Pr} = 6$, $\text{Bt} = 0.2$, $R_a = 4$, $Q1 = 0.5$, $P_1 = 0.01$, $\text{Re} = 5$, $S = 0.2$, $\text{De}_x = 0.2$, $P_3 = 0.01$, $l_0 = 3.7$ are consistently maintained throughout the investigation, except for variations highlighted in tables and graphical representations. The RSM is applied to optimize parameter interactions, enhancing the precision of numerical predictions. In graphical depictions, magenta (pink) lines represent nano nanofluids, cyan (light blue) lines represent hybrid nanofluids, and black lines indicate ternary nanofluids. Figures (5)–(14) illustrate the impact of different parameters on temperature distributions, thermophysical properties of ternary hybrid nanofluids are listed in Table 1. The experimental design matrix, along with the corresponding heat transfer results obtained from the RSM analysis is summarised in Table 2.

Figures 5 and 6 illustrate the influence of the Reynolds number (Re) on the velocity and temperature distributions

of nano, hybrid, and ternary nanofluids. As observed in Figure 5, the velocity profile decreases with an increment of the Reynolds number. This occurs because an increase in Re enhances the dominance of inertial forces over viscous forces, leading to improved momentum transport and reduced resistance to fluid motion. The ternary nanofluid (black line) exhibits the lowest velocity compared to hybrid (cyan) and nano (magenta) fluids due to its higher viscosity and density, which increase internal resistance and slow down the flow. In contrast, the nano fluid demonstrates the highest velocity since it has lower viscosity and experiences less resistance. Figure 6 reveals that as Re increases, the temperature profile decreases significantly. This trend is attributed to the enhanced convective heat transfer at higher Re , which results in a thinner thermal boundary layer and more efficient heat dissipation.

Figures 7 and 8 depict the impact of the magnetic field parameter on the velocity and temperature distributions of nano, hybrid, and ternary nanofluids. Figure 7 reveals that an increase in the magnetic field parameter causes a noticeable reduction in velocity across all nanofluid categories. This reduction arises from the Lorentz force generated by the applied magnetic field, which resists the fluid motion. A stronger magnetic field intensifies this opposing force, thereby producing a more significant decline in velocity. In contrast, Figure 8 illustrates that the temperature distribution increases with higher magnetic field values, indicating that magnetic effects contribute to enhanced thermal energy within the fluid. This is a direct consequence of the magnetic field-induced resistance, which slows down the fluid and increases energy dissipation, resulting in enhanced heat retention within the system.

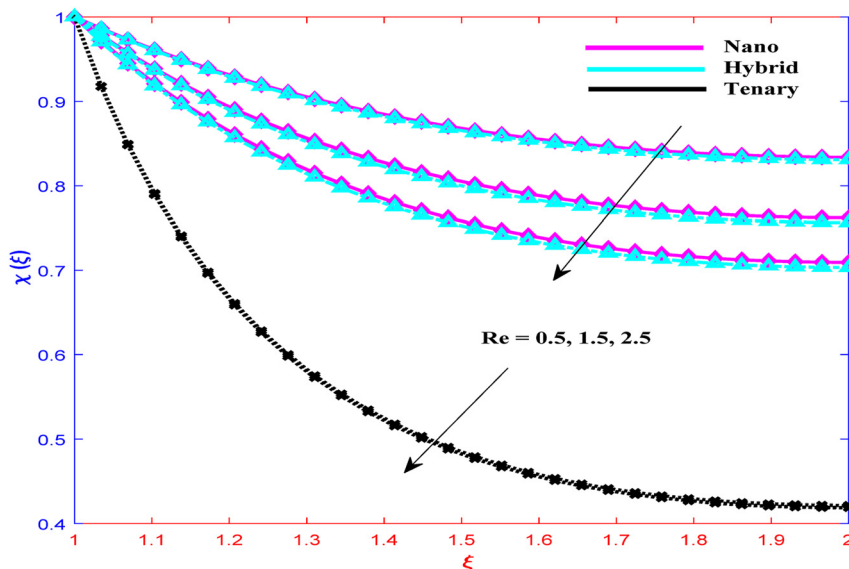
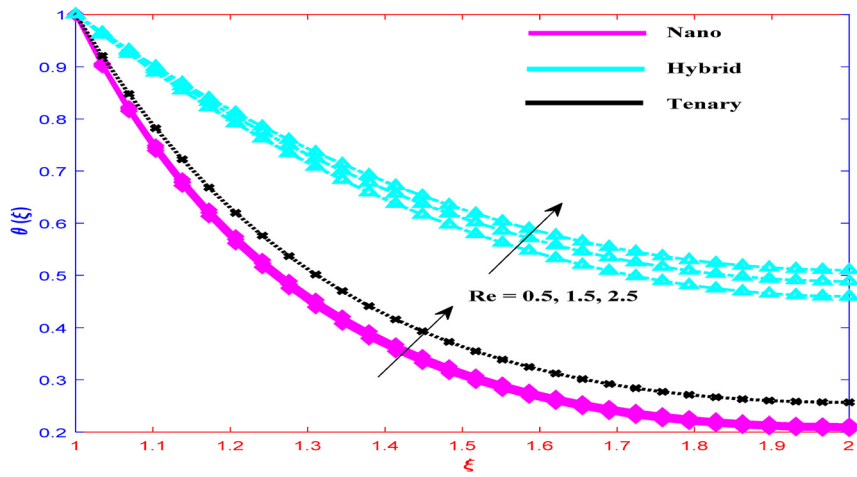
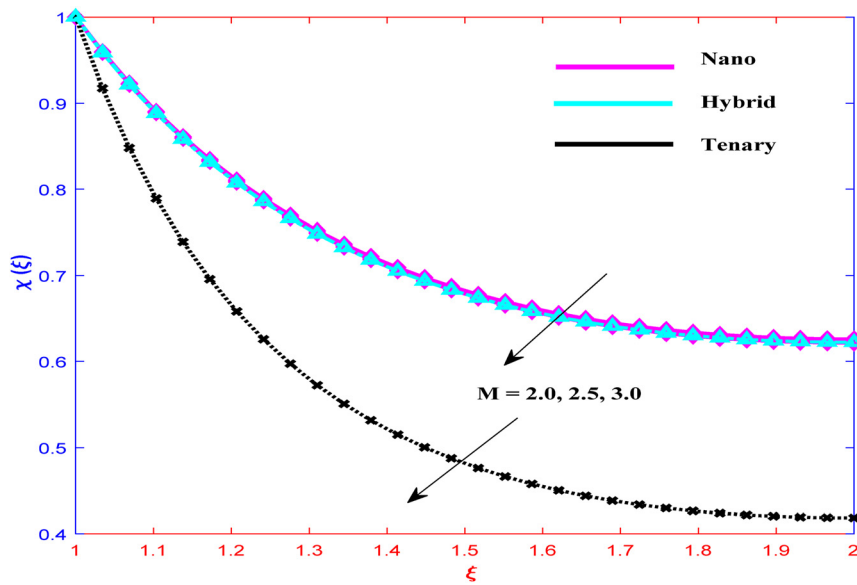
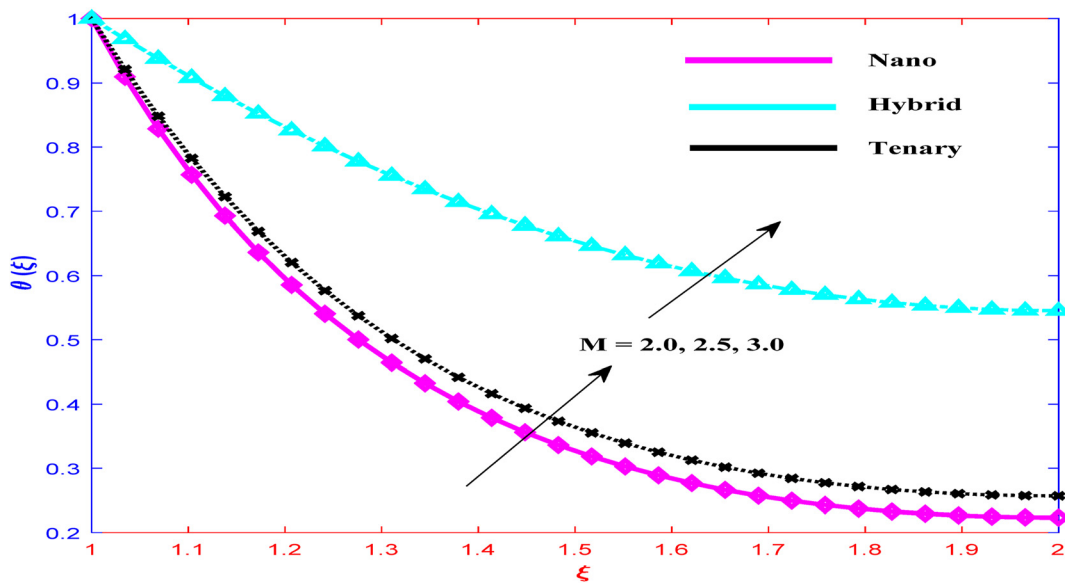


Figure 5: Velocity profile of Re .

Figure 6: Temperature profile of Re .Figure 7: Velocity profile of M .Figure 8: Temperature profile of M .

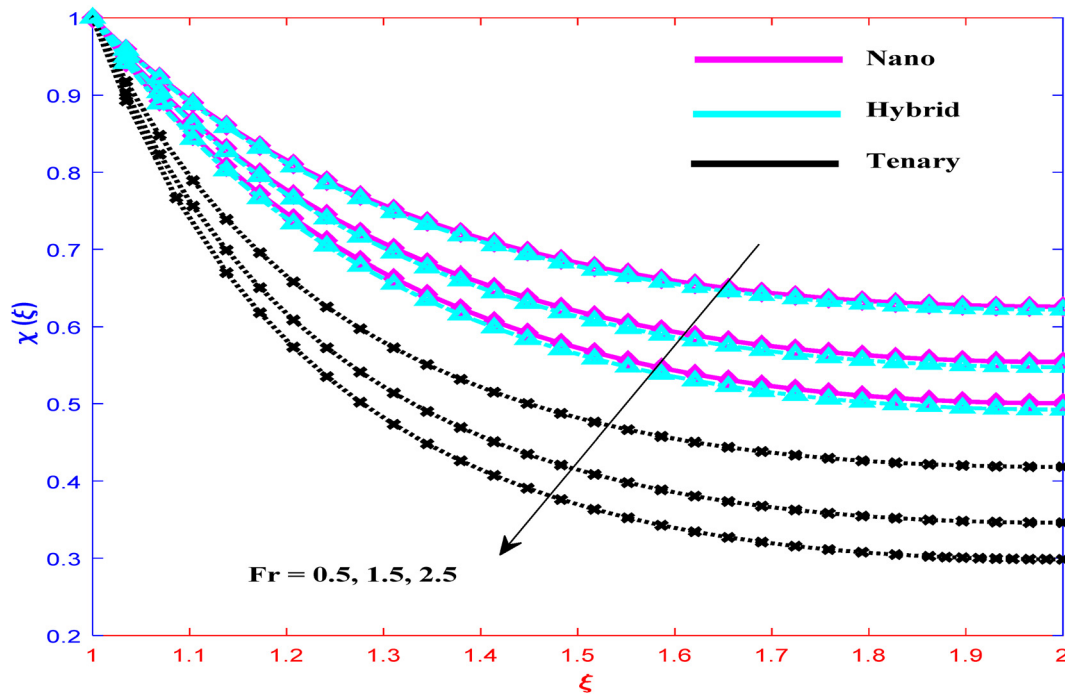


Figure 9: Velocity profile of Fr.

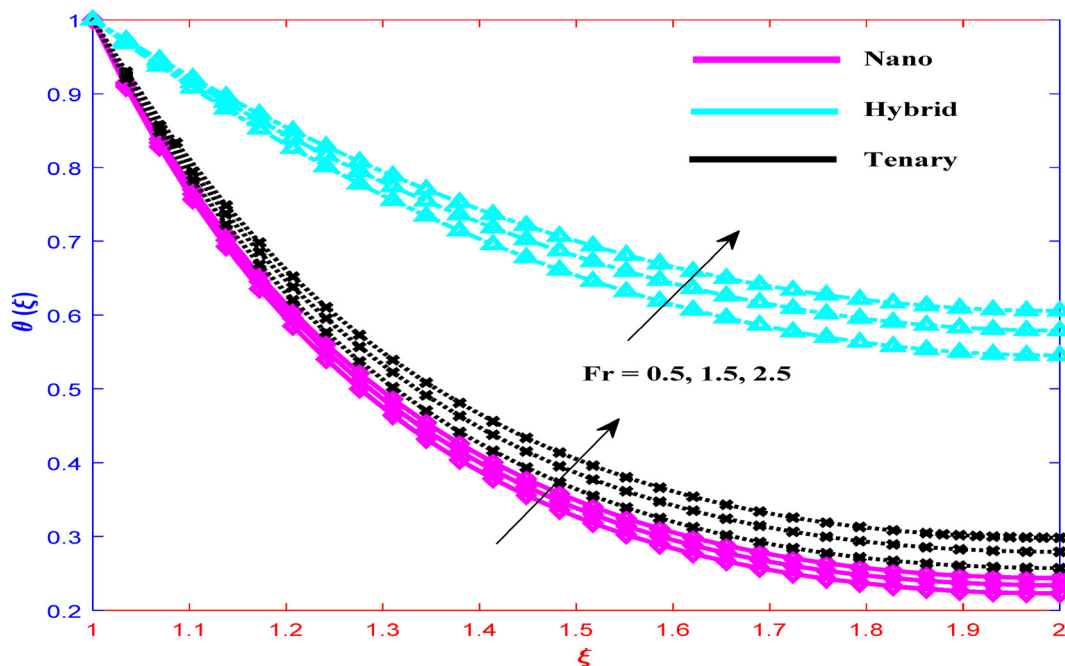


Figure 10: Temperature profile of Fr.

Figures 9 and 10 illustrate the effect of the Forchheimer number (Fr) on the velocity and temperature profiles for nano, hybrid, and ternary nanofluids. As seen in Figure 9, an increase in Fr leads to a significant reduction in velocity across all fluid types. In contrast, Figure 10 shows that as Fr

increases, the temperature profile rises. This is because the increased drag resistance at higher Fr slows down the fluid, restricting convective heat transfer and causing more heat to accumulate within the system. The ternary nanofluid again shows the highest temperature due to its lower

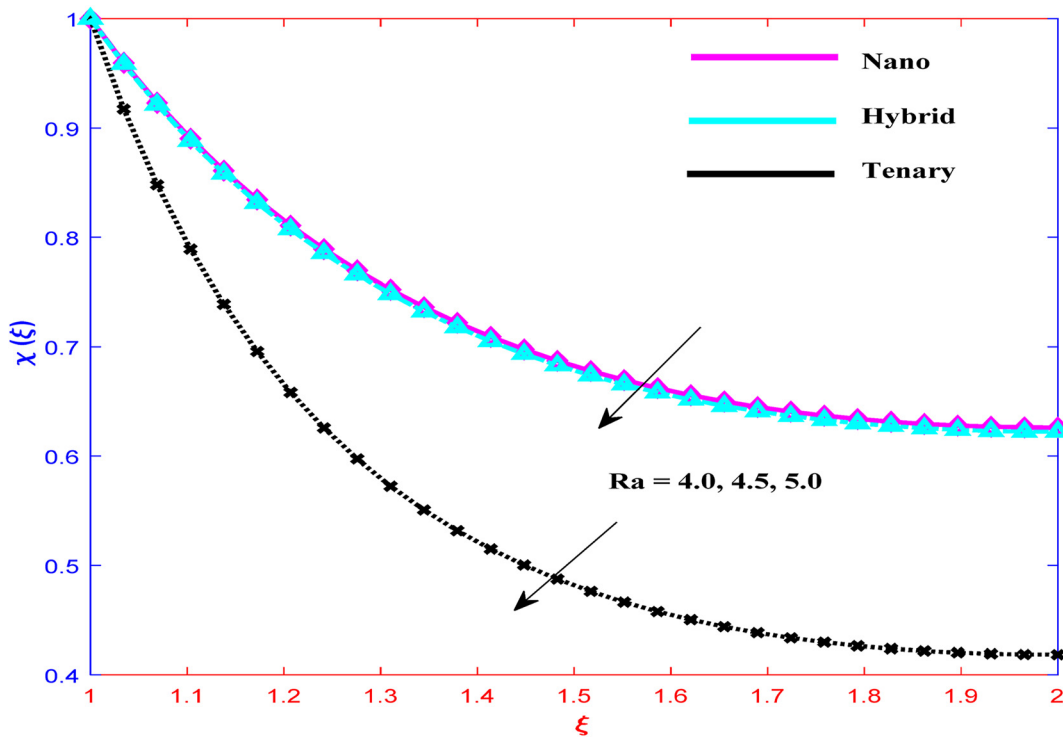


Figure 11: Velocity profile of Ra.

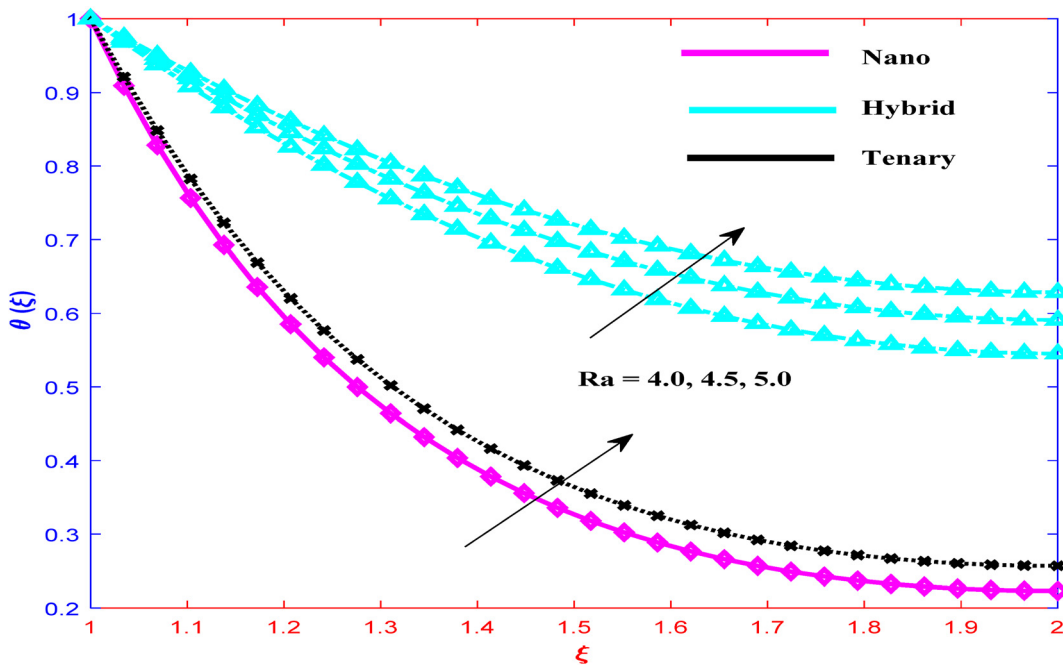


Figure 12: Temperature profile of Ra.

convective efficiency, while the nano fluid, benefiting from lower resistance, exhibits the lowest temperature.

Figures 11 and 12 illustrate the impact of the radiation parameter (Ra) on the velocity and temperature profiles. In

Figure 11, the velocity profile decreases as Ra increases across all fluid types. This is because thermal radiation raises the energy of the fluid, reducing viscosity effects and promoting fluid motion. Figure 12 shows that as Ra increases,

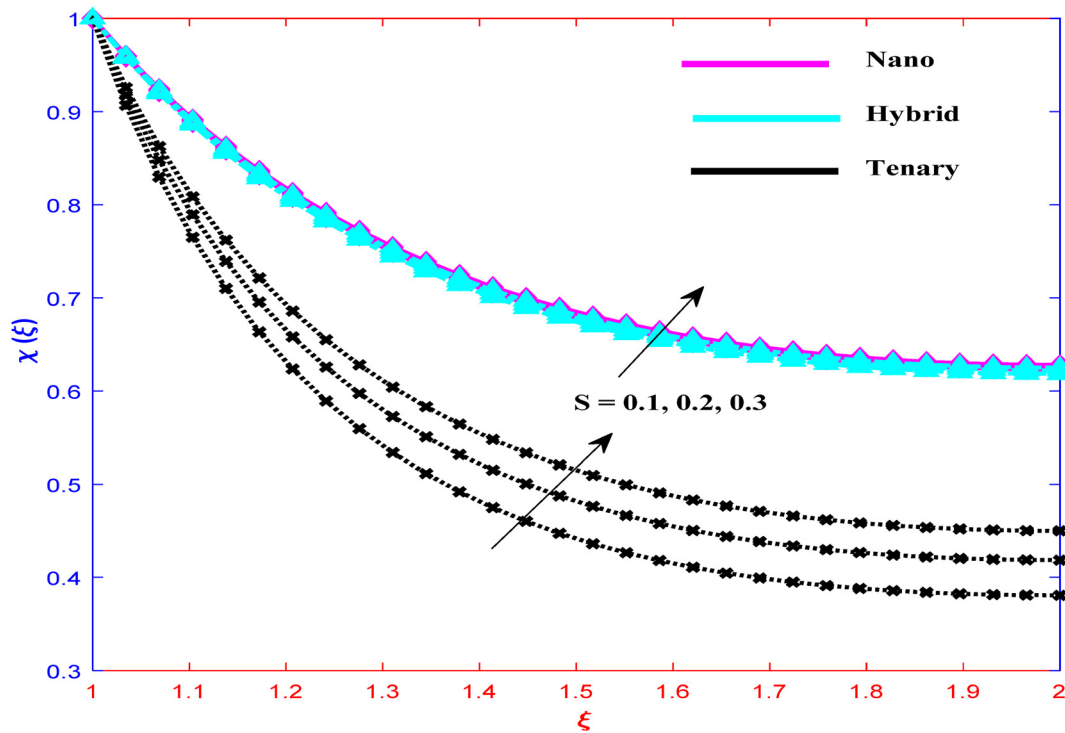


Figure 13: Velocity profile of S .

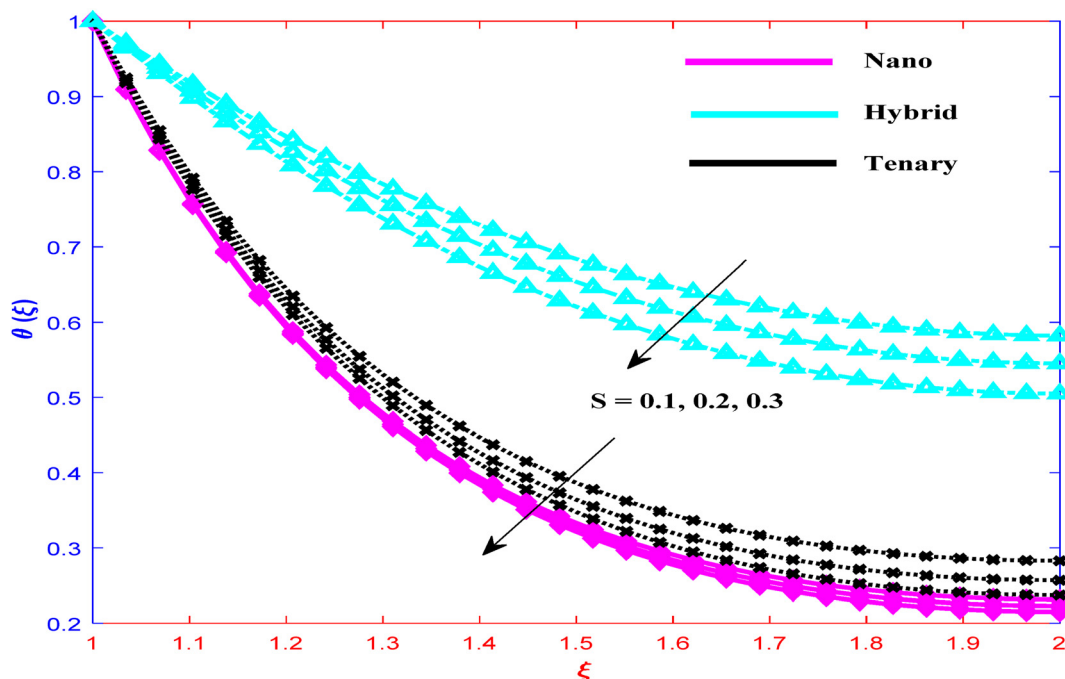


Figure 14: Temperature profile of S .

the temperature profile rises significantly for all nanofluid types. Radiation acts as an additional heat source, increasing thermal energy in the system and reducing heat dissipation through convection.

Figures 13 and 14 illustrate the influence of the unsteadiness parameter (S) on the velocity and temperature distributions for nano, hybrid, and ternary nanofluids. In Figure 13, as S increases ($S = 0.1, 0.2, 0.3$), the velocity profile

Table 1: Thermophysical properties of ternary hybrid nanofluids [12, 36, 40].

Properties	Water (H ₂ O)	Cobalt ferrite (CoFe ₂ O ₄)	Magnetite (Fe ₃ O ₄)	Titanium dioxide (TiO ₂)
K (w/mk)	0.6071	3.7	9.7	8.953
ρ (kg/m ³)	997	4,907	5,180	4,250
C_p (J/kgk)	4,179	700	670	686.2
σ (s/m)	5.5×10^7	1.1×10^7	0.74×10^6	2.4×10^6

decreases for all nanofluid types. This indicates that higher unsteadiness levels lead to a reduction in momentum transfer within the flow. Physically, this occurs because an increase in S represents stronger time-dependent effects, which reduce the influence of convective acceleration and cause a slower fluid motion. In Figure 14, an increment in S results in a decline in temperature across all nanofluids. This behavior arises because higher unsteadiness causes the fluid to undergo faster temporal variations, reducing the thermal boundary layer thickness and enhancing heat dissipation. The nano fluid (magenta line) exhibits the lowest temperature, benefiting from improved heat transfer, whereas the ternary nanofluid (black line) maintains the highest temperature due to its lower thermal diffusivity. The hybrid nanofluid (cyan line) shows intermediate behavior, balancing heat dissipation and thermal conductivity.

Table 2: Experimental design and heat transfer results.

Runs	Coded values			Actual values			Output		
	Y_1	Y_2	Y_3	M	Ra	Fr	Nus-1	Nus-2	Nus-3
1	-1	-1	-1	2	4	0.5	32.401876	11.168424	28.786413
2	1	-1	-1	3	4	0.5	32.326058	11.168414	28.786413
3	-1	1	-1	2	5	0.5	39.861993	10.761156	35.187089
4	1	1	-1	3	5	0.5	39.768720	10.761145	35.187090
5	-1	-1	1	2	4	2.5	30.148073	9.487535	25.167158
6	1	-1	1	3	4	2.5	30.081687	9.487527	25.167158
7	-1	1	1	2	5	2.5	37.089281	9.092955	30.763092
8	1	1	1	3	5	2.5	37.007610	9.092947	30.763092
9	-1	0	0	2	4.5	1.5	34.762621	10.002595	29.737052
10	1	0	0	3	4.5	1.5	34.684086	10.002586	29.737049
11	0	-1	0	2.5	4	1.5	31.103495	10.228371	26.761799
12	0	1	0	2.5	5	1.5	38.308031	9.826098	32.712301
13	0	0	-1	2.5	4.5	0.5	36.089695	10.939955	31.986751
14	0	0	1	2.5	4.5	2.5	33.581690	9.265957	27.965125
15	0	0	0	2.5	4.5	1.5	34.723383	10.002591	29.737049
16	0	0	0	2.5	4.5	1.5	34.723383	10.002591	29.737049
17	0	0	0	2.5	4.5	1.5	34.723383	10.002591	29.737049
18	0	0	0	2.5	4.5	1.5	34.723383	10.002591	29.737049
19	0	0	0	2.5	4.5	1.5	34.723383	10.002591	29.737049
20	0	0	0	2.5	4.5	1.5	34.723383	10.002591	29.737049

3.1 Response surface methodology (RSM)

In fluid dynamics, RSM is a powerful tool for optimizing flow parameters, analyzing interactions between governing variables, and improving the efficiency of computational or experimental studies. RSM is particularly useful when dealing with nonlinear and multivariable systems, such as turbulence modeling, heat transfer enhancement, or aerodynamic shape optimization. By constructing surrogate models, typically using second-order polynomial regression, RSM helps in approximating complex relationships between input parameters such as the Reynolds number, the Mach number, viscosity, or thermal conductivity and key response variables like drag coefficient, heat transfer rate, or velocity profiles. Through systematic experimental design techniques, such as central composite design (CCD) or Box–Behnken design (BBD), RSM reduces computational cost while maximizing insight into system behavior. It enables researchers to identify optimal conditions, study sensitivity effects, and refine numerical simulations without requiring an exhaustive set of high-fidelity simulations or experiments.

3.1.1 RSM, optimization outcomes

This study focuses on three key parameters: the magnetic field, radiation parameter, and local inertial parameter,

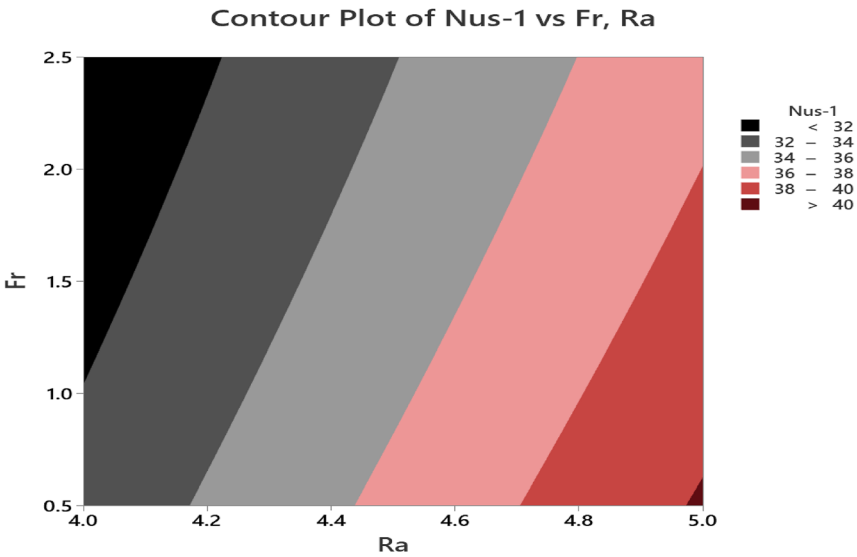


Figure 15: Contour visualization of Nus-1 with respect to Fr and Ra.

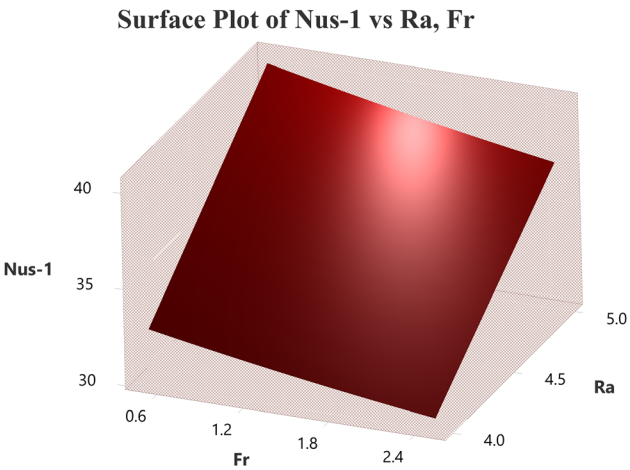


Figure 16: Surface visualization of Nus-1 with respect to Ra and Fr.

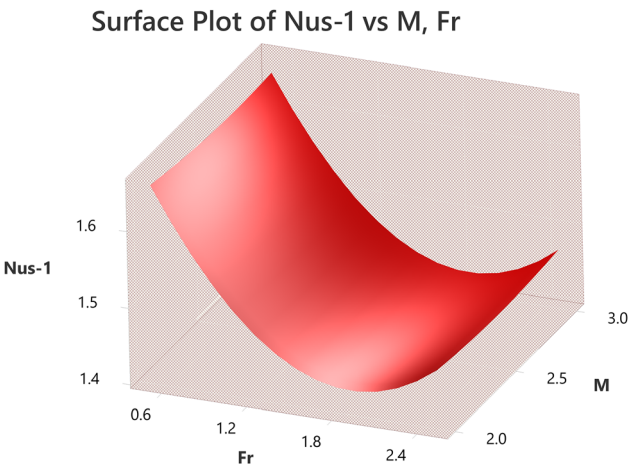


Figure 18: Surface visualization of Nus-1 with respect to M and Fr.

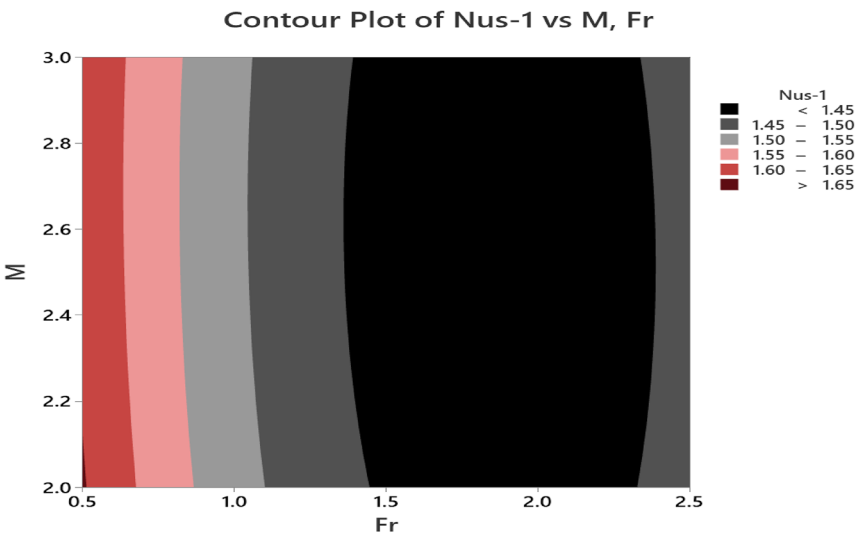


Figure 17: Contour visualization of Nus-1 with respect to M and Fr.

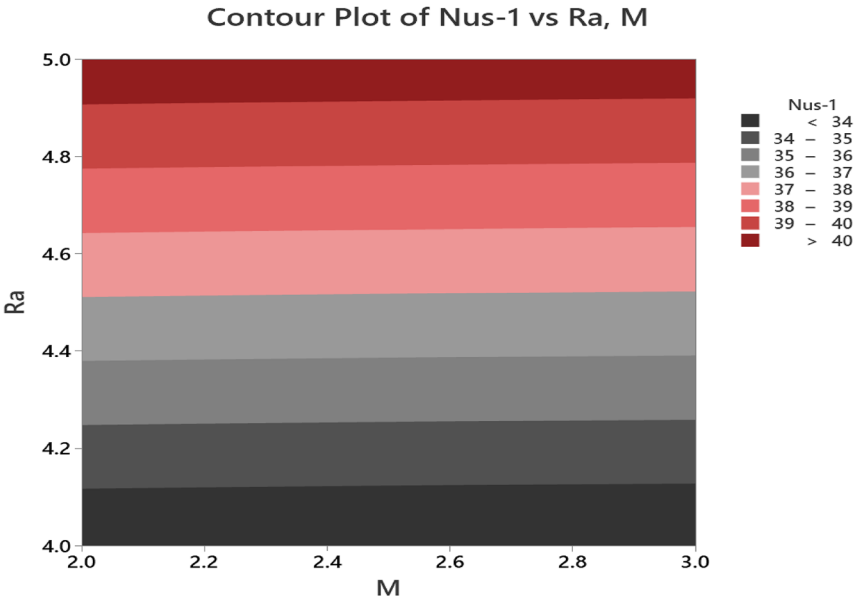


Figure 19: Contour visualization of Nus-1 with respect to Ra and M .

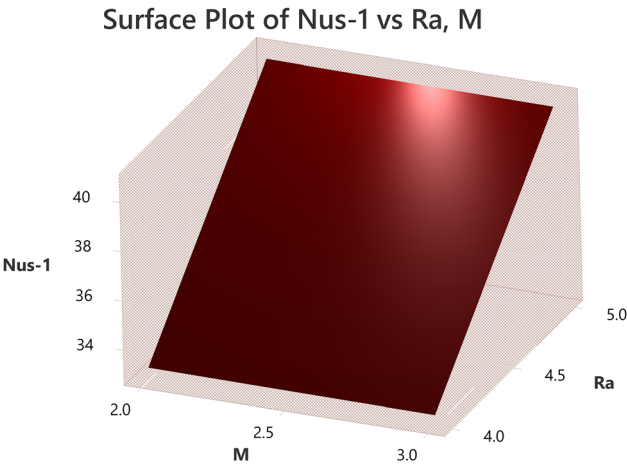


Figure 20: Surface visualization of Nus-1 with respect to Ra and M .

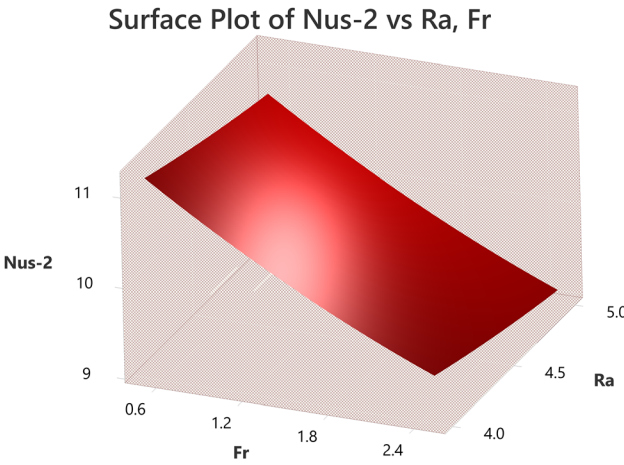


Figure 22: Surface visualization of Nus-2 with respect to Ra and Fr .

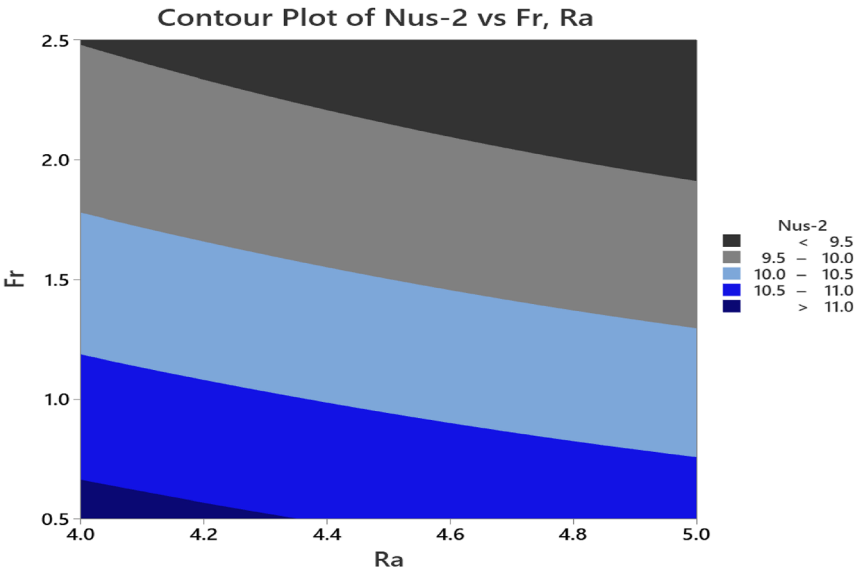


Figure 21: Contour visualization of Nus-2 with respect to Fr and Ra.

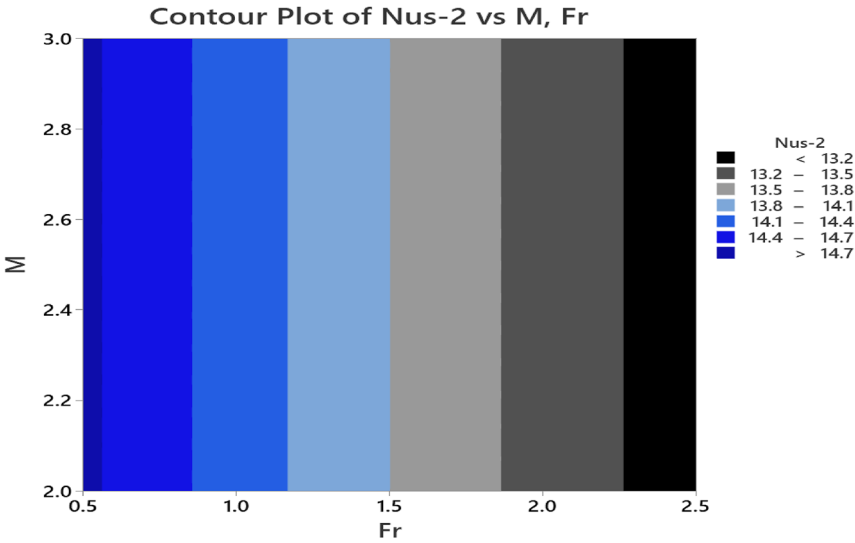


Figure 23: Contour visualization of Nus-2 with respect to M and Fr .

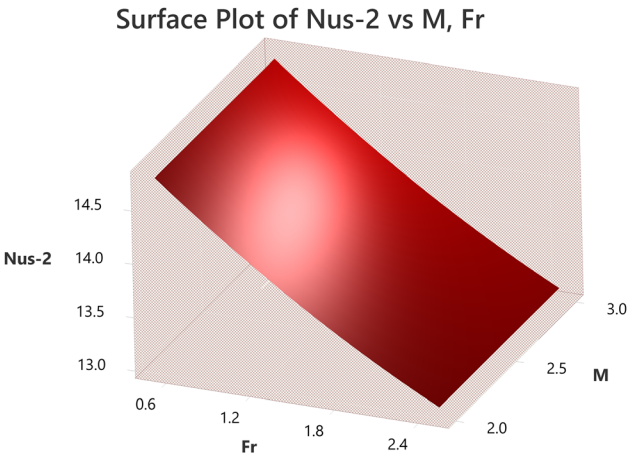


Figure 24: Surface visualization of Nus-2 with respect to M and Fr .

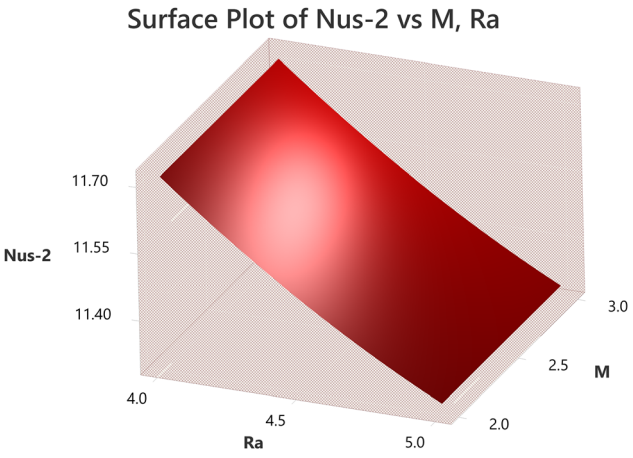


Figure 26: Surface visualization of Nus-2 with respect to M and Ra .

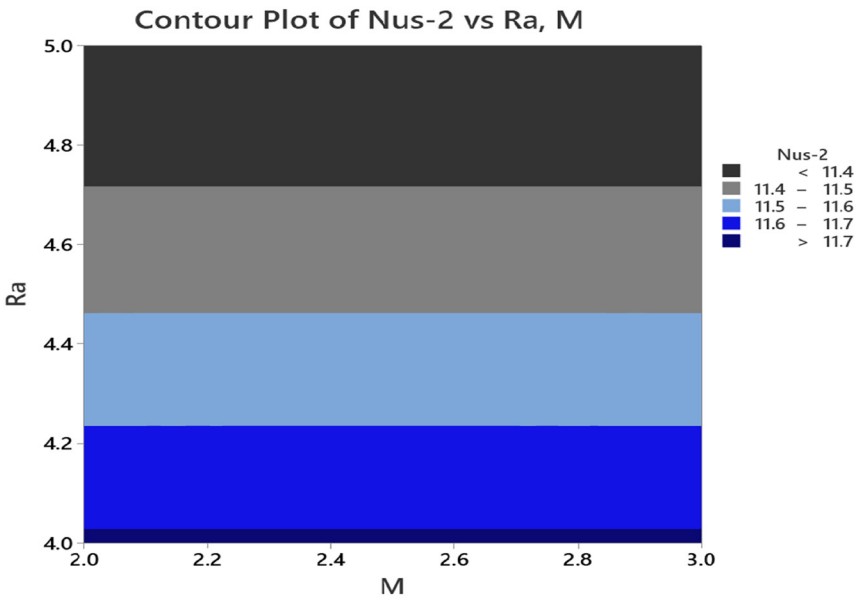


Figure 25: Contour visualization of Nus-2 with respect to Ra and M .

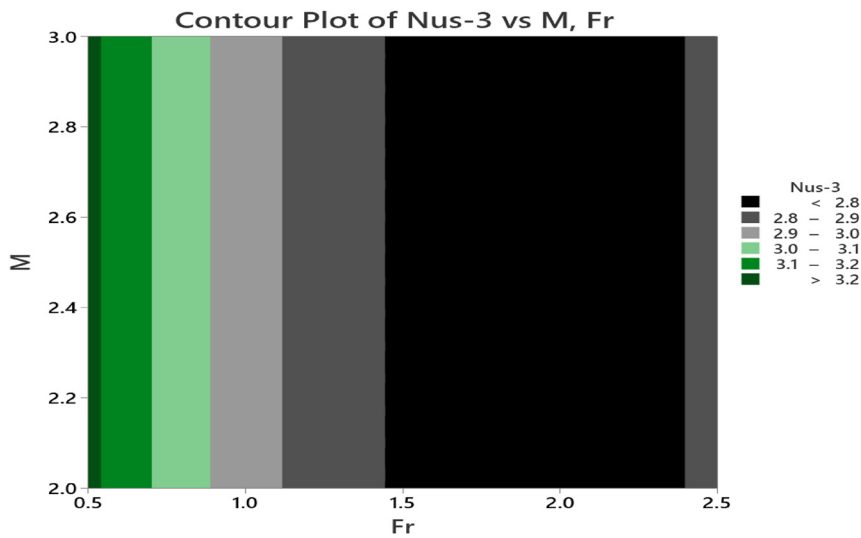


Figure 27: Contour visualization of Nus-3 with respect to M and Fr .

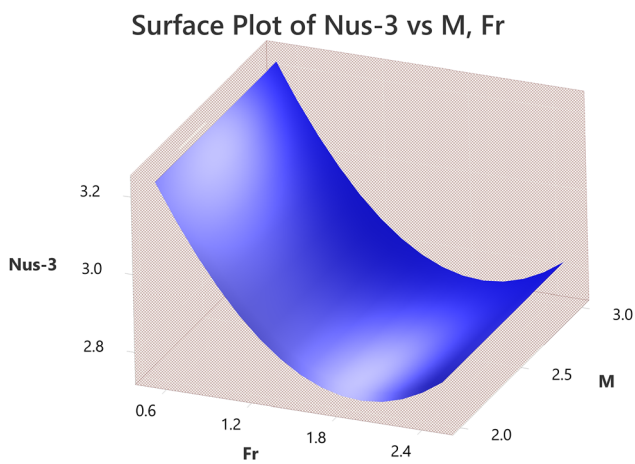


Figure 28: Surface visualization of Nus-3 with respect to M and Fr .

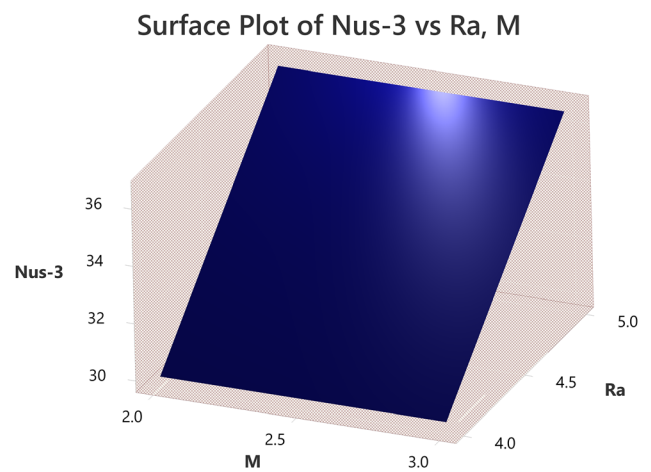


Figure 30: Surface visualization of Nus-23 with respect to Ra and M .

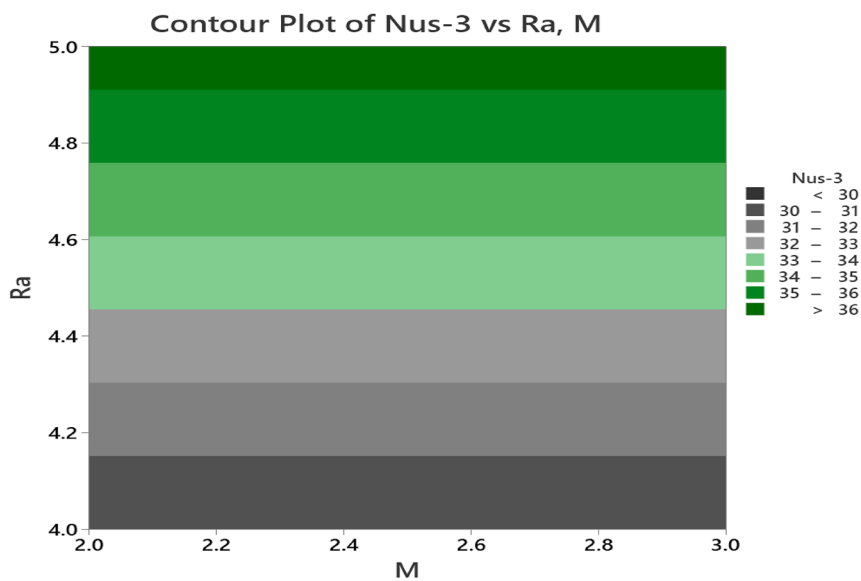


Figure 29: Contour visualization of Nus-3 v with respect to Ra and M .

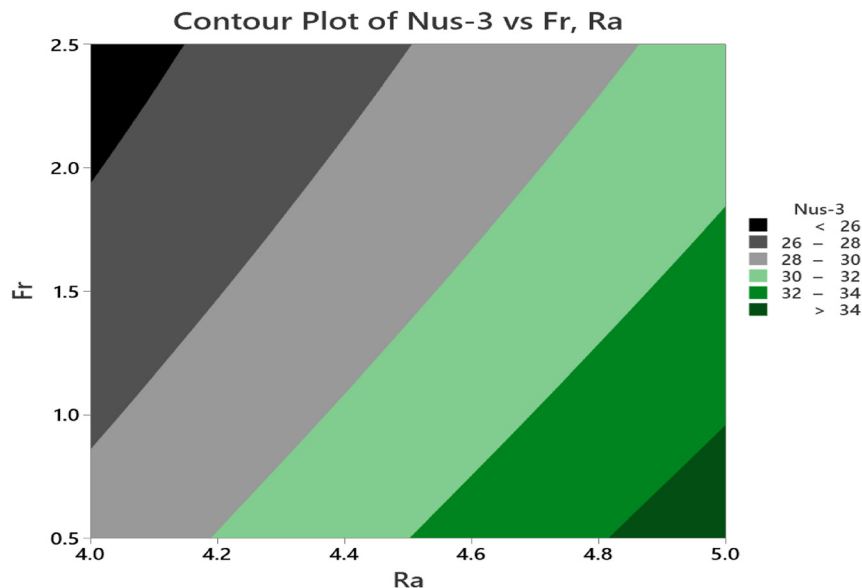


Figure 31: Contour visualization of Nus-3 with respect to Fr and Ra.

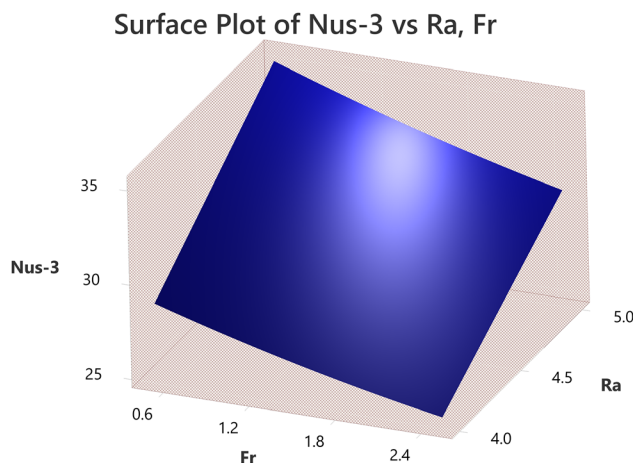


Figure 32: Surface visualization of Nus-3 with respect to Ra and Fr.

which are analyzed to determine their optimal influence on heat transfer enhancement. Each parameter is examined at three levels: low (−1), medium (0), and high (1). The relevant factors, their corresponding levels, and coded values are systematically summarized in Table 3.

3.1.2 RSM, graphical representation

Surface and contour plots are essential tools for visualizing the relationship between multiple parameters and their

impact on heat transfer characteristics in fluid flow. In this study, these plots provide insight into how essential governing parameters, such as the magnetic field (M), Forchheimer number (Fr), and radiation parameter (Ra), influence the Nusselt number (Nu), which quantifies the rate of heat transfer. From Figures 15–32, the surface plots offer a three-dimensional (3D) perspective, demonstrating how the heat transfer rate varies under different parameter combinations. These visualizations highlight the nonlinear interactions between various factors affecting convective heat transfer. In contrast, the contour plots provide a 2D representation of the same data, clearly distinguishing regions of high and low heat transfer rates. These graphical analyses offer a comprehensive understanding of how different parameters affect heat distribution, assisting in optimizing operating conditions for enhanced thermal performance. The Nusselt number (Nu) for nano ($Nu-1$), hybrid ($Nu-2$), and ternary ($Nu-3$) nanofluids is examined to compare heat transfer performance under different conditions.

Figures 15 and 16 illustrate the influence of the Forchheimer number (Fr) and the radiation parameter (Ra) on the Nusselt number ($Nu-1$) for nano nanofluids. In Figure 15 (contour plot), as Ra increases, the heat transfer rate ($Nu-1$) increases significantly, indicated by the transition from darker to lighter shades. Figure 16 provides a surface plot of the same relationship, offering a 3D visualization of the variations in ($Nu-1$). The upward slope in the surface

Table 3: Key RSM parameters, symbols, and corresponding levels.

Key parameters	Symbols	Levels		
		−1	0	1
M	Y_1	2	2.5	3
Ra	Y_2	4	4.5	5
Fr	Y_3	0.5	1.5	2.5

confirms that higher Ra results in better heat transfer, whereas an increase in Fr reduces the Nusselt number because of enhanced resistive forces within the porous medium. Figures 17 and 18 illustrate the impact of the Forchheimer number (Fr) and the magnetic field parameter (M) on the Nusselt number (Nus-1) for nano nanofluids. In Figure 17, the contour plot shows that with simultaneous increase in Fr and M , the heat transfer rate (Nus-1) significantly decreases, with the darkest regions indicating the lowest values. Figure 18 presents a surface plot of the same relationship, providing a 3D view of the variation in Nus-1. The downward slope in the surface confirms that as M and Fr increase, the Nusselt number declines, reinforcing the observation that both parameters act as opposing forces to convective heat transfer. Figures 19 and 20 illustrate the effect of the radiation parameter (Ra) and the magnetic field parameter (M) on the Nusselt number for nano nanofluids. The contour plot in Figure 19 shows a distinct trend where increasing Ra leads to a significant enhancement in Nus-1, indicated by the transition from darker to lighter shades. The surface plot in Figure 20 further confirms this observation by providing a 3D view of the relationship between Ra , M , and Nus-1. The plot exhibits a clear upward slope along the Ra axis, reinforcing the idea that higher Ra values correspond to increased heat transfer.

Figures 21 and 22 illustrate the influence of the Forchheimer number (Fr) and the radiation parameter (Ra) on the Nusselt number (Nus-2) for hybrid nanofluids. The contour plot in Figure 21 shows a distinct trend where Nus-2 increases with higher Ra but decreases with increasing Fr . The transition from dark to lighter blue shades indicates that lower Fr values result in improved heat transfer, while higher Fr values suppress it. The surface plot in Figure 22 provides a 3D representation of the interaction between Ra , Fr , and Nus-2. The plot exhibits an increasing trend along the Ra axis, confirming that higher radiation levels contribute to better heat transfer in hybrid nanofluids. Conversely, along the Fr axis, a downward slope is observed, indicating that

greater flow resistance due to increased Fr reduces the Nusselt number. Figures 23 and 24 illustrate the impact of the magnetic field parameter (M) and the Forchheimer number (Fr) on the Nusselt number (Nus-2) for hybrid nanofluids. The contour plot in Figure 23 shows a distinct decreasing trend in Nus-2 as Fr increases, indicated by the transition from blue to darker shades. The surface plot in Figure 24 provides a 3D visualization of these interactions, further confirming that Nus-2 decreases with increasing Fr . The downward slope along the Fr axis emphasizes that higher inertial resistance in the porous medium significantly restricts heat transfer. Figures 25 and 26 illustrate the relationship between the Nusselt number (Nus-2), the radiation parameter (Ra), and the magnetic field parameter (M) for hybrid nanofluids. The contour plot in Figure 25 demonstrates that Nus-2 increases with Ra , as indicated by the transition from darker to lighter shades along the vertical axis. This suggests that higher radiation enhances heat transfer by increasing thermal energy within the fluid. The surface plot in Figure 26 provides a 3D perspective of these interactions, confirming that Nus-2 follows an increasing trend with Ra . The plot also reveals a slight variation with M , though its impact is not as significant as Ra .

Figures 27 and 28 depict the impact of the magnetic field parameter (M) and the Forchheimer number (Fr) on the Nusselt number (Nus-3) for ternary nanofluids. The contour plot in Figure 27 shows a clear trend where Nus-3 decreases significantly as Fr increases. The surface plot in Figure 28 further visualizes this relationship in a 3D perspective. The downward curvature along the Fr axis confirms that as Fr increases, Nus-3 decreases sharply, reinforcing the idea that higher inertial resistance suppresses heat transfer. Figures 29 and 30 illustrate the effect of the radiation parameter (Ra) and the magnetic field parameter (M) on the Nusselt number (Nus-3) for ternary nanofluids. The contour plot in Figure 29 shows a clear increasing trend in Nus-3 with higher Ra , indicated by the transition from darker to lighter shades along the Ra axis. The surface plot in Figure 30 provides a 3D

visualization of the interaction between Ra , M , and $Nus-3$. The plot confirms that $Nus-3$ increases steadily with Ra , reinforcing the observation that radiation significantly enhances heat transfer in ternary nanofluids. Figures 31 and 32 illustrate the impact of the Forchheimer number (Fr) and the radiation parameter (Ra) on the Nusselt number ($Nus-3$) for ternary nanofluids. The contour plot in Figure 31 reveals a distinct trend where $Nus-3$ increases with Ra , indicated by the transition from darker to lighter shades along the Ra axis. This suggests that as Ra increases, heat transfer improves due to enhanced radiative energy absorption. The surface plot in Figure 32 provides a 3D visualization of the interaction between Ra , Fr , and $Nus-3$. The plot exhibits an upward slope along the Ra axis, reinforcing the idea that increasing radiation enhances the heat transfer rate.

Table 4 presents the heat transfer rates in terms of the Nusselt number (Nus) for nano ($Nus-1$), hybrid ($Nus-2$), and ternary ($Nus-3$) nanofluids under varying parameters. As the Reynolds number (Re) increases, the heat transfer rate (Nus) decreases for all three nanofluids, with the most noticeable decline in ternary nanofluids. Conversely, increasing the unsteadiness parameter (S) leads to an increase in the Nusselt number, indicating an enhancement in heat transfer for all fluid types. For the magnetic field

parameter (M), an increase results in a reduction in the heat transfer rate, with the effect being more pronounced in ternary nanofluids, which already experiences stronger resistive forces. A similar decreasing trend is observed with the Forchheimer number (Fr), where the heat transfer rate declines as Fr increases, particularly in ternary nanofluids due to its greater flow resistance. The radiation parameter (Ra) shows an opposite effect, where increasing Ra leads to an improvement in the heat transfer rate, with ternary nanofluids experiencing the highest enhancement due to the strong absorption of radiative heat. Similarly, the heat source parameter (Q) significantly enhances the Nusselt number across all nanofluids, with ternary nanofluids benefiting the most, indicating that an additional heat input plays a significant role in improving its thermal performance. Overall, Re , M , and Fr reduce the heat transfer rate, while S , Ra , and Q enhance it. Among the nanofluids, nano nanofluid maintains the highest heat transfer rate, followed by hybrid, while the ternary nanofluid consistently exhibits the lowest heat transfer performance.

Grid independence test:

Figure 33 illustrates that at medium and fine mesh both results are matching. It shows that present study results are satisfying the grid independence test.

Table 4: Heat transfer rates for $Nus-1$ (nano), $Nus-2$ (hybrid), and $Nus-3$ (ternary).

Re	S	M	Fr	Ra	De_x	De_d	Q	$Nus-1$	$Nus-2$	$Nus-3$
0.5								35.474363	13.631099	28.878255
1.5								34.519058	12.811812	28.810419
2.5								33.756765	12.190647	28.796719
	0.1							32.337543	12.403904	27.393082
	0.2							32.401876	11.168424	28.786413
	0.3							32.367510	10.088877	29.722672
		2.0						32.401876	28.786413	11.168424
		2.5						32.363997	28.786413	11.168419
		3.0						32.326058	28.786413	11.168414
			0.5					32.401876	11.168424	28.786413
			1.5					31.173923	10.228380	26.761801
			2.5					30.148073	9.487535	25.167158
				4.0				32.401876	11.168424	28.786413
				4.5				36.131935	10.939960	31.986751
				5.0				39.861993	10.761156	35.187089
					0.1			31.731576	10.600840	21.309264
					0.2			32.401876	11.168424	28.786413
					0.3			32.840396	11.537177	31.215149
						0.1		31.918301	10.825540	28.784272
						0.4		32.077640	10.937398	28.784914
						0.7		32.176657	11.007452	28.785342
							1.0	32.401876	12.059795	28.786414
							3.0	32.401876	15.304368	28.786416
							5.0	32.401876	18.147411	28.786419

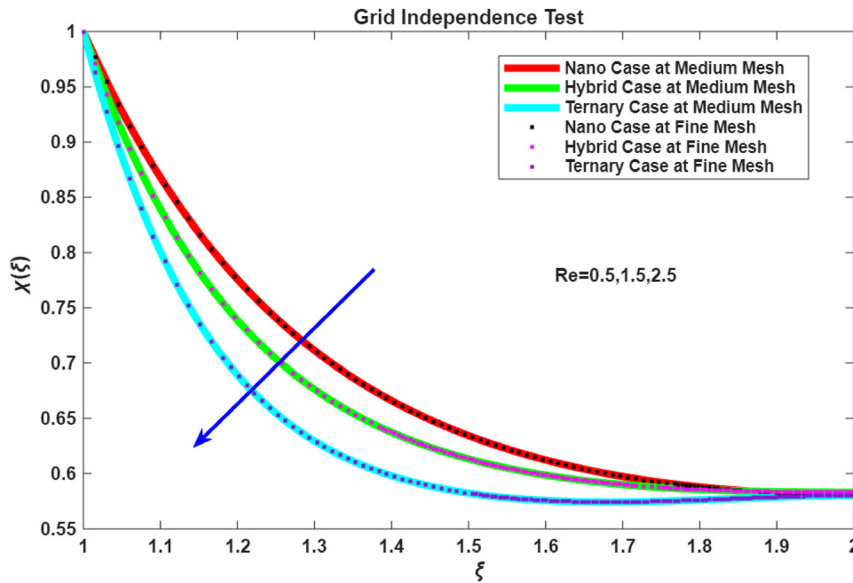


Figure 33: Grid independence test.

4 Conclusions

This research carries out a computational investigation into the Oldroyd-B ternary nanofluid flow incorporating TiO_2 , Fe_3O_4 , and CoFe_3O_4 nanoparticles in a Darcy–Forchheimer porous medium, considering thermal radiation, MHD, heat source/sink, viscous dissipation, relaxation, and retardation effects. The governing equations, originally expressed as a set of nonlinear partial differential equations, were reduced to ordinary differential form and subsequently solved using the BVP4C technique. Additionally, RSM was utilized to analyze parameter interactions and optimize heat transfer performance. The findings demonstrate that nano nanofluids exhibit the highest heat transfer performance, followed by hybrid nanofluids, while ternary nanofluids show the lowest thermal efficiency due to higher resistance. These results make nano nanofluids highly suitable for heat exchangers, MHD cooling systems, biomedical thermal applications, and energy-efficient nanofluid systems, while hybrid and ternary nanofluids may be considered in applications where enhanced stability or specific material properties are required. This work bridges theoretical modeling with real-world applications, contributing to advancements in fluid dynamics, industrial cooling, and sustainable energy technologies.

A concise summary of the main results is presented as follows:

- Velocity decreases for all parameters (Reynolds number, magnetic field, Forchheimer number, and radiation parameter).
- Velocity increases only with the unsteadiness parameter (S).
- Temperature increases with the Reynolds number, magnetic field, Forchheimer number, and radiation parameter.
- Temperature decreases with the unsteadiness parameter (S).
- Nanofluid: Has the highest velocity and lowest temperature due to superior convective heat transfer.
- Hybrid nanofluid: Exhibits moderate velocity and temperature behavior.
- Ternary nanofluid: Has the lowest velocity and highest temperature due to higher resistance to flow and reduced convective cooling.
- Temperature is observed to decline with the rising values of Re , M , and Fr , while it improves with S , Ra , and Q , with nano fluid exhibiting the highest and ternary nanofluid the lowest heat transfer rate.
- Surface and contour plots provide deeper insights into the variations of the Nusselt number, demonstrating the nonlinear interactions between thermal and flow resistance effects.
- Contour plots identify regions of high and low heat transfer efficiency, showing that increasing the Forchheimer number (Fr) reduces heat transfer, while higher radiation parameter (Ra) enhances it.
- Surface plots provide a 3D perspective, confirming the nonlinear interaction between thermal and flow resistance effects, with ternary nanofluids exhibiting the lowest heat transfer rate.
- The combined influence of these governing parameters plays a significant role in real-life applications, such as the optimization of heat exchangers, MHD-based cooling

devices, and nanofluid-driven thermal management technologies for biomedical and energy systems.

Nano nanofluid exhibits the best heat transfer performance due to its lower resistance and higher convective efficiency, while ternary nanofluid demonstrates the weakest performance due to elevated resistance and reduced convective effects.

4.1 Limitations of the study

– Skin friction coefficient limitations

The present study does not evaluate the skin friction coefficient, since the primary focus is placed on comparative thermal performance rather than wall shear behavior.

– Flow regime limitation

The analysis is restricted to steady, laminar flow conditions; thus, transient or turbulent behavior is not captured.

– Property assumptions limitation

All thermophysical properties of the fluid and nanoparticles are assumed constant, neglecting temperature-dependent variations that may occur in practical applications.

– Nano particle dispersion limitation

Nanoparticles are considered uniformly dispersed without agglomeration, sedimentation, or Brownian motion effects, which may influence heat transfer performance in real systems.

– Validation limitation

Validation is numerical only, and no experimental or industrial data are included to verify the findings.

– Geometry simplification limitation

The geometry considered is idealized and 2D, limiting direct application to complex 3D engineering designs.

Funding information: This work was supported by the Korea Institute of Energy Technology Evaluation and Planning (KETEP) and the Ministry of Climate, Energy & Environment (MCEE) of the Republic of Korea (No. RS-2025-02315209).

Author contribution: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: All data generated or analysed during this study are included in this published article.

References

1. Cengel YA, Cimbala JM. Fluid mechanics fundamentals and applications, 3rd ed. New York: Mc Graw Hill; 2013.
2. White FM. Viscous fluid flow. New York: McGraw-Hill Education, third; 2005:656 p.
3. Lin W, Zhang P, Lin J. Flow and heat transfer property of Oldroyd-B-Fluid-Based nanofluids containing cylindrical particles in a pipe. Processes (MDPI) 2021;9:647.
4. Fetecau C, Imran M, Fetecau C, Burdujan I. Helical flow of an Oldroyd-B fluid due to a circular cylinder. Z Angew Math Phys 2010;61:959–69.
5. Kamran M, Athar M, Imran M. Critical study on rotational flow of a fractional Oldroyd-B fluid through an infinite circular cylinder. Int Scholarly Res Network 2012 (2011):1–15. <https://doi.org/10.5402/2012/835398>.
6. Sanchez HAC, Jovanovic MR, Kumar S, Morozov A, Shankar V, Subramanian G, et al. Understanding viscoelastic flow instabilities: Oldroyd-B fluids. J Non-Newtonian Fluid Mech 2022;302:1047425.
7. Shakeel A, Ahmad S, Khan H, Shah NA, Haq SU. Flows with slip of Oldroyd-B fluids over a moving plate. Adv Mathematic Phys 2016;1–9. <https://doi.org/10.1155/2016/8619634>.
8. Jamil M, Khan NA, Imran MA. New exact solutions for an Oldroyd-B fluid with fractional derivatives: Stokes' first problem. Int J Nonlinear Sci Numer Simulation 2014;14:7–8.
9. Si X, Lei X, Xu B, Li B, Zhu J, Cao L. The periodic secondary flow of Oldroyd-B fluids driven by direct electric field in a rectangular curved channel. Phys Fluids 2023;35:032005.
10. Ganji DD, Malvandi A. Heat transfer enhancement using nanofluid flow in microchannels: simulation of heat and mass transfer. Oxford, UK: William Andrew; 2016.
11. Ghassemi M, Shahidian A. Nano and bio heat transfer and fluid flow. London, UK: Academic Press; 2017.
12. Priyadharshini P, Archana MV, Shah NA, Alsheri MH. Ternary hybrid nanofluid flow emerging on a symmetrically. Symmetry 2023;15:1225.
13. Abrar MN. Thermal investigation of casson hybrid nanoparticles over a porous stretchable plate: a Cattaneo–Christov heat flux model. J Therm Anal Calorim 2024;150:1–12.
14. Rehman M, Turkyilmazoglu M, Mushtaq Z. Effects of multiple shapes for steady flow with transformer oil+Fe₃O₄+TiO₂ nanofluids. Appl Math Mech 2024;45:373–88.
15. Revathi G, Rao DP, Rao SR, Babu KS, Prasad TV, Babu MJ. Entropy generation optimization in a ree-yring ternary hybrid nanofluid flow over an elastic surface with non-fourier heat flux. East Eur J Phys 2024: 122–33. <https://doi.org/10.26565/2312-4334-2024-4-11>.
16. Narayana Naik R, Suneetha S, Srinivasa Babu KS, Jayachandra Babu M. Entropy optimization in ternary hybrid nanofluid flow over a convectively heated bidirectional stretching sheet with Lorentz forces and viscous dissipation: a Cattaneo–Christov heat flux model. Proc Inst Mech Eng Part E J Process Mech Eng 2024:09544089241275779.
17. Naik RN, Suneetha S, Babu KS, Babu MJ. Impact of exponential heat source and thermal radiation on the flow of hybrid nanofluid across a bi-directional stretching surface with activation energy J Phys Conf 2024;2765:012003.
18. Acharya N, Maity S, Kundu PK. Framing the hydrothermal features of magnetized TiO₂–CoFe₂O₄ water-based steady hybrid nanofluid flow over a radiative revolving disk. Multidiscip Model Mater Struct 2020;16: 765–90.
19. Nasir S, Sirisubtawee S, Juntharee P, Berrouk AS, Mukhtar S, Gul T. Heat transport study of ternary hybrid nanofluid flow under magnetic dipole effect. Appl Nanosci 2022;12:2777–88.

20. Guedri K, Khan A, Sene N, Raizah Z, Saeed A, Galal AM. Thermal flow for radiative ternary hybrid nanofluid over nonlinear stretching sheet subject to darcy–forchheimer phenomenon. *Math Probl Eng* 2022;2022:14.
21. Adun H, Kavaz D, Dagbasi M. Review of ternary hybrid nanofluid: synthesis, stability, thermophysical properties, heat transfer applications, and environmental effects. *J Clean Prod* 2021;328:129525.
22. Rasheed T, Hussain T, Anwar MT, Ali J, Rizwan K, Bilal M, et al. Hybrid nanofluids as renewable and sustainable colloidal suspensions for potential photovoltaic/thermal and solar energy applications. *Front Chem* 2021;9:737033.
23. Yasmin H, Giwa So., Noor S, Sharifpur M. Experimental exploration of hybrid nanofluids as energy-efficient fluids in solar and thermal energy storage applications. *Nanomaterials* 2023;13:278.
24. Soudagar MEM, Ramesh S, Khan TMU, Almakayel N, Ramesh R, Ghazali NNN, et al. An overview of the existing and future state of the art advancement of hybrid energy systems based on PV-solar and wind. *Int J Low Carbon Technol* 2024;19:207–16.
25. Mukhtar S, Gul T. Solar radiation and thermal convection of hybrid nanofluids for the optimization of solar collector. *Mathematics* 2023;11:11758–75.
26. Freedman JP, Wang H, Prasher RS. Analysis of nanofluid-based parabolic trough collectors for solar thermal applications. *J Sol Energy Eng* 2018;140:051008–1.
27. Abrar MN, Kosar S, Razzaq R. Thermal investigation of micro-polar tangent hyperbolic flow using a hybrid nanofluid approach. *J Appl Phys* 2025;137:034302.
28. Razzaq R, Khan Z, Abrar MN, Almohsen B, Farooq U. Chemical reaction and radiation analysis for the MHD casson nanofluid fluid flow using artificial intelligence. *Chaos Solitons Fractals* 2025;190:115756.
29. Menni Y, Chamkha AJ, Lorenzini G, Kaid N, Ameer H, Bensafi M. Advances of nanofluids in solar collectors - a review of numerical studies. *Mathematic Modell Eng Problems* 2019;6:415–27.
30. Abdelrazik AS, Osama A, Allam AN, Shboul B, Sharafedin MA, Elwardany M, et al. ANSYS-fluent numerical modeling of the solar thermal and hybrid photovoltaic-based solar harvesting systems. *J Therm Anal Calorim* 2023;148:11373–424.
31. Jamshed W, Sirin C, Selimefendigil F, Shamsuddin MD, Altowairqi Y, Eid MR. Thermal characterization of coolant maxwell type nanofluid flowing in parabolic trough solar collector (PTSC) used inside solar powered ship application. *Coatings* 2021;11:1552.
32. Reddy MG, Sandeep N. Enhanced heat transfer in hydrodynamic non-Newtonian film flows past an extending cylinder: a hydrogen energy application. *Waves Random Complex Media* 2022;35:1–14.
33. Khuri AI, Mukhopadhyay S. Response surface methodology. *Wiley Interdiscip Rev: Comput Stat* 2010;2:128–49.
34. Maddina DK, Díaz Palencia JL. Response surface optimisation on non-uniform shapes ternary hybrid nanofluid flow in stenosis artery with motile gyrotactic microorganisms. *Chem Phys* 2024;590. <http://hdl.handle.net/20.500.12226/2592>.
35. Yasir M, Ahmed A, Khan M, Alzahrani AK, Malik ZU, Alshehri AM. Mathematical modelling of unsteady Oldroyd-B fluid flow due to stretchable cylindrical surface with energy transport. *Ain Shams Eng J* 2023;14:101825.
36. Lu D, Li Z, Ramzan M, Shafee A, Chung JD. Unsteady squeezing carbon nanotubes-based nano-liquid flow with Cattaneo–Christov heat flux and homogeneous–heterogeneous reactions. *Appl Nanosci* 2019;9:169–78.
37. Khan NS, Gul T, Islam S, Khan I, Alqahtani AM, Alshomrani AS. Magnetohydrodynamic nano liquid thin film sprayed on a stretching cylinder with heat transfer. *Appl Sci* 2017;7:271.
38. Kumar J, Sandeep N. Computational analysis to examine the role of nanoparticle shape on operative usage of solar energy. *Int J Hydrogen Energy* 2024;56:937–48.
39. Negi AS, Kumar B. Nano-ferrofluid (Mn–Zn, CoFe₂O₄, Fe₃O₄) flow in a permeable channel with induced magnetic field. *Waves Random Complex Media* 2023;1–28. <https://doi.org/10.1080/17455030.2023.2165193>.
40. Jan A, Mushtaq M, Hussain M. Heat transfer enhancement of forced convection magnetized cross model ternary hybrid nanofluid flow over a stretching cylinder: non-similar analysis. *Int J Heat Fluid Flow* 2024;106:109302.