

Unveiling Elementary School Students' Thinking Processes in Conquering The Natural Number Bias

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Abstract

This study sought to understand how students activate number sense in determining the position of fractions on a number line and identify how the natural number bias and number sense influences students' thinking processes. The study utilized the Cognitive Task Analysis (CTA), involving four fifth-grade elementary students as the research subjects. The findings from this study suggest that students activate number sense in many ways, i.e. by comprehending the value and denominator of a fraction, dividing the number line into equal parts, and measuring distances based on fractions. However, the natural number bias could hinder students' number sense, leading to their misunderstanding of the relationship between the fraction value and denominator. This misunderstanding may result in errors in comparing fractions, adding or subtracting fractions, and solving problems involving fractions. This study contributes to understanding the role of number lines in developing students' number sense and formulating recommendations for mathematics teachers on how to identify and address the natural number

Keywords:

Natural number bias; number sense; number line; Cognitive Task Analysis (CTA)

Introduction

Understanding whole numbers and fractions is one of the fundamental challenges in elementary mathematics education. Whole numbers have simple and straightforward characteristics that are easy to understand, as each number has a distinct successor (e.g., 4 follows 3), and their values are discrete (Sun et al., 2019). In contrast, fractions are more complex because they can be represented in various equivalent forms, such as $\frac{1}{2}$ being equal to $\frac{2}{4}$, a characteristic that does not occur with whole numbers. This complexity often causes difficulties for students in understanding fractions, especially when asked to determine the position of fractions on a number line (Meert, Grégoire, & Noël, 2010).



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Previous studies have shown that students often fall into natural number bias, which is the tendency to apply whole number concepts when dealing with fractions. This bias occurs because students tend to apply their understanding of simple whole number properties to the more complex context of fractions (Christou et al., 2020; Van Hoof et al., 2020). Natural number bias becomes evident when students incorrectly compare fractions, such as focusing only on the denominator. Students frequently assume that fractions with larger denominators have smaller values, without considering the relationship between the numerator and denominator. This misunderstanding of fraction concepts can lead to errors when comparing fractions and determining their positions on a number line (Obersteiner et al., 2016).

The use of number lines has proven effective in improving students' understanding of fraction concepts and developing their number sense (Van de Walle, Karp, & Bay-Williams, 2014). However, students' understanding of number lines is often still influenced by natural number bias, which hinders their ability to correctly place fractions. This bias arises because students tend to rely on their whole number understanding when positioning fractions on a number line, even when they possess a strong intuition about number values. For instance, students may misinterpret the proportional relationship between the numerator and denominator, relying instead on simplistic rules such as "the larger the denominator, the smaller the value," which applies to whole numbers but not fractions (Ni & Zhou, 2005).

Thus, it is important to explore how students activate their sense of number when determining the position of fractions on a number line and how natural number bias influences their thinking processes. Understanding these patterns will provide insights into the cognitive mechanisms behind students' errors and can inform effective teaching strategies that target and reduce natural number bias in fraction learning.

This study aims to explore students' thought processes in placing fractions on a number line and to analyze the influence of natural number bias on their number sense. By understanding these patterns, it is hoped that the results of this research can provide more effective recommendations for mathematics teachers to address natural number bias and enhance students' understanding of fractions.

This study seeks to answer two main questions:

1. How do students activate their number sense when determining the position of fractions on a number line?
2. How do natural number bias and number sense influence students' thought processes?

Literature Review

The Whole Number Bias and Natural Number Bias

The whole number bias refers to students' tendency to apply whole number concepts when working with fractions, a phenomenon that has been observed at various levels of education, from elementary school to university (Christou et al., 2020; Obersteiner et al., 2016; Van Hoof et al., 2020). This bias becomes particularly evident when students are asked to compare simple fractions with the same numerator. In these instances, students influenced by whole number bias tend to assume that the fraction with the larger denominator has the larger value, neglecting the proportional relationship between the numerator and denominator. This phenomenon arises from students' excessive focus on one element of the fraction typically the denominator (Vamvakoussi et al., 2012).

While whole number bias primarily involves using whole number logic to interpret fractions, a more fundamental cognitive challenge students face is rooted in the natural number bias. Natural number bias is a more general cognitive tendency where students over-rely on their intuitive understanding of whole numbers (natural numbers) when confronted with fractions. In simple terms, students apply their natural number understanding to fractions, assuming the rules and patterns of whole numbers apply universally, even in the context of more complex fraction relationships.

The natural number bias in fractions becomes especially evident when students try to compare or position fractions. For instance, when comparing fractions like $\frac{1}{3}$ and $\frac{1}{5}$, students influenced by natural number bias might wrongly assume that $\frac{1}{5}$ is larger because the numerator is numerically smaller, even though the actual value of $\frac{1}{3}$ is larger. Similarly, when fractions with different numerators are compared, students might still cling to the belief that the fraction with the larger numerator is always larger, much like with natural numbers (Obersteiner et al., 2013).

The natural number bias is reinforced when students use fractions with numerators or denominators that are easy to understand, such as $\frac{1}{2}$, or simple multiples of 1. These simplified fractions often encourage students to focus on whole number differences, rather than truly understanding the proportional nature of fractions. To address this bias, research suggests that more complex fractions, with varied numerators and denominators, should be used in teaching. This approach would allow for a deeper examination of how natural number bias affects students' understanding of fractions (Van Hoof et al., 2020).

Students who are prone to natural number bias are more likely to make errors when comparing fractions that do not align with whole number relationships.

For example, when comparing $\frac{2}{9}$ and $\frac{1}{4}$, students influenced by natural number bias may make errors, assuming the larger denominator means a smaller value, ignoring the relational dynamics between the numerator and denominator (Deliyianni et al., 2016). These errors reflect the cognitive gap between students' whole number logic and their ability to grasp fractional equivalence and relative values.

A better understanding of natural number bias and its impact on fraction comparison processes is crucial for educators. This understanding will help educators design more effective teaching strategies that enable students to overcome their reliance on whole number reasoning. By focusing on the specific challenges that natural number bias presents, teachers can help students improve their conceptual understanding of fractions. One recommended strategy is the use of various fraction representations, such as number lines, which allow students to visualize the relationships between fractions and see beyond the whole number-based assumptions that may be clouding their understanding (Michaelidou & Gagatsis, 2005).

Though natural number bias is a widespread cognitive challenge, the degree to which this bias diminishes over time is a subject of ongoing debate. Some researchers argue that exposure to more fraction concepts over time gradually reduces natural number bias (Van Hoof et al., 2015). However, studies show that this bias does not significantly decrease in students until later grade levels. For example, natural number bias may still be prevalent in 4th and 6th graders, but it diminishes gradually in higher grades, such as in 8th graders, who demonstrate a better understanding of fractions that are not governed by whole number rules. Notably, experts in mathematics do not exhibit natural number bias, further indicating that this bias can be overcome with deeper understanding and experience (Obersteiner et al., 2013).

The consequences of natural number bias are significant and long-lasting. If students do not grasp the fundamental principles of fractions early on, they are more vulnerable to this bias, which can persist for years. Even students who have learned the procedural rules for fraction operations may still struggle with conceptual understanding, leading to difficulties in more advanced mathematical topics that involve fractions (Deliyianni et al., 2016). To address these issues, number lines are often employed to evaluate and reduce natural number bias. By visually representing fractions on number lines, students can better understand the proportional relationships that underlie fractional values and how these relationships differ from the properties of whole numbers. A systematic and structured approach, along with appropriate visual tools, will help educators identify natural number bias in students' thinking and take steps to overcome it, ultimately fostering a deeper

understanding of fractions and providing a strong foundation for future mathematical learning.

Number Lines: Unlocking Number Sense and Overcoming Natural Number Bias

Number lines have long been recognized as an effective tool for helping students visualize mathematical concepts, including fractions. With number lines, students can directly observe the relationship between the value of a fraction and its position within a numerical range, enabling them to develop number sense or number intuition. A strong number sense is a critical foundation for deeper mathematical understanding and the ability to solve problems involving numbers (Van de Walle, Karp, & Bay-Williams, 2014).

Despite their great potential, many previous studies have focused on the effectiveness of number lines in strengthening students' conceptual understanding of fractions (Barbieri et al., 2020; Rahayuningsih et al., 2025; Rahayuningsih et al., 2021) without specifically exploring how natural number bias affects students' ability to place fractions on a number line. Natural number bias, the tendency of students to apply whole number logic when working with fractions, remains a significant obstacle in mathematics education (Christou, Pollack, Van Hoof, & Van Dooren, 2020; Obersteiner, Van Hoof, Verschaffel, & Van Dooren, 2016). This bias becomes evident in the context of number lines when students struggle to place fractions correctly. Students influenced by natural number bias may incorrectly divide the number line or fail to understand the relationship between the numerator and denominator, leading to errors in fraction placement (Vamvakoussi, Van Dooren, & Verschaffel, 2012).

This study aims to address this gap by deeply exploring how students activate their number sense through number lines and how natural number bias influences their thought processes in determining the position of fractions. The study emphasizes the use of number lines not only as a visual aid but also as a diagnostic tool that can identify students' conceptual errors caused by natural number bias. By doing so, the research provides a new contribution to the field of mathematics education by linking the role of number lines with an analysis of students' thought processes related to natural number bias.

The novelty of this study lies in its systematic focus on understanding the interaction between number lines, number sense, and natural number bias. Unlike previous studies that have assessed the general effectiveness of number lines in teaching fractions (Barbieri et al., 2020), this research offers new insights into how number lines can be strategically used to overcome natural number bias and improve students'

understanding of fraction positions on a number line. The results of this research are expected to provide a strong foundation for developing more effective and targeted teaching strategies in the future.

Methods

Design and Research Setting

This study uses the Cognitive Task Analysis (CTA) approach to investigate students' thought processes in understanding the position of fractions on a number line and to analyze the influence of whole number bias on their understanding. CTA was chosen because it effectively breaks down complex thought processes into more structured and systematic stages (Clark et al., 2008). CTA has been widely applied across various fields, including education, to understand individuals' cognitive behavior when completing tasks that require deep thinking (Schraagen et al., 2000; Merriënboer, 2010). However, its application in elementary mathematics education, particularly related to whole number bias, remains limited, giving this study its novelty.

The initial phase begins with gathering preliminary information. This information is obtained through a literature review focusing on previous studies about whole number bias (Christou et al., 2020; Van Hoof et al., 2020) and the importance of number sense in understanding number lines (González-Forte et al., 2022; Van de Walle et al., 2014). In addition, the researcher conducts exploratory interviews with elementary mathematics teachers at the research site. These interviews aim to gather information about the characteristics of students prone to whole number bias, indications of difficulties students face in placing fractions on a number line, and the teaching strategies teachers have used to address these issues. This approach aligns with Schraagen et al. (2000), who emphasize the importance of understanding the context before starting cognitive analysis.

Based on the preliminary information, the second phase is identifying students' knowledge representations. At this stage, the researcher develops a conceptual framework that maps the stages of students' thought processes when solving fraction tasks. This framework includes four main components: understanding the problem, selecting solution strategies, applying mathematical operations, and evaluating answers. This analysis focus aligns with the systematic approach of CTA, which aims to identify the skills and knowledge underlying individual performance (Clark et al., 2008). To validate this framework, the researcher designs an instrument consisting of fraction tasks specifically structured to trigger whole number bias. The tasks include comparing fractions with different denominators and placing fractions on a number line, such as asking students to determine the positions of

$\frac{2}{3}$ and $\frac{3}{5}$ on the same number line (Meert, Grégoire, & Noël, 2010).

The next phase, eliciting focused knowledge, is conducted using the think-aloud protocol technique. This technique allows students to verbally express their thought processes while completing the given tasks. In practice, students are asked to divide the number line into appropriate segments, place fractions in specific positions, and explain the logic they used. The think-aloud method was chosen because it is effective in uncovering students' real-time and in-depth thought processes (Ericsson & Simon, 1993). During these sessions, the researcher provides minimal instructions, such as "Tell me what you are thinking when dividing this line" or "Why did you choose this position for the fraction?" to encourage students to articulate their reasoning. This open-ended approach allows the researcher to capture authentic, unfiltered insights into students' cognitive strategies.

After the think-aloud session, the researcher conducts semi-structured interviews to further explore the strategies students used, the challenges they faced, and indications of natural number bias (as well as any other relevant cognitive biases). The interview instrument was validated through expert judgment by two mathematics education experts, who reviewed the clarity, appropriateness, and alignment of the questions with the study's objectives (Creswell & Miller, 2000). To ensure the reliability of the instrument, it was tested through trials with students outside the study participants. This additional step was essential to assess whether the instrument consistently measured the intended constructs across different individuals and contexts, ensuring that the questions were appropriately understood and the data collected would be reliable.

The fourth phase involves data analysis and verification. The think-aloud and interview recordings are transcribed and analyzed using thematic analysis to identify thought patterns, common strategies, and indications of whole number bias (Braun & Clarke, 2006). The analysis focuses on how students understand the relationship between the numerator and denominator in the context of number lines and how whole number bias influences their decisions. Data triangulation is conducted by comparing transcripts, direct observations, and student task results to ensure the consistency of findings. The researcher also performs member checking with participants, where preliminary analysis results are shared with students to validate the accuracy of the interpretations made by the researcher (Stalmeijer et al., 2014).

In the final phase, developing a thought process model, the analysis results are used to construct a model that illustrates the stages of students' thinking when solving

fraction tasks on a number line. This model visualizes how whole number bias emerges in students' thought processes and the role of number sense in helping them understand the relative values of fractions. The model is presented as a descriptive narrative and flowchart that can be used by teachers as a guide to identify and address whole number bias in the classroom. Additionally, this study provides practical recommendations for mathematics teachers to integrate number lines effectively as a teaching tool, using a more concrete and interactive approach (Van de Walle et al., 2014).

Throughout this study, ethical procedures are implemented in adherence to responsible research principles. Student participation is carried out with written consent (informed consent) from parents and students, after they receive detailed explanations about the study's objectives, procedures, and participants' rights. Students' data confidentiality is strictly maintained, and the right to withdraw participation without consequence is explicitly stated. With this approach, the research is conducted ethically, systematically, and with a focus on students' needs and real-world contexts.

The research instrument consists of a fraction estimation task on a number line designed to explore students' understanding of placing fractions and to identify indications of whole number bias. The instrument design is adapted from previous studies focusing on using number lines as a tool to understand fraction values (Iuculano & Butterworth, 2011; Meert, Grégoire, & Noël, 2010). In this study, the instrument is systematically developed, starting from preparation, implementation, to the validation process, to ensure it possesses adequate validity and reliability.

The instrument consists of a 20 cm number line on A4 paper, marked with 0 on the left end and 1 on the right end, representing the range of fractions used in the tasks. Additionally, the researcher prepares fraction cards containing 15 target fractions that have been predetermined. These fractions include:

- Values less than 1 to focus students on parts of a whole,
- Denominators varying from 2 to 10, with an additional denominator of 20,
- Equivalent fractions, such as $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, to test the understanding of fractions representing 1.
- Target fractions such as $\frac{4}{9}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{4}{8}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{5}{6}$, $\frac{1}{4}$, $\frac{8}{10}$, $\frac{1}{20}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{3}{6}$, and $\frac{5}{7}$ are chosen to create task complexity variation. This variation allows the researcher to observe the consistency of students' understanding and the extent to which whole number bias influences the strategies they use.

The task is carried out individually in the following stages: (1) The researcher provides simple verbal instructions to students to estimate the position of fractions on a number line; (2) Each fraction card is shown one at a time, and students are asked to mark the position of the fraction on the number line using a pencil; (3) The marked positions are recorded on a data sheet by the researcher, and observations are conducted to note students' behavior during the marking process.

To ensure the instrument's validity and reliability, the researcher conducts a validation process, including content validity and instrument reliability. Content validity is tested through expert judgment involving two mathematics education experts and one elementary school mathematics teacher. This evaluation aims to assess the extent to which the instrument measures the research objectives, namely: (1) Students' understanding of fraction values; (2) Activation of students' sense of number; and (3) Indications of whole number bias in determining the position of fractions. The experts provide feedback regarding: (1) Clarity of instructions; (2) Relevance of the fraction cards to the research objectives; and (3) Task complexity level appropriate for fifth-grade students.

The results of the expert evaluation are analyzed using Aiken's V to calculate the instrument's validity score. The obtained Aiken's V value is 0.85, indicating high validity (Azwar, 2015). This value exceeds the minimum threshold of 0.70, meaning the instrument is appropriate for use in the study.

Subsequently, the instrument's reliability is tested through a field trial with fifth-grade students who are not part of the study participants. This trial aims to assess the consistency of students' responses in marking the position of fractions on the number line. The trial data is analyzed using reliability coefficients through the test-retest method, and a reliability score of 0.82 is obtained. This value indicates good reliability, as it exceeds the 0.70 threshold (Creswell, 2014).

With high content validity and good reliability, this instrument is of sufficient quality to reveal students' thought processes in placing fractions on a number line. Furthermore, this instrument was developed contextually based on previous studies proven effective in identifying students' understanding and indications of whole number bias (Meert et al., 2010; Van Hoof et al., 2020).

The validation and reliability testing processes ensure that the instrument can be accurately used to evaluate students' understanding of fraction positions and provide consistent data within the context of this study. Thus, this instrument serves not only as a measurement tool but also as an in-depth exploration

tool for identifying students' thought patterns and challenges in understanding the concept of fractions.

Data Analysis

Data collection and analysis were conducted systematically and iteratively, using a triangulation approach and validation through member checking to ensure the accuracy of data interpretations. Member checking was carried out by verifying the research findings with the study participants, allowing data interpretations to be directly confirmed by their sources (Creswell & Miller, 2000; Stalmeijer et al., 2014). The data analysis used in this study is comprehensive, aiming to uncover in depth the thought processes of students experiencing whole number bias and how their sense of number plays a role in solving fraction problems.

The data analysis process began with student data, focusing on the transcription of audio recordings from the think-aloud protocol sessions. These transcriptions were analyzed using thematic analysis (Braun & Clarke, 2006) to identify patterns, themes, and categories in students' thought processes. The main themes identified included: problem-solving strategies, understanding of fraction concepts and number operations, and the influence of whole number bias on students' thinking. In this process, each student response was coded to map similarities and differences in the approaches used. For example, students with whole number bias tended to focus on the denominator's value as the determinant of magnitude, while students with a strong number sense were more likely to compare the relationship between the numerator and denominator.

The initial analysis results were then verified through member checking, where the researcher discussed preliminary findings with several study participants. Through member checking, the researcher ensured that data interpretations matched the actual experiences expressed by the students. This step is crucial for enhancing the credibility of the data obtained (Creswell & Miller, 2000). Additionally, data triangulation was conducted, where data from multiple sources, including think-aloud sessions, student interviews, direct observations, and task results, were compared and integrated. This triangulation helped the researcher build a more comprehensive and valid understanding of students' thought processes when tackling fraction tasks.

After analyzing student data, teacher data were analyzed, focusing on interviews with mathematics teachers. This analysis aimed to understand teachers' perspectives on the occurrence of whole number bias in the classroom and the instructional strategies they implemented to help students overcome this bias. Teacher interviews were analyzed using a

similar thematic approach, focusing on teachers' experiences in: (1) identifying whole number bias among students, (2) teaching strategies used to improve students' understanding of fractions, and (3) teachers' efforts to develop students' number sense through number-line-based approaches.

Results

How students activate number sense when determining the position of fractions on a number line

Case Analysis

The following cases will illustrate fascinating examples of the results of a mathematics learning project on fractions. These cases are centered around the problem-solving processes employed by three students who activated their number sense when dealing with fractions.

Case 1:

"On a number line, Student 1 placed $\frac{4}{9}$ in the left side of $\frac{1}{2}$. The value of $\frac{4}{9}$ is smaller than $\frac{1}{2}$ because the denominator is larger. When a number line is divided into 9 parts, then $\frac{4}{9}$ represents four of nine segments. The position of the fraction will be between 0 and $\frac{1}{2}$, closer to 0. Student 1 may be inclined to choose the fraction with the smaller denominator or a value closer to 0 or 1, even though the fraction does not fit the problem. The decision made by the student may be due to his familiarity with the whole numbers and it is easier to understand fractions with smaller denominators. When asked to find a fraction smaller than $\frac{1}{2}$, the student chose $\frac{1}{3}$ rather than $\frac{4}{9}$, even though the value of $\frac{4}{9}$ is smaller than $\frac{1}{3}$. The student may be trapped by the simple rule that "the larger the denominator, the smaller the value". The student neglected the fact that this rule is only applicable when the numerators are the same."

Case 2:

"Student 2 was trying to put $\frac{3}{5}$ on a number line. First, he divided the number line into five equal parts, where the third part starts at 0 and is $\frac{3}{5}$ of the total length of the number line. When measuring the distance from 0 to 1, $\frac{3}{5}$ is located at the point that is $\frac{3}{5}$ of the way. However, when the student was asked to compare $\frac{3}{4}$ and $\frac{5}{6}$, he would say that $\frac{5}{6}$ was greater than $\frac{3}{4}$ since the denominator is larger although $\frac{3}{4}$ is larger in value."

Case 3:

"According to Student 3, $\frac{1}{1}$ is an interesting fraction because the denominator and numerator are the same, namely 1. On a number line, $\frac{1}{1}$ is located at the far right, namely at point 1. This shows that $\frac{1}{1}$ represents a whole, just like the number 1. When asked to find a fraction that represents a whole, Student 3 chose $\frac{1}{1}$, even though other fractions such as $\frac{10}{10}$ or $\frac{20}{20}$ also represent a whole."

Focusing on the denominator is one of the common misconceptions that often occurs when students learn fractions. The tendency for students to assume that fractions with larger denominators have smaller

values is a simplification that is not always true. When students only focus on the size of the denominator without considering the numerator, they often make incorrect conclusions. For example, when comparing $\frac{4}{9}$ and $\frac{1}{2}$, students may automatically assume $\frac{4}{9}$ is smaller because 9 is greater than 2. However, in the illustration where a cake is divided into 9 pieces, 4 pieces of 9 are certainly larger than half of the cake. This misconception indicates that students do not fully understand the concept of fractions in depth. To overcome this, it is important for educators to emphasize that both the numerator and denominator play an important role in determining the value of a fraction. The use of concrete models such as pictures or real objects can help students visualize the concept of fractions and compare them more effectively. In addition, sufficient practice in comparing fractions with different denominators is also very important to strengthen students' understanding of fractions.

The difficulties faced by students when comparing fractions with different denominators often arise due to a lack of basic understanding of fractional equivalence and how to convert fractions into equivalent forms. When dealing with two fractions with different denominators, students tend to fall into the simple assumption that the fraction with the larger denominator is always greater. In fact, to be able to compare two fractions accurately, students need to convert them into fractions that have the same denominator. The process of converting fractions into equivalent forms involves an understanding of the concepts of multiples and Greatest Common Divisor. A lack of understanding of these concepts often hinders students in solving fraction comparison problems. In addition, difficulty in visualizing the equal parts of two different wholes can also make it difficult for students to compare fractions. In other words, students may struggle to imagine how $\frac{3}{4}$ of one pizza compares to $\frac{5}{6}$ of another pizza. Therefore, the use of visual models such as pie charts or number lines can greatly assist students in understanding this concept. In addition, it is crucial for students to engage in sufficient practice to enhance their comprehension of determining the denominator of Least Common Multiple and converting fractions into equivalent forms.

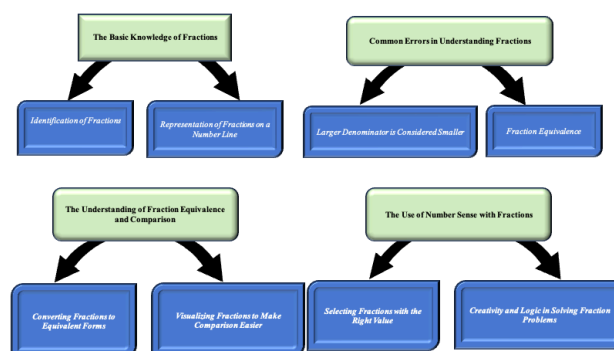
A common challenge encountered by students when comprehending fractions is associated with the representation of the whole. Students frequently focus on the general form of a fraction, which is $\frac{a}{b}$, and assume that the denominator and numerator must be different. As a result, they only know fractions such as $\frac{1}{2}$, $\frac{3}{4}$, or $\frac{5}{6}$ as representations of smaller parts of a whole. In fact, fractions can also be used to represent whole parts or wholes. For example, when asked about a fraction that represents an entire pizza, many students will immediately answer $\frac{1}{1}$. They tend to ignore other possibilities such as $\frac{2}{2}$, $\frac{3}{3}$, or

even $\frac{10}{10}$ which also represent the entire pizza. This misunderstanding shows that students do not fully understand that a fraction is a comparison between a part and a whole, and that the value of a fraction does not always have to be less than one.

This misunderstanding stems from students' initial understanding of whole numbers, in which larger numbers are associated with larger values. However, this concept does not always apply to fractions. Failures in addressing this misconception might impede students' understanding of more complex fraction operations, such as adding, subtracting, multiplying, and dividing fractions.

The students' answers to the fraction problem in this study showed how they activated their number sense to understand fractions and place them on a number line. Student 1 understood the concept of value and denominator of fractions to determine which was smaller than $\frac{1}{2}$. On the other hand, Student 2 was able to comprehend how to divide a number line into equal parts and measure distance based on fractions. Student 3 possessed knowledge of the relationship between $\frac{1}{1}$ and the number 1, as well as its position on a number line. These three cases demonstrated that students with strong number sense could solve problems involving fractions and number lines in creative and logical ways. Figure 1 is a diagram pattern that illustrates how students activate their number sense when determining the position of fractions on a number line.

Figure 1.
Students' Thinking Process in Understanding Fractions



1. The Basic Knowledge of Fractions

Understanding fractions begins with a clear grasp of their components and their representation on a number line. One of the first challenges students face is identifying fractions. Often, students focus on the denominator of a fraction and mistakenly assume that the fraction with the larger denominator always represents a smaller value. For example, when asked to compare $\frac{1}{3}$ and $\frac{1}{4}$, students influenced by this misconception might assert that $\frac{1}{4}$ is larger because its denominator is larger, even though $\frac{1}{3}$ is actually the larger fraction.

The representation of fractions on a number

line is another essential step in building fraction understanding. In this process, students use number lines to determine where fractions are positioned by dividing the line into equal parts. For instance, when placing $\frac{1}{2}$ on a number line, students must divide the line into two equal parts and locate $\frac{1}{2}$ exactly halfway between 0 and 1. However, many students find this challenging, especially when the fractions have more complex denominators.

2. Common Errors in Understanding Fractions

Among the most common errors in fraction comprehension is the assumption that fractions with larger denominators are always smaller. This larger denominator means smaller value misconception arises from students' reliance on their understanding of whole numbers, where larger numbers are seen as greater. For instance, students may incorrectly assume that $\frac{1}{5}$ is smaller than $\frac{1}{2}$, simply because 5 is larger than 2.

Another frequent misunderstanding involves fraction equivalence. Students often struggle with the concept that fractions with the same numerator and denominator represent the same value, such as $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ all equating to 1. This inability to recognize equivalent fractions hinders their understanding of how fractions relate to each other.

3. The Understanding of Fraction Equivalence and Comparison

An important skill for fraction comparison is the ability to convert fractions to equivalent forms. For example, when comparing $\frac{1}{2}$ and $\frac{2}{4}$, students need to recognize that these fractions are equivalent. Converting fractions to have the same denominator allows students to more easily compare their sizes. In this way, converting fractions to equivalent forms is an essential step in fraction comparison and understanding.

Visual aids like pie charts and number lines can be extremely helpful in making this comparison easier for students. By visualizing fractions, students can see their relative sizes more clearly. For example, a pie chart representing $\frac{1}{2}$ versus $\frac{2}{4}$ shows that both fractions cover half of the whole, reinforcing the concept of equivalence. These visual tools help students move beyond rote learning and develop a deeper understanding of fraction relationships.

4. The Use of Number Sense with Fractions

Students with strong number sense can use logic to determine the relative value of a fraction, even when the fractions are not expressed with the same denominator. For instance, when asked to compare $\frac{3}{5}$ and $\frac{2}{3}$, a student with well-developed number sense can recognize that $\frac{3}{5}$ is larger, even though the fractions have different denominators, by understanding the proportional size of the fractions on a number line.

Moreover, students apply creativity and logic in solving fraction problems. Strong number sense allows them to explore different strategies for solving fraction-related problems. For example, students might use strategies like finding common denominators or using visual tools to help solve addition, subtraction, or comparison

problems involving fractions. The ability to approach fraction problems in creative ways not only strengthens their understanding but also builds their problem-solving skills in mathematics more broadly.

How the natural number bias hinders the thinking processes of students with strong number sense, leading to incorrect responses to the fraction problem

A task involving fractions stated that there is a number line with 10 equal parts. Students were asked to place $\frac{3}{4}$ on the number line. Students with strong number sense could understand that $\frac{3}{4}$ represents 3 of the 4 equal parts on the number line. However, these students showed a simplicity bias. They were tempted to place $\frac{3}{4}$ in the middle of the number line, as this is the simplest and easiest way to remember position. Then, they placed $\frac{3}{4}$ in the 5th part of the number line, right in the middle. In other words, they focused on the simplicity of the position rather than the actual value of the fraction. These students assumed that $\frac{3}{4}$ should be placed in the middle because it is "half" of 4 parts. This case strongly suggests that the natural number bias can hinder students' ability to solve fraction problems accurately, although they possess strong number sense.

When comparing $\frac{1}{3}$ and $\frac{2}{5}$, students with strong number sense could comprehend that fractions represent parts of a whole. They could divide whole numbers into equal parts and understand the relative value of fractions. However, these students had difficulty solving the problem due to the natural number bias. They concentrated more on the denominator of the fraction than on its value and assumed that the fraction with the larger denominator always has a larger value. These students mentioned that $\frac{2}{5}$ was greater than $\frac{1}{3}$ because the denominator was larger ($5 > 3$). They merely focused on the denominator of the fraction (5 and 3) rather than its values ($\frac{1}{3}$ and $\frac{2}{5}$). Therefore, they failed to consider that, compared to $\frac{1}{3}$, $\frac{2}{5}$ represents a larger part of the whole.

Student biases might lead to a misinterpretation of the correlation between fraction values and their denominators. This misinterpretation can result in errors in comparing, adding, or subtracting fractions as well as solving problems that involve fractions.

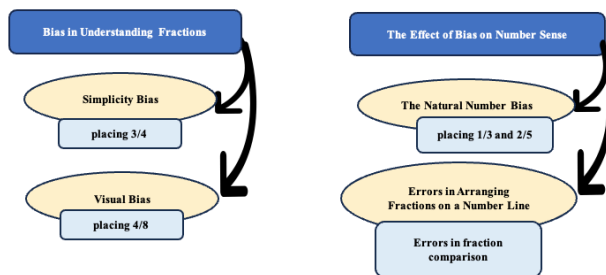
A math problem reads: "A number line has 10 equal parts. Place the fraction $\frac{4}{8}$ on the number line." Students with strong number sense would understand that $\frac{4}{8}$ represents 4 of 8 equal parts on a number line. They would divide a whole number into equal parts and understand the relative value of fractions. However, the students' ability to solve the problem might be hampered by visual bias. Students relied on the visual representation of the number line to understand fractions, but they did not always understand the relationship between the distance on the number

line and the value of the fraction. As a result, students placed $\frac{4}{8}$ in the 4th section of the number line, right in the middle. Students focused on the equal distance on the number line for each section. Students assumed that $\frac{4}{8}$ should be placed in the 4th section because it is the "middle" of the 8 sections. Visual bias can lead to students misinterpreting the relationship between the value of a fraction and its visual representation on the number line. This misinterpretation of fractions can result in inaccuracies when arranging fractions on a number line, comparing fractions, and solving problem involving fractions.

Figure 2 presents a diagram that illustrates the influence of the natural number bias on students' understanding of fractions and use of number sense. This diagram depicts the different types of bias that affect students' understanding of fractions and how they can lead to errors in solving fraction problems.

Figure 2.

The Impact of Bias on Fraction Understanding and Number Sense



1. Bias in Understanding Fractions

A common tendency among students is to place fractions in positions that seem simpler to them, even when those positions do not correspond to the correct fractional values. For example, when asked to place $\frac{3}{4}$ on a number line, many students place it directly in the middle of the segment between 0 and 1. This happens because they perceive it as the "simplest" fraction close to 1, even though the actual value of $\frac{3}{4}$ is slightly less than 1. Students tend to apply this simplicity bias without considering the precise relationship between the numerator and the denominator, leading to misplacement on the number line.

Visual Bias: Another observed bias is the visual bias, where students rely heavily on the visual representation of fractions on the number line but fail to understand how the distance between points corresponds to the value of the fraction. For instance, when placing $\frac{4}{8}$ on the number line, students may incorrectly position it in the 4th place of the 8 equal divisions, simply because they see it as "in the middle" without considering that $\frac{4}{8}$ is equivalent to $\frac{1}{2}$ and should be placed halfway between 0 and 1. This bias suggests that students often misinterpret the visual cues from number lines without a full understanding of the fractional value they represent.

2. The Effect of Bias on Number Sense

The Natural Number Bias: Natural number bias occurs when students mistakenly apply their understanding of whole numbers to fractions. For example, when comparing $\frac{1}{3}$ to $\frac{2}{5}$, many students believe that $\frac{2}{5}$ is larger because 5 is greater than 3, assuming the fraction with the larger denominator must always have a smaller value, just like with whole numbers. This misconception leads students to incorrectly compare fractions based solely on the size of the denominator, rather than understanding the true proportional relationship between the numerator and denominator.

Errors in Arranging Fractions on a Number Line: Errors in arranging fractions on a number line are common and often stem from the biases discussed above. For example, when asked to compare $\frac{1}{2}$ and $\frac{3}{4}$, students influenced by natural number bias might incorrectly place $\frac{1}{2}$ further to the right of $\frac{3}{4}$ simply because $\frac{1}{2}$ is perceived as smaller based on their intuitive understanding of whole numbers. These misplacements and errors in comparison can lead to a lack of understanding of fraction size and their relative positions on the number line, hindering students' ability to make accurate fraction comparisons.

Discussion

The present study sought to understand how students activate their number sense when determining the position of fractions on a number line and identify how the natural number bias and number sense affect students' thinking processes. The results of this study are corroborated by past studies demonstrating that both whole numbers and fractions have different characteristics and require different cognitive involvement in determining them on a number line (Klein et al., 1998).

The findings of this study suggest that fraction value estimation develops differently from whole number value estimation. According to a study by Yuan and Chen (2020), 41.3% of fourth graders adopted a nonlinear model in fraction estimation, while 21.2% of fifth graders demonstrated a linear estimation pattern. These findings indicate that fraction value estimation develops from a nonlinear to a linear pattern. Similarly, (Braithwaite & Siegler, 2018) reported that fraction estimation on a number line can be used to assess whole number bias. Furthermore, (Namkung & Fuchs, 2016) revealed that attention and processing speed were significant predictors for whole numbers and fraction calculations. These results align with the findings in the first study which showed that attention to the concept of fractions may reduce whole number bias.

Case 1: Student 1 placed the fraction $\frac{4}{9}$ to the left of $\frac{1}{2}$, demonstrating an understanding that $\frac{4}{9}$ is smaller than $\frac{1}{2}$. This shows that the student understood that the numerator and denominator

of a fraction can affect its relative value. This understanding is in line with cognitive development theory, where children at the concrete operational stage begin to understand mathematical concepts through the manipulation of concrete objects. In line with the results of research conducted by Judd and Klingberg (2021), spatial cognitive training, especially working memory and visuospatial reasoning, has a significant impact on mathematics learning in children aged 6-8 years. Furthermore, Fyfe et al. (2015) mentioned that fading concreteness, where problems are presented with concrete material and fade into abstract representations, is beneficial for children's understanding of mathematical concepts.

Case 2: Student 2 chose $\frac{3}{5}$ and was able to divide a number line into 5 equal parts. Student 2, however, made an error when comparing $\frac{3}{4}$ and $\frac{5}{6}$ by stating that $\frac{5}{6}$ is larger because its denominator is larger. This error can be explained by the theory of "whole number bias," in which children are more likely to rely on more familiar whole numbers than on more abstract fractional values. Children's difficulties with fractions and rational numbers are associated with their knowledge of whole numbers (Ni & Zhou, 2005).

Case 3: Student 3 showed a good understanding of the fraction $\frac{1}{1}$ representing the whole, but only chose $\frac{1}{1}$ as the representation of the whole without considering other fractions such as $\frac{10}{10}$ or $\frac{20}{20}$. According to Siegler's conceptualization theory, children tend to use the simplest and most familiar examples to understand new concepts (Siegler & Pyke, 2013)

Natural number bias is the tendency of students to use whole numbers as the primary reference in understanding and processing fractions. The natural number bias can hinder students' deeper understanding of the relative value of fractions. According to the dual process theory of Kahneman (2011), students often use "system 1" (fast and intuitive thinking) than "system 2" (slow and analytical thinking) when processing fractions.

The results of this study are in line with the findings of Van Hoof et al. (2015) which concluded that the whole number bias in fraction estimation takes 4 years to fade. However, in this study, half of the students managed to correct their whole number bias within one year. This could happen due to a more effective teaching method in introducing the concept of fractions to students. Mazzocco et al. (2013) suggested that students with low mathematical scores may learn fractions from one and a half parts first with a gradual shift to more difficult fractions. These students have the potential to achieve the same level of performance as high-achieving students within a year.

The current study also found that the nonlinear group exhibited a clear whole number bias in estimating unit fractions and equivalent fractions. However, in experiments involving fractions with the same denominator, students did not estimate large values when the target fraction had a large numerator. This suggests that the estimation behavior of nonlinear group fractions is mainly influenced by their denominators. This finding supports the results of the study conducted by Braithwaite and Siegler (2018).

To address bias in fraction learning, teachers need to apply various learning approaches that can help students develop a deeper understanding of fractions. Theory from research conducted by Getenet and Callingham (2021) suggests that teachers should focus on connecting constructs and procedures and forming classroom discussions that support students' learning about fractions. For example, teachers can use concrete property, such as pieces of paper divided into equal parts, to help students understand the relative value of fractions.

Multisensory learning approaches, as proposed by Dewey's active learning theory, can help students overcome visual and natural number biases (Crollen et al., 2019). By using a variety of media and props, students can develop a more holistic understanding of fractions. For example, using a number line that can be touched and physically manipulated can help students understand the relationship between the value of a fraction and its position on the number line.

Conclusion

The findings from this study indicate that students may experience various difficulties and misinterpretations in understanding the concept of fractions, especially in the context of positioning fractions on a number line. Although students demonstrate strong number sense with fractions, they are often trapped in biases related to whole numbers that can interfere with their understanding of the value and comparison of fractions.

First, this study has identified that students tend to assume that fractions with larger denominators always have smaller values, indicating a lack of in-depth understanding of the interaction between the numerator and denominator in fractions. This is seen in their tendency to compare fractions based on the value of the denominator alone, without evaluating the numerator.

In addition, students frequently face difficulty recognizing fractions that also represent whole numbers, such as $\frac{1}{1}$, $\frac{2}{2}$, or $\frac{3}{3}$. Students typically comprehend $\frac{1}{1}$ as a representation of a whole and do not realize that other fractions can also represent the same value.

This study also highlights the importance of using concrete and visual models in teaching fractions. By utilizing appropriate teaching models and tools, students can better understand the relationship between fraction values, their position on a number line, and how to compare fractions more accurately. Discussions that involve repetition of concepts and additional practice in comparing fractions are essential to reinforce this understanding.

In summary, this study recommends that educators should focus on managing and reducing bias in fraction learning, by integrating multisensory approaches and utilizing concrete objects to help students develop a more comprehensive and in-depth understanding of fractions. In this way, students are expected to overcome the difficulties they face when dealing with fractions and build a strong foundation for comprehending fractions and more complex mathematical operations in the future.

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