



UNIVERSITI PUTRA MALAYSIA

**SECURE ACCESS TO AUTHORIZED RESOURCES BASED ON
FINGERPRINT AUTHENTICATION**

AHMED BABA ELMADANI

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FINGERPRINT AUTHENTICATION**

By

AHMED BABA ELMADANI

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfillment of the Requirement for the Degree of Doctor of Philosophy**

March 2003



DEDICATION

To the souls of my beloved father and mother in the heavens (Baba and Tabagort), who regretfully did not live to see this work, which resulted from their gift of many years of love, encourage and support to me.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment
of the requirement of the degree of Doctor of Philosophy

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Chairman: Veeraraghavan Prakash, Ph.D.

Faculty: Engineering

The Internet makes it convenient for anyone to access publicly available information on the servers. The increased functionality of computers and other technological equipment makes the connectivity easier than ever before, and the need for security more and more important. Passwords are frequently used to control access to restricted functions. Unfortunately, password or personal identification number (PIN) verification suffers an inherent problem. It cannot ensure that the user is the claimed individual. Higher security systems have now veered towards biometric verification in conjunction with passwords. Fingerprints are a practical bodily characteristic to use, as they are unique to each individual and easily collected using image-capture systems.

In this thesis, the methods of fingerprint recognition and classification are investigated. Then, the possible approaches to use are discussed while investigating



the subject. The final choice combines a feature-based and correlation-based approach.

The thesis proposes a novel method of allowing users access after authenticating them by their fingerprints. The method is based on a statistical approach, and is crafted in such a way that the authentication operations are inconspicuous to the user. All the required image processing techniques that make the extraction of the true fingerprint features easier are used: equalization, filtering, binarization and thinning.

A new method for constructing a unique key from the fingerprint image is also presented, with the fingerprint database (fingerprint features, unique fingerprint key, public and private keys) created shown. The database can be manipulated (insertion, retrieving, and deletion) rapidly using the Adelson Velskii and Landis (AVL) tree searching technique. The AVL tree is used to increase the compression ratio as its compression algorithm works efficiently for all types of data. The security is maintained and enhanced by adding a public key system.

Finally, a unique fingerprint based key is constructed using a user's fingerprint image and the key used to access and retrieve the private key. The method successfully recognizes the user fingerprint image even with noisy, translated or rotated images.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia bagi memenuhi syarat untuk mendapatkan ijazah Doktor Falsafah

**SECURE ACCESS TO AUTHORIZED RESOURCES BASED ON
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Internet dapat memudahkan seseorang mencapai maklumat yang terkandung di dalam pelayan menggunakan teknologi web. Fungsi komputer dan peralatan teknologi lain yang semakin canggih telah membuatkan sambungan ke Internet menjadi lebih mudah. Kemudahan ini memerlukan ciri-ciri keselamatan agar sesuatu perkhidmatan boleh dipercayai oleh para pengguna.

Kata kunci adalah suatu kaedah yang biasa digunakan untuk menghadkan capaian kepada pengguna yang dibenarkan. Malangnya, pengesahan kata kunci atau PIN mempunyai satu kelemahan yang ketara. Kaedah ini gagal memastikan seseorang yang memasukkan kata kunci atau PIN adalah individu yang dimaksudkan. Sistem keselamatan yang lebih canggih masa kini menggunakan kaedah pengesahan biometrik bersama kata kunci sebagai penyelesaian. Pengesahan cap jari adalah pilihan yang sesuai disebabkan keunikannya dan ia senang diperolehi melalui sistem penangkapan imej.

Tesis ini menyiasat kaedah pengenalan cap jari dan klasifikasi. Kemudian penerangan mengenai pendekatan kajian dilakukan. Tesis ini menggabungkan kaedah pengecaman dan pengelasan cap jari.

Tesis ini mencadangkan suatu kaedah baru untuk memberikan capaian kepada pengguna selepas proses pengesanan menggunakan cap jari. Kaedah yang dicadangkan adalah berdasarkan pendekatan statistik. Pendekatan ini membolehkan operasi pengesanan adalah tersembunyi daripada pengguna sistem. Teknik-teknik pemprosesan imej yang membolehkan pengambilan ciri-ciri dengan lebih mudah telah digunakan seperti penyamaan (*equalization*), penapisan (*filtering*), penduaan (*binarization*) dan penipisan (*thinning*).

Suatu kaedah baru yang membina kekunci unik daripada imej cap jari dipersembahkan dan pengkalan cap jari yang mengandungi ciri-ciri cap jari, kekunci cap jari unik, kekunci awam dan rahsia ditunjukkan.

Maklumat di dalam pengkalan data dicapai secara pantas menggunakan kaedah pencarian pepokok Adelson Velskii dan Landis (AVL). Pepokok AVL digunakan untuk meningkatkan nisbah mampatan secara berkesan untuk sebarang jenis data. Keselamatan dicapai dengan bantuan sistem kekunci awam.

Keputusan seterusnya ialah kekunci unik berdasarkan cap jari telah dibina menggunakan imej cap jari pengguna dan digunakan untuk mencapai kekunci rahsia. Kaedah yang dicadangkan telah berjaya mengenalpasti cap jari pengguna walaupun daripada sumber imej yang mengandungi hingar, teralih atau terpusing.

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I certify that an Examination Committee met on 23rd June 2003 to conduct examination of Ahmed Baba Elmadani on his Doctor of Philosophy thesis entitled "Secure Access to Authorized Resources Based on Fingerprint Authentication" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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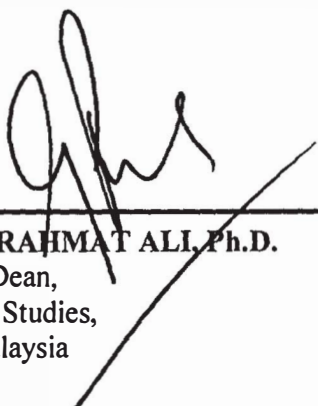
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I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.



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LIST OF SYMBOLS AND ABBREVIATIONS

FFT	Fast Fourier Transform
$F(u, v)$	Fourier Transform
$f(x, y)$	Inverse FFT
$g(x, y)$	The enhanced block
$G_x(i, j)$	Gradient of x-axis
$G_y(i, j)$	Gradient of y-axis
$\Phi(i, j)$	The direction of a block
I_c	Image with a length equals to 25 pixels
$S(i, j)$	Point in directional matrix
$C(i, j)$	Singular point
H	Directional core or delta point matrix
Pos	Position of delta with corresponding core
$Dist$	Distance between delta and corresponding core
ϖ	Mean feature
\mathfrak{R}	Standard deviation feature
η	Moment feature
δ	Variance feature
σ	Coherence feature
FP-Key	Unique fingerprint key
MSE	Mean square error
CC	Correlation Coefficient
$PR1$	Captured image

