UNIVERSITI PUTRA MALAYSIA

# APPLICATION OF THE DIFFERENTIAL QUADRATURE METHOD TO PROBLEMS IN ENGINEERING MECHANICS

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By

MD MOSLEMUDDIN FAKIR

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirement for the Degree of Master of Science

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**DEDICATION**

This Thesis is Dedicated To

**Our Parents, My Wife Dr. Sabira**

**&**

**Our Daughter Nusrat Jahan Shoumy**

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science.

**APPLICATION OF THE DIFFERENTIAL QUADRATURE METHOD TO PROBLEMS IN ENGINEERING MECHANICS**

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**MD. MOSLEMUDDIN FAKIR**

**April 2003**

**Chairman: Associate Professor Waqar Asrar, Ph.D.**

**Faculty: Engineering**

The numerical solution of linear and nonlinear partial differential equations plays a prominent role in many areas of engineering and physical sciences. In many cases all that is desired is a moderately accurate solution at a few grid points that can be calculated rapidly.

The standard finite difference method currently in use have the characteristic that the solution must be calculated with a large number of mesh points in order to obtain moderately accurate results at the points of interest. Consequently, both the computing time and storage required often prohibit the calculation. Furthermore, the mathematical techniques involved in the finite difference schemes or in the Fourier transform methods, are often quite sophisticated and thus not easily learned or used.

The differential quadrature method (DQM) is a numerical solution technique, which has been presented in this thesis. This method is a simple and direct technique, which can be applied in a large number of cases to circumvent the difficulties of programming complex algorithms for the computer, as well as excessive use of storage and computer time. The initial and/or boundary value problems can be solved by this method directly and efficiently. The accuracy of the differential quadrature (DQ) method depends mainly on the accuracy of the weighting coefficient computation, which is a vital key of the method. In this thesis, the technique has been illustrated with the solution of six partial differential equations arising in Heat transfer, Poisson and Torsion problem with accurate weighting coefficient computation and two types of mesh points distribution (equally spaced and unequally spaced). In all cases, the obtained DQ numerical results are of good accuracy with the exact solutions and hence show the potentiality of the method. It is also shown that the obtained DQ results in this thesis either agree very well or improved than those of some similar published results. This method is a vital alternative to the conventional numerical methods, such as finite difference and finite element methods. It is expected that this technique can be applied in a large number of cases in science and engineering to circumvent both the above-mentioned conventional difficulties.

Abstrak tesis dipersembahkan kepada Senat Universiti Putra Malaysia sebagai memenuhi syarat keperluan untuk ijazah Master Sains.

**APLIKASI KAEDAH PEMBEZAAN SUKUAN TERHADAP MASALAH KEJURUTERAAN MEKANIK**

**Oleh**

**MD. MOSLEMUDDIN FAKIR**

**April 2003**

**Pengerusi:** **Profesor Madya Waqar Asrar, Ph.D.**

**Fakulti:** **Kejuruteraan**

Kaedah penyelesaian berangka bagi persamaan pembezaan lelurus separa dan tidak lelurus memainkan peranan penting dalam pelbagai bidang kejuruteraan dan sains fizik. Dalam kebanyakan kes, kaedah-kaedah ini memerlukan penyelesaian yang tepat pada titik-titik grid dan bentuk pengiraannya boleh dilakukan secara berulangan.

Kaedah pembezaan terhingga yang sering digunakan pada masa ini kebiasaannya mempunyai penyelesaian yang perlu dikira bersama jumlah pada titik setara bagi mendapatkan keputusan yang tepat pada titik yang diingini. Hasilnya, kedua-dua pengiraan simpanan dan masa yang diperlukan tidak termasuk di dalam pengiraan tersebut. Tambahan lagi teknik matematik yang terlibat di dalam skema ini (Pembezaan Terhingga) atau di dalam kaedah Jelmaan Fourier agak kompleks dan tidak mudah dipelajari serta digunakan.

Tesis ini memperkenalkan satu teknik penyelesaian berangka yang dikenali sebagai Kaedah Pembezaan Sukuan. Kaedah ini sangat ringkas dan tepat. Ia boleh diapgunakan dalam pelbagai kes bagi mengatasi masalah-masalah pengaturcaraan algorithma yang kompleks pada komputer. Contohnya seperti penggunaan masa dan penyimpanan memori yang terlalu lama dan banyak di dalam komputer. Dengan menggunakan kaedah ini juga masalah pada peringkat awal dan/atau pada nilai sempadan dapat diselesaikan secara terus dengan rapi. Ketepatan kaedah Pembezaan Sukuan ini adalah bergantung sepenuhnya kepada ketepatan pengiraan pekali pemberat, di mana ia merupakan kunci utama dalam kaedah ini. Di dalam tesis ini juga membincangkan teknik di mana penyelesaian bagi enam Persamaan Pembezaan Sebahagian yang diterbitkan di dalam masalah Perpindahan Haba, Poisson dan Torsion telah dimasukkan bersama pengiraan pekali pemberat yang tepat dan juga berserta dua jenis pembahagian titik setara (jarak yang sama dan jarak yang tidak sama). Dalam semua kes, keputusan berangka Pembezaan Sukuan yang diperolehi adalah baik dari segi ketepatan berserta penyelesaian yang betul dan ini menunjukkan keupayaan kaedah ini. Keputusan Pembezaan Sukuan yang diperolehi dalam tesis ini juga memberi persetujuan di mana keputusan yang diperoleh adalah lebih baik daripada semua keputusan yang seakan-akan sama diterbitkan. Kaedah ini merupakan alternatif kepada kaedah berangka yang biasa digunakan iaitu Pembezaan Terhingga dan Kaedah Unsur Terhingga. Ia dijangka dapat digunakan di dalam pelbagai perkara dalam bidang sains dan kejuruteraan bagi mengatasi masalah-masalah yang berkaitan dengan kedua-dua kaedah yang disebutkan di atas.

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# CHAPTER 1

# INTRODUCTION

## 1.1 Background

Presently there are many numerical solution techniques known to the computational mechanics community. Differential Quadrature Method (DQM) is one of those numerical solution techniques to solve initial and/or boundary value problems which arise in problems of engineering and physical sciences. The essence of the DQM is that a partial derivative of a function is approximated by a weighted linear sum of the function values at given discrete points. Richard Bellman and his associates developed this numerical solution technique in the early 1970s and since then, the technique has been successfully employed in a variety of problems in engineering and physical sciences. This relatively recent origin numerical technique has been projected by its proponents as a potential alternative numerical solution technique to the conventional numerical solution techniques such as finite difference method and finite element method. Compared with those methods, the DQM requires less computational times and computer storage.

Due to its rather recent origin, the DQM is possibly not well known to the computational mechanics community. However, Bellman and Casti (1971), in their introductory paper, proposed the Differential Quadrature Method (DQM) as a new technique for the numerical solution of initial and/or boundary value problems of ordinary and partial differential equations. The paper was apparently aimed toward offering an alternative solution technique in view of the problems of numerical stability. But the paper included no details such as the determination of weighting coefficients and example application of the method. The proposed new technique was fully illustrated in a subsequent paper by Bellman *et al* (1972) where they solved numerically some partial differential equations arising in different models of fluid flow and turbulence.

Here we focus on the accurate determination of weighting coefficients, which is a vital need to solve engineering problems numerically using DQM technique. Hence the efficiency of the method along with weighting coefficient is investigated by solving some example application engineering problems in mechanics.

## 1.2 The Reasons for Using Differential Quadrature Method (DQM) as a Numerical Solution Technique

The Differential Quadrature Method has been used due to the following reasons:

(i) The method is very efficient to find the accurate numerical solution even with fewer number of grid points.

(ii) Efficient technique in terms of memory consumption and computational time.

(iii) There is no need of coordinate transformation from physical domain to computational domain.

(iv) The method is mathematically less cumbersome.

(v) In recent years, the DQ method has become increasingly popular in the numerical solution of initial and boundary value problems.

(vi) It is still under developing stage and has wide scope of applications.

## 1.3 Research Objective

The research objective is to apply the numerical solution technique DQM to solve accurately initial and/or boundary value problems of ordinary and partial differential equations, which arise in problems of engineering mechanics.

In this thesis, six engineering problems have chosen to solve accurately and efficiently the following six problems by the differential quadrature method (DQM). The problems are:

(i) Temperature distribution in a triangular fin

(ii) Torsion of a rectangular cross-section shaft

(iii) Solution of Poisson equation in a rectangular domain

(iv) Temperature distribution in an Insulated tip rectangular fin

(v) Temperature distribution in a Convection tip rectangular fin

(vi) Temperature distribution in a very long rectangular fin

In order to solve the above problems accurately, the main objectives of this thesis are:

(i) To compute the weighting coefficients accurately

(ii) To develop a computer code for the DQM

(iii) To apply it to solve problems in engineering mechanics

(iv) To verify the accuracy of the results by comparing with the exact solution and with the published numerical solution.

## 1.4 Contribution of the Thesis

In the thesis, all the six application problems of engineering mechanics, which are mentioned in section 1.2, have been solved independently by the method of differential quadrature (DQM). The results are found to agree very well with the exact solution and either agree or sometimes better than the published numerical solution (whenever available) in the literature.

To the best of author's knowledge, the solution of poisson equation in a rectangular domain and the solutions of rectangular fin problems (insulated tip fin, convection tip fin and long fin) have not been solved earlier by the differential quadrature method (DQM).

It is expected that this thesis will contribute something additional with the potentiality of the differential quadrature method to the computational mechanics community.

In order to meet the objectives of this work, the main contributions of this thesis are:

(i) The weighting coefficients are determined accurately to obtain relatively accurate DQ numerical solution.

(ii) The computer codes are developed for DQM to solve problems in engineering mechanics.

(iii) Six application problems in heat transfer (temperature distribution), Poisson and elasticity are solved numerically using DQM.

(iv) The performance of DQ numerical results is evaluated comparing with exact results and some other published similar results.

## 1.5 Organization of the Thesis

The thesis is consisted of six Chapters. Following an introduction, the chapter 2 discusses a through literature review on past and present research of DQM which has been presented in detail. From the very beginning to present development of the DQM inclusive of the areas of interest covered by the method has been discussed.

In Chapter 3, the quadrature rule and determination of the weighting coefficients of DQM are discussed and formulated. The computation of weighting coefficients is a vital task for the method as the accuracy of the DQ numerical solution depends on the accuracy of the weighting coefficients. The formulae for percent error calculation, equally and unequally spaced sampling points distribution are also presented in this chapter.

In chapter 4, mathematical formulations of application problems of differential quadrature method have been illustrated in detail including exact solutions. The example application problems in engineering mechanics are: heat distribution in a triangular fin, torsion of a rectangular cross-section shaft, solution of Poisson equation in rectangular domain, temperature distribution in an insulated tip fin, temperature distribution in a convection tip fin and temperature distribution in a very long fin. Quadrature analog equations of the governing equations, exact equations and boundary conditions are presented in this chapter too.

The results and discussions of the application problems are presented in chapter 5. Results for both equally spaced and unequally spaced sampling points are shown in terms of tables and graphs. Cubic spline interpolation results are presented wherever necessary. Convergence and comparison of the solutions for equally and unequally spaced and for odd and even number of sampling points are depicted in the figures. Maximum percent errors for equal and unequal spacing sampling points are shown. In two-dimensional torsion problem and poisson problem, surface graphs are presented for exact and numerical solutions.

Chapter 6 concludes the thesis by highlighting the efficiency and accuracy of our DQ numerical solutions and future research directions. Finally, references and biodata of the author are added at the end of the thesis.

# CHAPTER 2

# LITERATURE REVIEW

## 2.1 Differential Quadrature Method (DQM)

The Differential Quadrature Method (DQM) was first proposed by Bellman *et al* (1972) who solved some initial and boundary value problems of ordinary differential equation (ODE) and partial differential equation (PDE). The method has a relatively recent origin and is being gradually employed as a separate solution technique for the initial and boundary value problems of engineering and physical sciences. Areas of the problems in which the applications of DQM may be found in the literature include fluid mechanics, bioscience, statics and dynamics of structural mechanics, transport processes, static aero-elasticity and lubrication mechanics. It has been found that the DQM has a better capability of producing highly accurate solutions with minimal computational effort. A comprehensive literature review on DQM is given in Bert and Malik (1996). Here, most part of literature review is taken from that review paper.

Bellman and Casti (1971), in their preliminary paper, formulated the quadrature rule for a derivative as an analogous extension of quadrature for integrals. The paper did not include any details such as the determination of weighting coefficients nor provide any application of the method although the work apparently aimed at offering an alternative technique for solution in view of problems of numerical stability and large computation times involved with long-