Modified Mann-Kendall with Higher-Order Statistics for Trend Analysis

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Abstract

Trend analysis of rainfall events is important to understand changes in precipitation patterns over time, which may affect climate adaptation planning. The Mann–Kendall (MK) trend test is preferred by researchers for its robustness in handling non-normal data with extreme outliers. However, the assumption of independence is often not fulfilled. Modifications have been introduced to improve MK performance under autocorrelation, but the issue of nonlinearity is still not widely discussed. In this study, a modified MK called Mann–Kendall with Third-Order Cumulant (MKC3) is proposed. A simulation study was conducted with comparisons made against MK and other MK variants to provide practical guidance for practitioners, followed by a case study of rainfall in Peninsular Malaysia. The results show that MK, TFPW, SMK, MKRD, and MKC3 have different strengths: MK for independent data, TFPW preserves trends in autocorrelated series, SMK performs well for weak autocorrelation with small sample size, MKRD is robust for strong autocorrelation and sinusoidal models, and MKC3 performs well in bilinear and nonlinear models. The case study shows increasing trends during the Northeast Monsoon (NEM) and decreasing trends during the Southwest Monsoon (SWM). Overall, MKC3 shows promising robustness, but the selection remains a trade-off.

Keywords: Mann-Kendall, Trend analysis, Rainfall, Nonlinearity

1 Introduction

Trend detection in climate data, particularly on extreme rainfall event, is always the famous topic which has been researched lately. Trend analysis is crucial to provide informed decisions to authorities. One of the most famous trend detection tests used by researchers is the Mann-Kendall (MK) trend test (**??**), a nonparametric statistical test created by Henry B. Mann and Maurice G. Kendall.

Taking Malaysia as an example, **?** used MK to detect trend of extreme rainfall indices for two monsoons and two intermonsoon, with significant evidence of trend detection discovered on intensity, frequency and cumulative amounts in Northeast Monsoon (NEM) and March-April (MA), while extreme intensity and frequency in Southwest Monsoon (SWM) and September-October (MO). **?** used MK, Seasonal MK (SMK) and Scaling Hypothesis MK (SHMK) in trend analysis of rainfall and rainfall extremes in Sarawak, Malaysia. The result revealed differences of significance of trend with MK and its modifications, revealing the presence of scaling effects in influencing the MK's interpretation. Similar research was also conducted by **?**, who used the MK to interpret the significance of trends for a total of seven extreme rainfall indices across five stations in Peninsular Malaysia. All the extreme rainfall indices showed an increasing trend, except for Kota Bharu, which is located on the east coast of Peninsular Malaysia. Similar study was also done by **?**, using MK to interpret trend of extreme rainfall indices in east coast of Peninsular Malaysia.

According to **?**, despite having less robustness in accuracy of results compared to parametric tests, MK is suitable to be applied for trend analysis in climate data, as it uses the ranks of observations to detect the trend rather than using their exact values. This makes MK robust in handling different types of distributions and outliers, which are commonly found in climate data. However, **?** mentioned that positive serial correlation, also generally known as positive autocorrelation, increases the chance of a significant result even when no trend is present.

The previous researchers understood the impact of positive autocorrelation in affecting the interpretation of trends in climate data. This has led to the development of various modified versions of the MK. In general, they can be classified into three groups. The first method is preprocessing the data before applying MK, which includes Pre-whitening (PW) by **?**, Trend-Free Pre-whitening (TFPW) by **?**, and Block Bootstrap Mann-Kendall (BBSMK) by **?**. The second method is modifying the MK, particularly its variance, which includes Seasonal MK (SMK) by **?**, Partial MK (PMK) by **?**, and Scaling Hypothesis MK (SHMK) by **?**. The last method is the fusion of the first and second methods, which includes Mann-Kendall Rank Detrended (MKRD) by **?** and Mann-Kendall Data Detrended (MKDD) by **?**.

From the traditional MK test proposed to handle non-normal data, to the later stage when autocorrelation was discovered to cause false trend detection, various modified versions of the MK test have been developed to minimize the impact of autocorrelation. As mentioned in the previous paragraph, these modified approaches share a common goal of improving the reliability of trend detection under autocorrelation conditions. For example, PW aims to decorrelate the data at lag 1 before applying the MK test, while TFPW preserves the trend before performing decorrelation. However, both methods have been debated. **?** stated that PW could theoretically remove part of the trend during the decorrelation process, which reduces the statistical power. On the other hand, **?** argued that TFPW contradicts the rationale of the PW procedure, as TFPW focuses on maintaining the trend to avoid a loss in statistical power. Similarly, **?** disagreed with TFPW, emphasizing that the primary objective should be to control the type I error rate, while statistical power should only be a secondary concern.

Apart from that, SMK was reported by **?** to have weak statistical power, and similar findings were also reported for BBSMK, PMK, and SHMK by their respective authors. Meanwhile, MKRD and MKDD employ nearly similar techniques for computing the corrected variance formula, with the main difference lying in the data used to identify the correction factor. MKRD applies the correction to rank-detrended data, whereas MKDD uses classically detrended data. **?** compared both MKRD and MKDD in terms of type I and type II errors, and their simulation results showed comparable outcomes, with MKDD performing slightly better in reducing type I error and MKRD showing an advantage in minimizing type II error.

The common issue faced by all types of modifications can be summarized simply. Although these methods effectively reduce type I errors, they also tend to decrease statistical power, except for TFPW, which preserves the trend beforehand. This explains why many researchers have emphasized the trade-off mechanism in their approaches, recognizing that no single modification is universally optimal and that each method is more suitable for particular situations. Therefore, it would be beneficial to include practical guidance in the discussion section to assist practitioners in selecting between the conventional MK test and its various modified versions.

This selection is further complicated by the fact that autocorrelation is far from the only issue present in real-world climate data. **?** have verified the presence of nonlinear correlation of El Niño 3.4 index, where $x(t)^2$ is correlated with $x(t+4)$ at -0.24 by bicovariance, which cannot be detected by lower-order statistics such as correlation and covariance. **?** had also shown the presence of nonlinearity of the daily rainfall data at several selected stations in the United States. **?** also verified the existence of nonlinearity in temporal rainfall, which urges the need to consider nonlinearity in high-resolution temporal rainfall data.

An important question that arises is whether explainable linear dependencies, such as autoregressive (AR) processes which exhibit strong autocorrelation, can have the type I errors reduced through MK modification. However, what if a future value cannot be explained by a linear function, such as nonlinear or chaotic behavior? To the best of authors' knowledge, there is currently limited or no published research discussing the influence of nonlinearity on MK's interpretation. Nevertheless, it is reasonable to hypothesize that if MK is known to be influenced by positive serial correlation, then similar effects might also arise from positive nonlinear dependencies.

Therefore, we propose a modification of the MK, which will be named as Mann-Kendall with Third-Order Cumulant (MKC3) that aims to reduce the impact of both positive serial correlation and nonlinear dependencies. A Monte Carlo simulation will be conducted to evaluate type I and type II error rates. The results will be compared against existing MK modifications to develop practical guidance for practitioners. In addition, a case study will be conducted to assess trends in daily rainfall, the monthly maximum 1-day precipitation (Rx1day), and the monthly maximum consecutive 5-day precipitation (Rx5day), all measured in millimeters (mm).

Both Rx1day and Rx5day are well-established extreme rainfall indices developed by the World Meteorological Organization (WMO) Expert Team on Climate Change Detection and Indices (ETCCDI). These indices represent the wettest single day (Rx1day) and the wettest consecutive five days (Rx5day) within each month. Trend analysis of these indices allows for the identification of stations across Peninsular Malaysia experiencing worsening extreme rainfall, serving as an indicator for assessing potential risk regions.

2 Data

2.1 Data simulation

We simulated nine types of time series data with 10,000 replications, with sample size of 30, 50, 100 and 200 respectively to examine the type I and type II error rates of MK, TFPW, SMK, MKRD and MKC3. Table **??** showed the types of simulated data, with details shown in Appendix **??**.

Table 1 Type of simulated data

No.	Time series model	Trend	Autocorrelation	Nonlinearity
1	White Noise	No	No	No
2	Autoregressive Model	No	Yes	No
3	Nonlinear Autoregressive	No	Yes	Yes
4	Bilinear Model	No	Yes	No
5	Sinusoidal Model	No	Yes	Yes
6	Chaotic	No	Yes	Yes
7	Deterministic Linear Trend	Yes	No	No
8	Autoregressive with Trend	Yes	No	Yes
9	Sinusoidal with Trend	Yes	Yes	Yes

Note: Details of each simulated time series models is shown in Appendix **??**
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2.2 Case study for application

Peninsular Malaysia, located between 1° to 6° North and 100° to 103° East. According to **?**, the rainfall patterns are influenced by two monsoon seasons. The NEM, which occurs from November to February, brings heavy rainfall to the east coast, especially from November to January. The SWM, which takes place from May to August, is generally associated with drier conditions, with June and July being the driest months in most regions.

In the central and northern parts of the Peninsular Malaysia, rainfall typically shows two peaks. The first peak usually occurs in April to May and the second in October to November, while the minimum rainfall is commonly recorded in June to July and February. In the southwest coastal area, the pattern of double peaks and minima is less clear due to the influence of early morning Sumatras during the Southwest Monsoon. As a result, October and November record the highest rainfall, and February records the lowest (**?**).

Daily rainfall data recorded by rainfall stations across Peninsular Malaysia, obtained from the Malaysia Drainage and Irrigation Department (DID), were used. The selection criterion was data adequacy in which stations with more than 5% missing values were excluded. After evaluation, 15 stations were selected, as shown in Table **??**, and their distribution is illustrated in Fig. **??**.

Table 2 Name of station with latitude and longitude

No.	Station	Name of station	Latitude (°)	Longitude (°)
1	1632096	Pintu Kawalan Tampok	1.63	103.20
2	2237164	Ladang Lendu	2.36	102.19
3	2723075	Ladang Juasseh	2.80	102.31
4	3022001	Kuala Krau	3.71	102.37
5	3214057	Kg. Pulau Besar	4.14	100.99
6	3723077	Gunung Brinchang	4.52	101.38
7	3933001	Empangan Jajar	5.67	102.49
8	4109095	Kuala Jambu	6.15	102.10
9	4313033	Kaki Bukit	6.64	100.21
10	5205052	Bagan Buaya	5.22	100.50
11	5529027	Kg. Merang	5.53	102.95
12	5624001	Bandar Tasik Puteri	3.30	101.49
13	5806066	Jeniang Klinik	5.81	100.63
14	6120014	Hulu Jabor	3.92	103.31
15	6602002	Bt. 42 Jalan Kluang	2.26	103.74

Source: Malaysia Drainage and Irrigation Department

For missing values, we will impute by Inverse Distance Weighting (IDW) interpolation, which is given by:

$$y_{j(m)} = \sum_{i=1}^{n}(d_{mi}^{-k} \cdot x_{j(i)}) / \sum_{i=1}^{n} d_{mi}^{-k} \tag{1}$$

where $x_{j(i)}$ is the observed rainfall at neighboring station i, d_{mi} is the euclidean distance between station m and station i while k is the power parameter. In this study, we used $k = 2$ as suggested by **?**.

For trend analysis, we used daily rainfall data and the derived extreme weather indices which are Rx1day and Rx5day, calculated from the daily rainfall records. The Rx1day and Rx5day for period j are defined as follows:

$$\text{Rx1day}_j = \max(RR_{ij}) \tag{2}$$

$$\text{Rx5day}_j = \max(RR_{kj}) \tag{3}$$

where RR_{ij} and RR_{kj} denote the precipitation amount for the one-day and five-day intervals ending on day i and k, respectively, in period j.

3 Methodology

In this section, we outline the methodology used to evaluate the performance of MKC3 in comparison with MK, TFPW, SMK, and MKRD. The proposed MKC3 is explained in detail in this section. In accordance with the application to climate data, MKC3, together with the mentioned MK modifications, is applied to rainfall and extreme rainfall indices.

A Python library named *pymannkendall* for MK and its variants by **?** is used in simulation and case study. This library provides a consistent framework for applying MK, TFPW, SMK, MKRD, and other modifications, suitable for trend analysis in climate data. The MKC3 function in this study is adapted from `mk.hamed_rao_modification_test`, with modifications to include the third-order cumulant in the variance correction formula of MKRD. Before obtaining the third-order cumulant, the rank-detrended data are standardized. This modification is intended to improve the handling of time series with both autocorrelation and nonlinearity.

Both type I and type II error rates will be tabulated to compare the performance of MK and its variants across various time series models. A flowchart providing practical guidance for practitioners on selecting among MK, TFPW, SMK, MKRD, and MKC3 in applied settings will be developed based on the simulation results.

In the case study, the Z-score statistics and p-values of MK, TFPW, SMK, MKRD, and MKC3 will be tabulated. To provide clearer insight into regions exhibiting significant trends in daily rainfall, Rx1day, and Rx5day, maps illustrating the Z statistics of each method for each rainfall station will be presented. The maps were produced using QGIS Desktop 3.44.1 **?**.

3.1 Mann-Kendall Trend Test

The MK trend test (**??**) evaluates the correlation between the ranks of the time series values and their chronological order. For a time series $X = \{x_1, x_2, \ldots, x_n\}$, the test statistic is defined as:

$$S = \sum_{i<j} a_{ij} \tag{4}$$

where

$$a_{ij} = \operatorname{sign}(x_j - x_i) = \operatorname{sign}(R_j - R_i) = \begin{cases} 1 & x_i < x_j \\ 0 & x_i = x_j \\ -1 & x_i > x_j \end{cases} \tag{5}$$

and x_i and x_j are the time series values whose ranks are given by R_i and R_j, respectively.

By assuming data independence and random ordering, the statistic S follows a normal distribution for large n, with mean and variance given by:

$$\mathbb{E}[S] = 0 \tag{6}$$

$$\operatorname{Var}[S] = \frac{n(n-1)(2n+5) - \sum_t^n t(t-1)(2t+5)}{18} \tag{7}$$

where t is the number of tied observations.

The standardized test statistics Z is computed by:

$$Z = \begin{cases} \frac{S-1}{\sqrt{\operatorname{Var}[S]}} & S > 0 \\ 0 & S = 0 \\ \frac{S+1}{\sqrt{\operatorname{Var}[S]}} & S < 0 \end{cases} \tag{8}$$

The significance of the trend is assessed using the value of Z. In a two-sided test, the null hypothesis H_0 is rejected if $|Z| > 1.96$ at the 5% level of significance, indicating the presence of a trend. The sign of the Z-value reflects the direction of the trend, with a positive value indicating an increasing monotonic trend and a negative value indicating a decreasing monotonic trend.

3.2 Trend Free Pre-whitening (TFPW)

? introduced Trend Free Pre-whitening (TFPW), a technique applied before conducting MK, which aims to detect a significant trend in a time series with strong autocorrelation.

Firstly, Theil-Sen Estimator (TSE) is conducted to identify linear trend, followed by a detrended procedure, which is given by:

$$X'_t = X_t - bt \tag{9}$$

where b is the slope estimated by TSE.

Next, the decorrelation of X_t is performed based on the lag-1 serial correlation coefficient, which is given by:

$$Y'_t = X'_t - r_1 X'_{t-1} \tag{10}$$

where r_1 is the coefficient of the first lag.

Lastly, the identified trend estimated by TSE is blended with Y'_t to get:

$$Y_t = Y'_t + bt \tag{11}$$

in which Y_t represents the modified time series data evaluated using the MK procedure, as detailed in Section **??**, to determine the significance of the trend, with Z-score statistics represented by Z_{TF}

3.3 Seasonal Mann-Kendall (SMK)

The SMK test, introduced by **?**, accounts for seasonality, missing values, and autocorrelation in time series data. As censored observations are common in hydrological records, particularly flood data, it is essential to adapt the MK test for seasonal and censored time series. The SMK test statistic is given by:

$$S' = \sum_{g=1}^{p} S_g \tag{12}$$

and approaches a normal distribution asymptotically, with a mean of 0 and the variance given by:

$$\mathrm{Var}[S'] = \sum_{g} \sigma_g^2 + \sum_{\substack{g,h \\ g \neq h}} \sigma_{gh} \tag{13}$$

where $\sigma_g^2 = \mathrm{Var}\,[S_g]$ and $\sigma_{gh} = \mathrm{Cov}\,[S_g, S_h]$.

To dive into details, σ_{gh} is estimated by a consistent estimator given by:

$$\hat{\sigma}_{gh} = \frac{K_{gh}}{3} + \frac{(n^3 - n) r_{gh}}{9} \tag{14}$$

where

$$K_{gh} = \sum_{i<j} \mathrm{sign}[(X_{jg} - X_{ig})(X_{jh} - X_{ih})] \tag{15}$$

$$r_{gh} = \frac{3}{n^3 - n} \sum_{i,j,k} \mathrm{sign}[(X_{jg} - X_{ig})(X_{jh} - X_{kh})] \tag{16}$$

The Z-score statistic of SMK, denoted as Z_S, is obtained in a similar manner to Eqn. **??**, using S' and $\mathrm{Var}[S']$ as defined in Eqn. **??** and Eqn. **??**, respectively.

3.4 Mann-Kendall Rank Detrended (MKRD)

? proposed the MKRD method, which applies a variance correction by using rank-detrended data, as defined by the following expression:

$$\mathrm{Var}^*[S] = \mathrm{Var}[S] \cdot \frac{n}{n_s^*} \tag{17}$$

where Var[S] is shown in Eqn. **??** and $\frac{n}{n_s^*}$ represent the correction due to autocorrelation, which is given by:

$$\frac{n}{n_s^*} = 1 + \frac{2\sum_{i=1}^{n-1}(n-i)(n-i-1)(n-i-2)\rho_s(i)}{n(n-1)(n-2)} \tag{18}$$

where n is the number of observations, n_s^* the effective sample size, and $\rho_s(i)$ the lag-i autocorrelation of the rank detrended data.

The standardized statistics of MKRD, defined as Z^* is obtained similarly as shown in Eqn. **??**, by replacing Var[S] to Var*[S].

3.5 Mann-Kendall with Third-Order Cumulant (MKC3)

The focus of this research is to incorporate a term into the variance correction formula that can detect the presence of nonlinearity. To achieve this, the third-order cumulant is selected. **?** mentioned that for a zero-mean, stationary real-valued time series $x(t)$, the third-order cumulant, which is also the third central moment for zero-mean processes, is defined as:

$$C_3(\tau_1, \tau_2) = \mathbb{E}[x(t) \cdot x(t+\tau_1) \cdot x(t+\tau_2)] \tag{19}$$

where τ_1 and τ_2 are the time lags. This expression captures third-order statistical dependencies, such as nonlinearity, which cannot be detected by covariance or correlation. If nonlinearity does not exist, the third-order cumulant is zero.

Given a discrete time series of $x(t)$ of length N, which has been detrended and standardized, the estimation of $C_3(\tau_1, \tau_2)$ is defined as:

$$\hat{C}_3(\tau_1, \tau_2) = \frac{1}{N-\mu} \sum_{k=1}^{N-\mu} x(t_k) \cdot x(t_k+\tau_1) \cdot x(t_k+\tau_2) \tag{20}$$

where $\mu = \max(\tau_1, \tau_2)$ is the maximum lag. In this research, $\tau_1 = 1$ and $\tau_2 = 2$ are used to capture the nonlinearity in the short term, as the focus is to capture the nonlinear dependency of the time series on a specific recent short-term lags, which is given by:

$$\Phi_3 = \frac{1}{N-2} \sum_{k=1}^{N-2} x(t_k) \cdot x(t_k+1) \cdot x(t_k+2) \tag{21}$$

The Φ_3 will be added into the variance correction formula of MKRD, as a further modification, which is given by:

$$\frac{n}{n_s'} = \frac{n}{n_s^*} + \Phi_3 = 1 + \frac{2\sum_{i=1}^{n-1}(n-i)(n-i-1)(n-i-2)\rho_s(i)}{n(n-1)(n-2)} + \Phi_3 \tag{22}$$

which gives the modified variance of

$$\mathrm{Var}'[S] = \mathrm{Var}[S] \cdot \frac{n}{n'_s} \tag{23}$$

The standardized Z statistics of MKC3, defined as Z' is obtained similarly as shown in Eqn. **??**, by replacing $\mathrm{Var}[S]$ to $\mathrm{Var}'[S]$.

The rationale for setting the time lags to 1 and 2 in the third-order cumulant is to reduce the impact of false positives. Selecting wider time lags does not significantly affect the false positive issue. This is consistent with the correction formula shown in Eqn. **??**, where the autocorrelation at larger time lags receives progressively lower weight in the adjustment.

According to **?**, who applied various types of MK tests to identify trends under both short and long-term persistence, short-term persistence tends to overestimate trends (increasing the likelihood of false positives), whereas long-term persistence may lead to underestimation (the opposite effect). These findings underscore the importance of accounting for short-term dependencies or nonlinearity in MK modifications. Consistent with the perspective of **?**, reducing type I errors should remain the primary priority when modifying the MK test. This consideration forms the basis for selecting these time lags in MKC3.

4 Results and discussion

This section evaluates the performance of MK and its modified versions (TFPW, SMK, MKRD, and MKC3) through comparisons of type I and type II error rates. Furthermore, practical guidance for practitioners in selecting appropriate methods is provided. The approaches are then applied to daily rainfall and the extreme rainfall indices of Rx1day and Rx5day, followed by maps to illustrate the Z statistics of each method for each rainfall stations.

4.1 Type I and Type II error rates

Tables **??** and **??** show the type I and type II error rates of MK, TFPW, SMK, MKRD, and MKC3. Overall, the results indicate that no single method is universally robust, as each performs best depending on the sample size or the type of time series model.

MK itself has proven its robustness when dealing with white noise data, particularly for large sample sizes of 100 and 200. It has shown that modifications are not required when the time series data do not exhibit autocorrelation.

TFPW is the only method that preserves the trend in advance. This leads to no outstanding performance in reducing type I error rates while showing its robustness in reducing type II error rates when dealing with time series models that exhibit trends and dependencies at the same time.

The findings of SMK are in line with the statement by **?**, where the robustness of SMK in reducing type I error rates leads to a sacrifice of statistical power. SMK is robust for intermediate autocorrelated data, sinusoidal and chaotic models with a small sample size of 30, but its robustness begins to vanish as autocorrelation becomes stronger and sample size increases. It is also concerning that SMK appears to be overcorrected for small sample sizes, as the type I error rate of white noise should always follow the nominal level of 0.05.

MKRD is robust for strongly autocorrelated data and sinusoidal models with larger sample sizes of 50, 100, and 200. The crossover point between SMK and MKRD occurs in the AR model with $\phi = 0.6$ for sample sizes of 50 and 100, and with $\phi = 0.3$ for sample size of 200. Meanwhile, the type I error rates for the chaotic model show nearly identical performance between the two methods. In context of type II error rates, MKRD is much robust in strong deterministic trend models.

MKC3 generally performs well in bilinear and nonlinear models for all types of sample sizes except for a sample size of 30. In the context of type II error rates, MKC3 is outstanding in weaker deterministic trend models, indicating robustness in detecting weaker trends.

4.2 Practical Guidance in MK Selection

Fig. **??** presents a detailed summary guiding practitioners on how to select the appropriate MK test for conducting trend analysis. In this paper, we do not provide an in-depth discussion on how climatological data should undergo Exploratory Data Analysis (EDA). Generally, identifying the data structure should include, but is not limited to, methods such as the TSE for slope valuation, Autocorrelation Function (ACF) for detecting autocorrelation, and Bispectrum Analysis for detecting nonlinear structures. Ideally, this should be demonstrated in the case study, which is left for future research. As for the case study, the priority is still the results comparison between the variants of MK.

From a general perspective, Fig. **??** clearly shows that no single method can be considered robust across all scenarios. Therefore, as a closing remark for this section, it is emphasized that both the data structure and sample size should be carefully considered when selecting the appropriate MK test.

4.3 Application in rainfall data

Table **??**, Table **??**, and Table **??** show the trend analysis of daily rainfall, Rx1day, and Rx5day for 15 selected stations across Peninsular Malaysia. Fig. **??**–**??** present maps illustrating the Z-score statistics for MK, TFPW, SMK, MKRD, and MKC3 for daily rainfall, Rx1day, and Rx5day.

Referring to the trend results of daily rainfall in Table ??, MK and SMK report 13 stations with significant increasing trends across the full period, followed by four stations identified by TFPW, and two stations by MKRD and MKC3. During the NEM, MK and SMK report 14 and 13 stations with significant increasing trends, respectively, followed by six stations identified by TFPW and four stations by MKRD and MKC3. For the SWM, MK and SMK report 2 significant increasing trends, TFPW reports one, while MKRD and MKC3 report none.

For Rx1day, as shown in Table ??, SMK, MKRD, and MKC3 report one station with a significant increasing trend. During the NEM, MK and TFPW report one significant increasing trend, MKRD and MKC3 report two and three, while SMK reports none. During the SWM, MK, TFPW, and MKRD report one significant increasing trend, SMK reports one significant decreasing trend, while MKC3 reports one significant increasing and one significant decreasing trend.

For Rx5day, MKRD and MKC3 report one and two significant increasing trends, respectively, while the other methods report none. During the NEM, MKRD and MKC3 report two stations with significant increasing trends. During the SWM, MK and MKRD report one significant increasing trend each, while MKC3 reports one significant increasing and one significant decreasing trend.

In a broader interpretation, referring to Table ??–??, generally MK and all the variants show that daily rainfall shows strong potential for an overall increase, especially during the NEM, while the SWM indicates a potential decrease. For both Rx1day and Rx5day during the NEM, although few stations exhibit significant trends, attention is warranted because the number of stations with relatively higher z-score statistics is high.

Another concern is the high coverage of negative Z-score statistics for the SWM in daily rainfall, Rx1day, and Rx5day across the northern and parts of the east coast regions, as shown in Fig. ??, Fig. ??, and Fig. ??. This finding is consistent with ?, whose significant results revealed a decline in rainfall in Perak, a state located in the northern region, during the SWM. This indicates a potential increase in rainfall discrepancies between the NEM and the SWM.

An additional finding in Table ?? and Table ?? indicates that the Z statistics of the SMK test produced identical Z-score values for a considerable number of stations in Rx1day and Rx5day. A similar outcome is also observed in the MK, TFPW, MKRD, and MKC3 tests, although the issue is not as severe as in SMK. This occurs due to the relatively small sample sizes in Rx1day and Rx5day, particularly during the NEM and SWM periods, which contain only 22 and 23 data points, respectively. The reason SMK shows a more severe outcome is that the limited number of ties within these seasonal periods provides insufficient information for computing the modified statistic S', as shown in Eqn. ??. Consequently, this leads to unusual outcomes for the SMK test in Rx1day and Rx5day. Therefore, the use of SMK is not recommended for small sample sizes to avoid unreliable results, consistent with the statement in Section ??, which indicates that SMK tends to overcorrect white noise for sample sizes of 30 and 50.

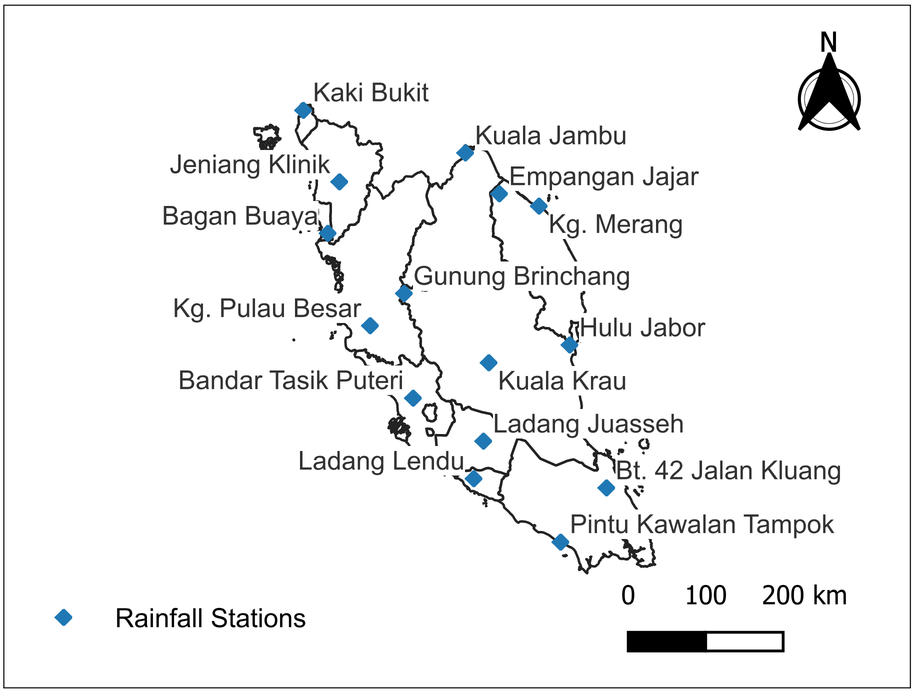

Fig. 1 Distribution of stations used in this study. Map generated using QGIS Desktop version 3.44.1 (**?**).

Table 3 Type I error rates of MK, MKRD, and MKC3 for each time series model

		Type I error rates				
n	Time series model	MK	TFPW	SMK	MKRD	MKC3
30	$y_t = w_t$	0.054	0.056	0.040*	0.090	0.094
	$y_t = 0.3y_{t-1} + w_t$	0.119	0.151	0.082*	0.147	0.149
	$y_t = 0.6y_{t-1} + w_t$	0.263	0.341	0.193*	0.222	0.224
	$y_t = 0.9y_{t-1} + w_t$	0.527	0.671	0.434	0.370*	0.373
	$y_t = 0.5y_{t-1} + 0.05y_{t-1}^2 + w_t$	0.215	0.281	0.156*	0.200	0.199
	$y_t = 0.9y_{t-1} + 0.5w_{t-1}y_{t-1} + w_t$	0.451	0.560	0.352	0.325	0.319*
	$y_t = \sin(y_{t-1}) + w_t$	0.263	0.302	0.178*	0.238	0.235
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + w_t$	0.238	0.323	0.143*	0.250	0.249
50	$y_t = w_t$	0.049	0.051	0.034*	0.089	0.093
	$y_t = 0.3y_{t-1} + w_t$	0.129	0.154	0.089*	0.147	0.151
	$y_t = 0.6y_{t-1} + w_t$	0.286	0.377	0.212	0.201*	0.204
	$y_t = 0.9y_{t-1} + w_t$	0.586	0.749	0.511	0.358*	0.359
	$y_t = 0.5y_{t-1} + 0.05y_{t-1}^2 + w_t$	0.234	0.298	0.172*	0.185	0.182
	$y_t = 0.9y_{t-1} + 0.5w_{t-1}y_{t-1} + w_t$	0.464	0.606	0.380	0.292	0.285*
	$y_t = \sin(y_{t-1}) + w_t$	0.270	0.322	0.202	0.199*	0.199*
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + w_t$	0.934	0.947	0.839*	0.906	0.904
100	$y_t = w_t$	0.051*	0.052	0.053	0.090	0.091
	$y_t = 0.3y_{t-1} + w_t$	0.136	0.158	0.123*	0.129	0.133
	$y_t = 0.6y_{t-1} + w_t$	0.297	0.394	0.273	0.184*	0.184*
	$y_t = 0.9y_{t-1} + w_t$	0.621	0.798	0.591	0.330*	0.331
	$y_t = 0.5y_{t-1} + 0.05y_{t-1}^2 + w_t$	0.250	0.320	0.222	0.158	0.152*
	$y_t = 0.9y_{t-1} + 0.5w_{t-1}y_{t-1} + w_t$	0.481	0.644	0.452	0.258	0.251*
	$y_t = \sin(y_{t-1}) + w_t$	0.275	0.342	0.249	0.173*	0.175
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + w_t$	0.649	0.668	0.575	0.382*	0.383
200	$y_t = w_t$	0.050*	0.050*	0.052	0.089	0.090
	$y_t = 0.3y_{t-1} + w_t$	0.143	0.167	0.137	0.111*	0.114
	$y_t = 0.6y_{t-1} + w_t$	0.319	0.418	0.306	0.172*	0.172*
	$y_t = 0.9y_{t-1} + w_t$	0.634	0.818	0.616	0.296*	0.297
	$y_t = 0.5y_{t-1} + 0.05y_{t-1}^2 + w_t$	0.262	0.338	0.246	0.149	0.144*
	$y_t = 0.9y_{t-1} + 0.5w_{t-1}y_{t-1} + w_t$	0.502	0.671	0.479	0.240	0.230*
	$y_t = \sin(y_{t-1}) + w_t$	0.273	0.345	0.258	0.154*	0.154*
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + w_t$	0.334	0.373	0.312	0.327*	0.332

*Lowest type I error rate

Table 4 Type II error rates of MK, MKRD, and MKC3 for each time series model

		Type II error rates				
n	Time series model	MK	TFPW	SMK	MKRD	MKC3
30	$y_t = \frac{0.25\sigma}{n}t + w_t$	0.933	0.932	0.954	0.879	0.873*
	$y_t = \frac{0.5\sigma}{n}t + w_t$	0.889	0.893	0.930	0.828	0.821*
	$y_t = \frac{0.75\sigma}{n}t + w_t$	0.801	0.811	0.881	0.741	0.737*
	$y_t = \frac{\sigma}{n}t + w_t$	0.693	0.701	0.810	0.631*	0.634
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + \frac{0.5\sigma}{n}t + w_t$	0.917	0.869*	0.948	0.900	0.898
	$y_t = 0.9y_{t-1} + \frac{0.5\sigma}{n}t + w_t$	0.466	0.326*	0.566	0.630	0.629
50	$y_t = \frac{0.25\sigma}{n}t + w_t$	0.922	0.921	0.949	0.866	0.860*
	$y_t = \frac{0.5\sigma}{n}t + w_t$	0.837	0.842	0.894	0.773	0.770*
	$y_t = \frac{0.75\sigma}{n}t + w_t$	0.698	0.707	0.800	0.629	0.626*
	$y_t = \frac{\sigma}{n}t + w_t$	0.518	0.535	0.656	0.456	0.452*
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + \frac{0.5\sigma}{n}t + w_t$	0.283	0.238*	0.430	0.313	0.314
	$y_t = 0.9y_{t-1} + \frac{0.5\sigma}{n}t + w_t$	0.412	0.250*	0.489	0.638	0.637
100	$y_t = \frac{0.25\sigma}{n}t + w_t$	0.896	0.897	0.907	0.838	0.836*
	$y_t = \frac{0.5\sigma}{n}t + w_t$	0.714	0.722	0.752	0.648	0.647*
	$y_t = \frac{0.75\sigma}{n}t + w_t$	0.458	0.470	0.524	0.403	0.400*
	$y_t = \frac{\sigma}{n}t + w_t$	0.204	0.211	0.268	0.173*	0.174
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + \frac{0.5\sigma}{n}t + w_t$	0.840	0.815*	0.858	0.919	0.917
	$y_t = 0.9y_{t-1} + \frac{0.5\sigma}{n}t + w_t$	0.378	0.197*	0.402	0.664	0.664
200	$y_t = \frac{0.25\sigma}{n}t + w_t$	0.834	0.834	0.844	0.769*	0.769*
	$y_t = \frac{0.5\sigma}{n}t + w_t$	0.493	0.496	0.526	0.433	0.430*
	$y_t = \frac{0.75\sigma}{n}t + w_t$	0.157	0.163	0.191	0.135*	0.136
	$y_t = \frac{\sigma}{n}t + w_t$	0.022	0.024	0.034	0.020	0.019*
	$y_t = \sin\left(\frac{2\pi t}{50}\right) + \frac{0.5\sigma}{n}t + w_t$	0.985	0.974	0.984	0.951	0.949*
	$y_t = 0.9y_{t-1} + \frac{0.5\sigma}{n}t + w_t$	0.362	0.181*	0.372	0.688	0.688

*Lowest type II error rate

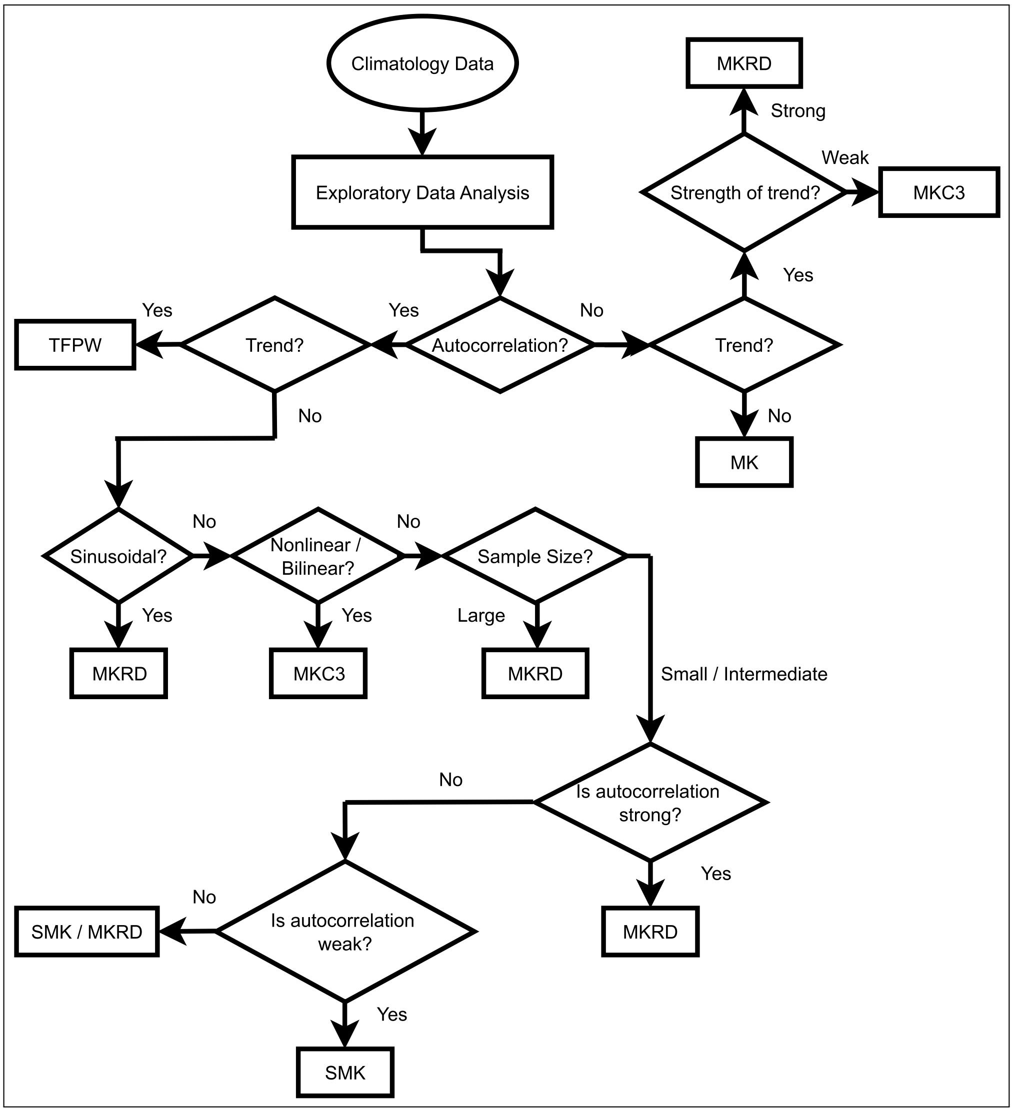

Fig. 2 Flowchart presenting practical guidelines for selecting between MK and various modified MK tests based on the characteristics of time series models

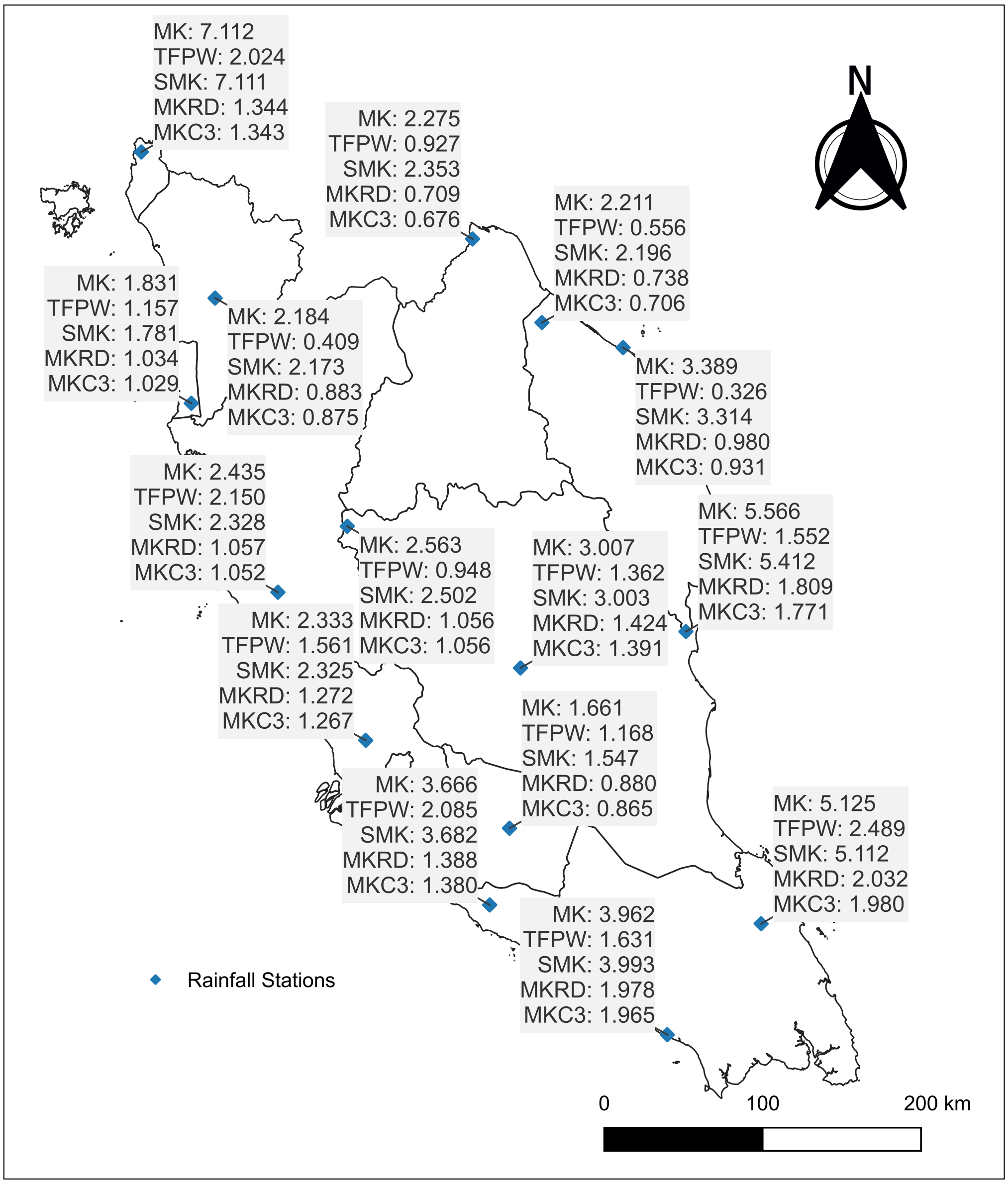

Fig. 3 MK Z-scores of daily rainfall for the full study period. Map generated using QGIS Desktop version 3.44.1 (?).

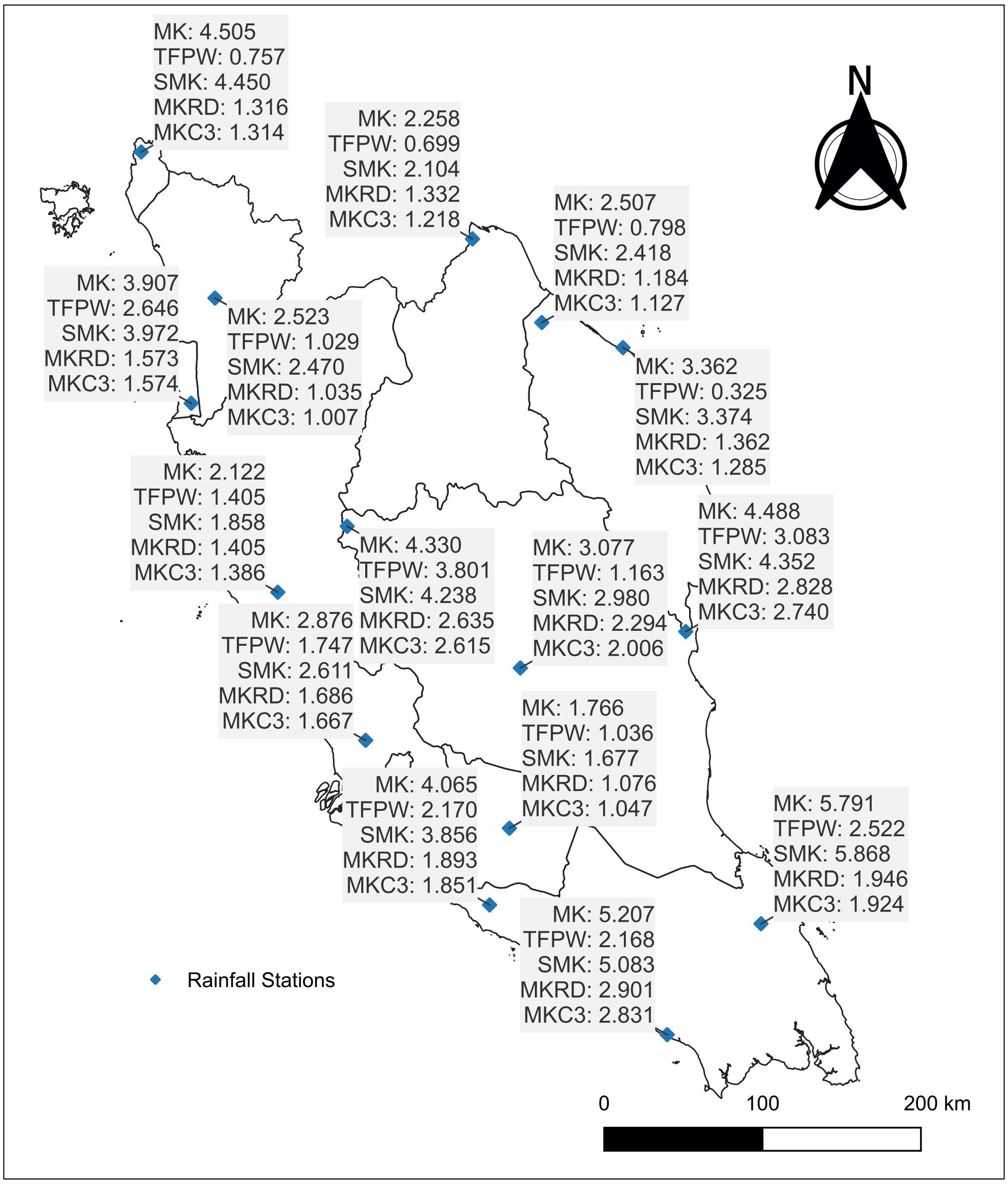

Fig. 4 MK Z-scores of daily rainfall for the NEM period. Map generated using QGIS Desktop version 3.44.1 (?).

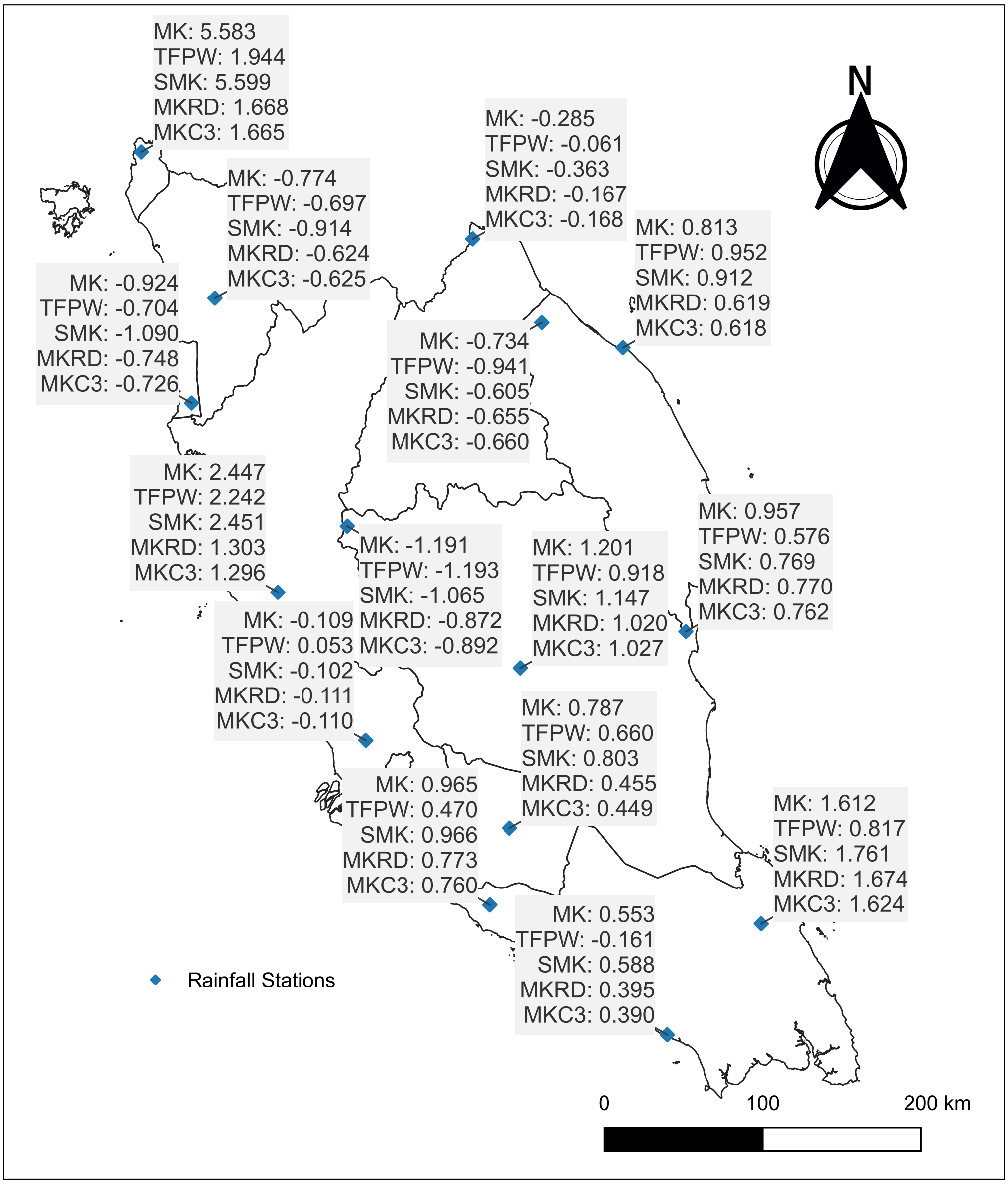

Fig. 5 MK Z-scores of daily rainfall for the SWM period. Map generated using QGIS Desktop version 3.44.1 (?).

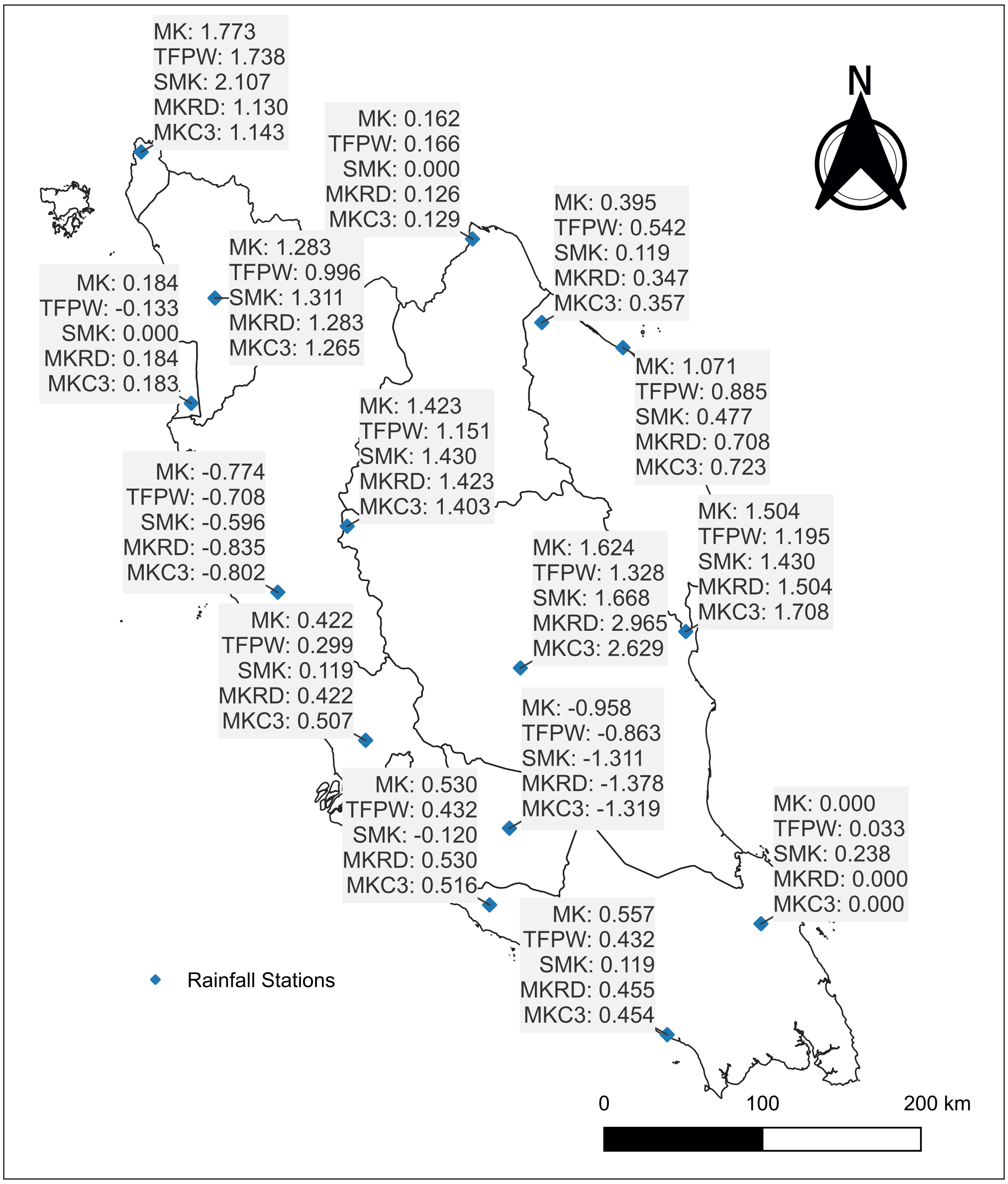

Fig. 6 MK Z-scores of Rx1day for the full study period. Map generated using QGIS Desktop version 3.44.1 (?).

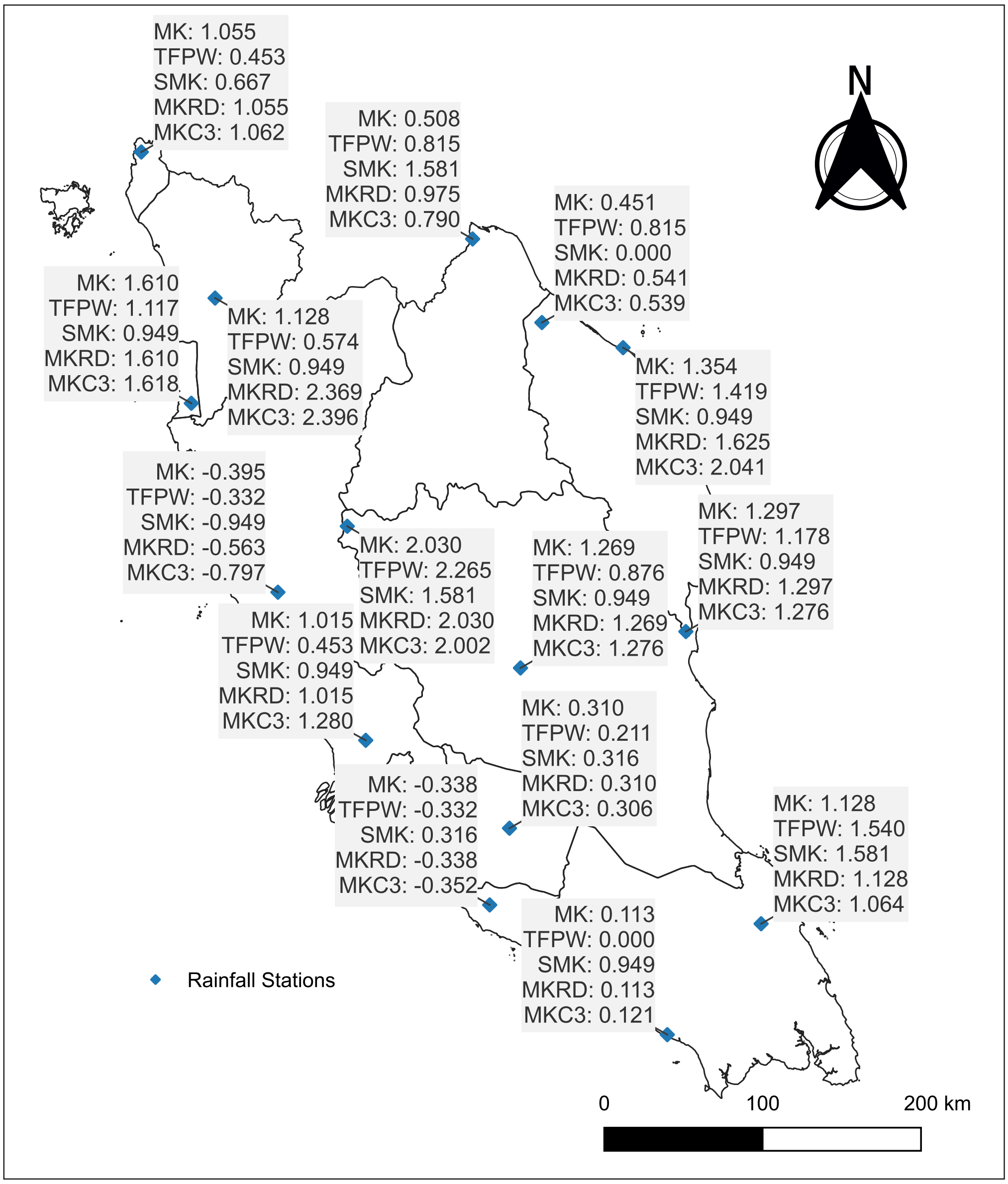

Fig. 7 MK Z-scores of Rx1day for the NEM period. Map generated using QGIS Desktop version 3.44.1 (**?**).

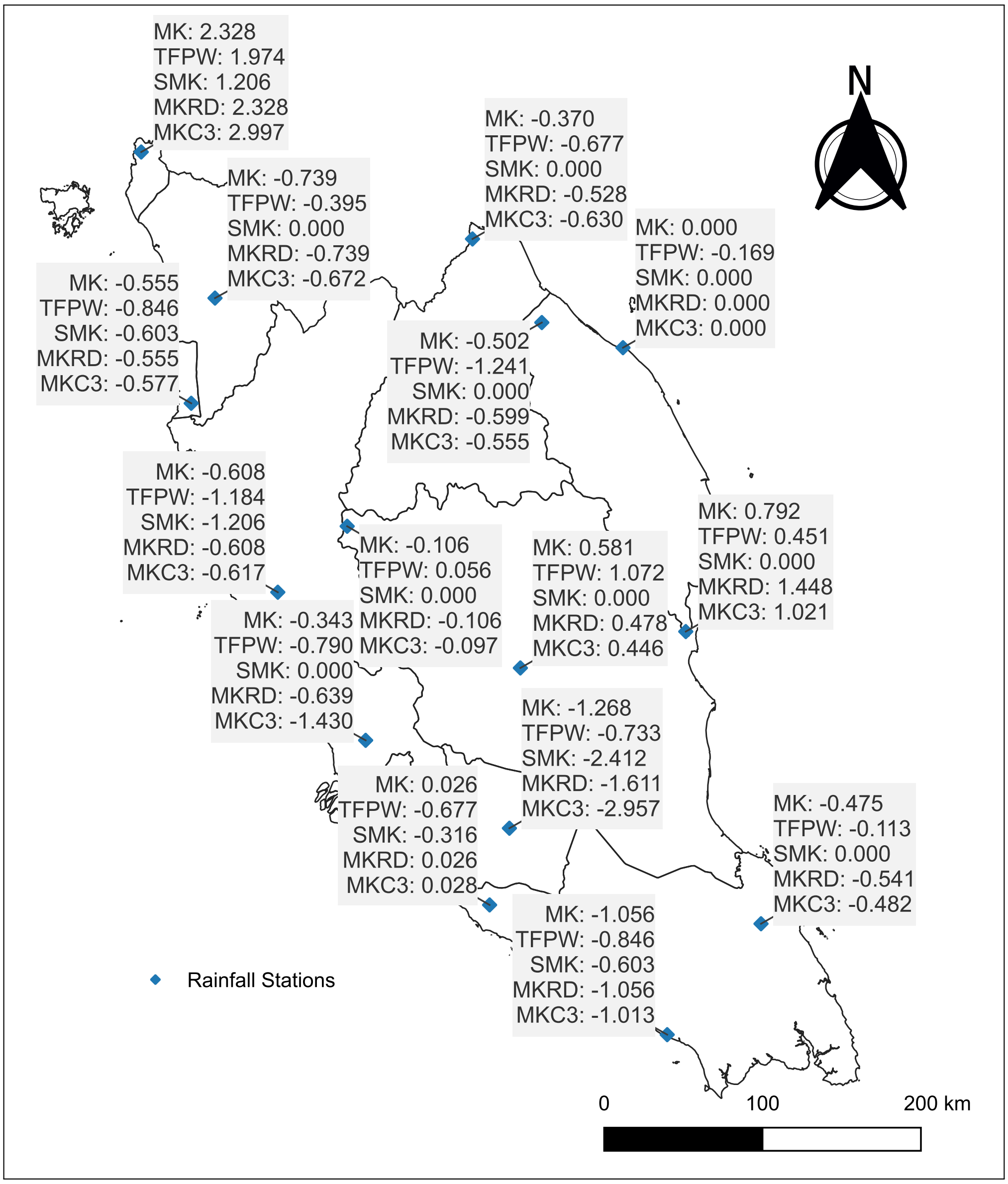

Fig. 8 MK Z-scores of Rx1day for the SWM period. Map generated using QGIS Desktop version 3.44.1 (**?**).

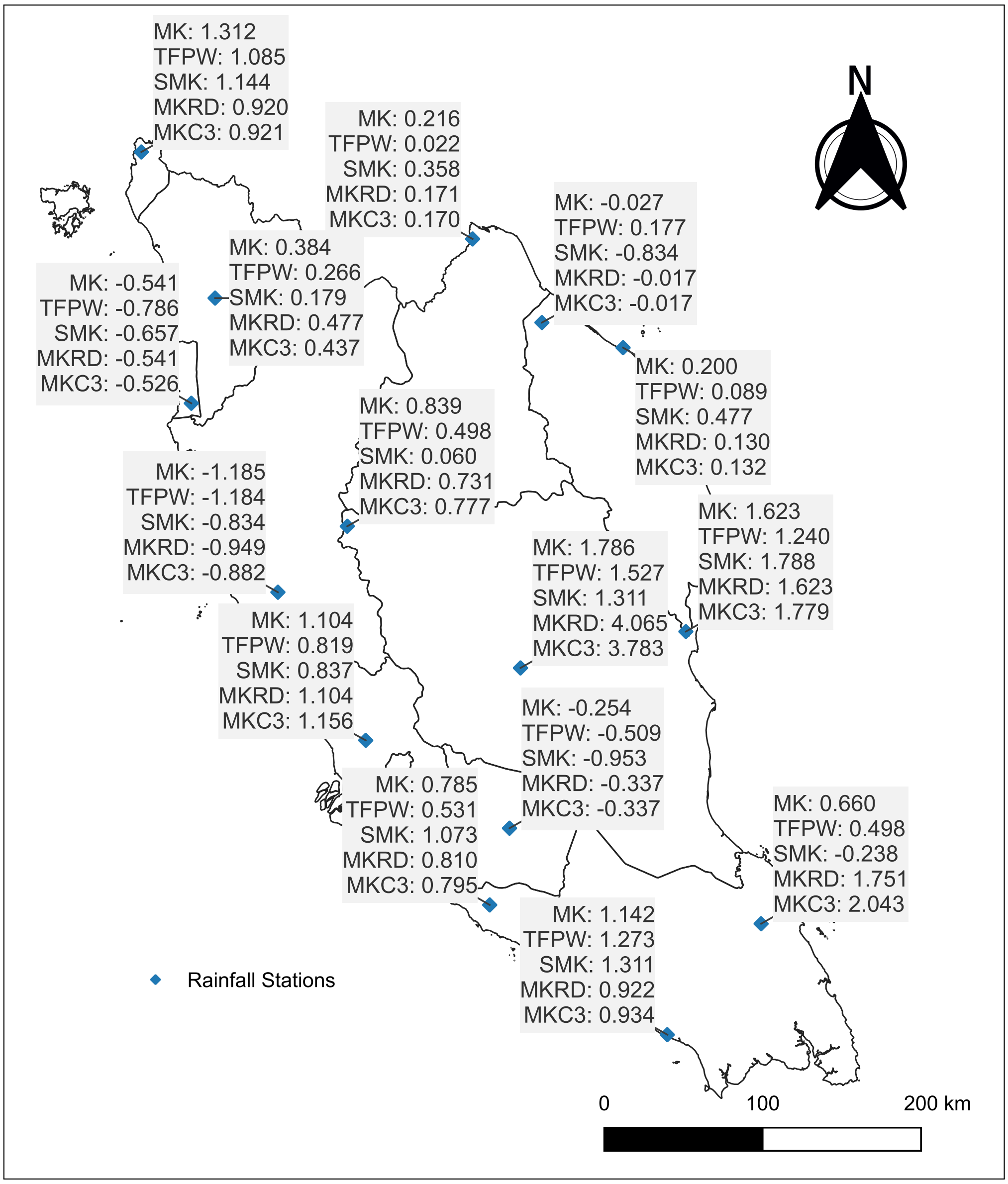

Fig. 9 MK Z-scores of Rx5day for the full study period. Map generated using QGIS Desktop version 3.44.1 (**?**).

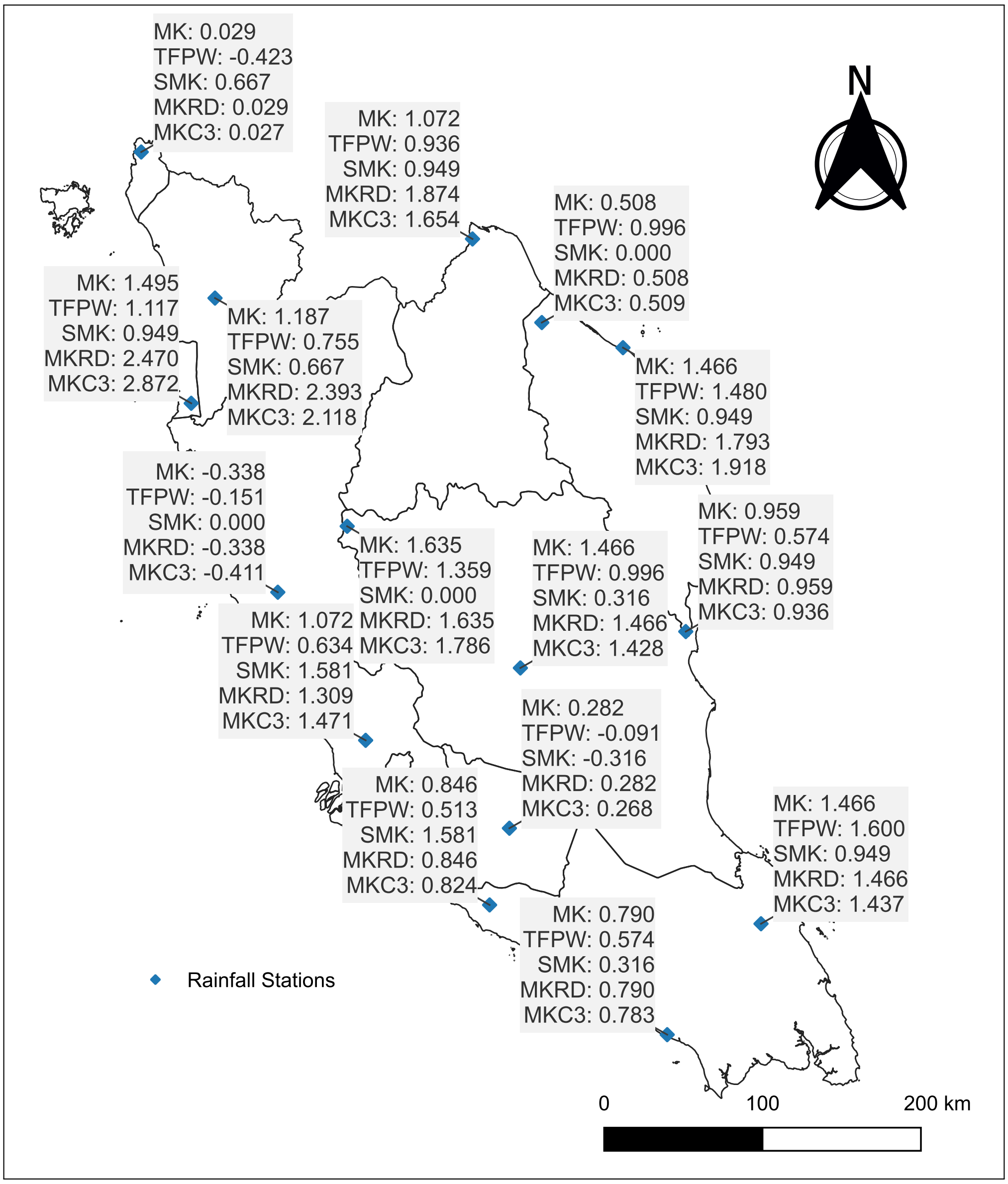

Fig. 10 MK Z-scores of Rx5day for the NEM period. Map generated using QGIS Desktop version 3.44.1 (?).

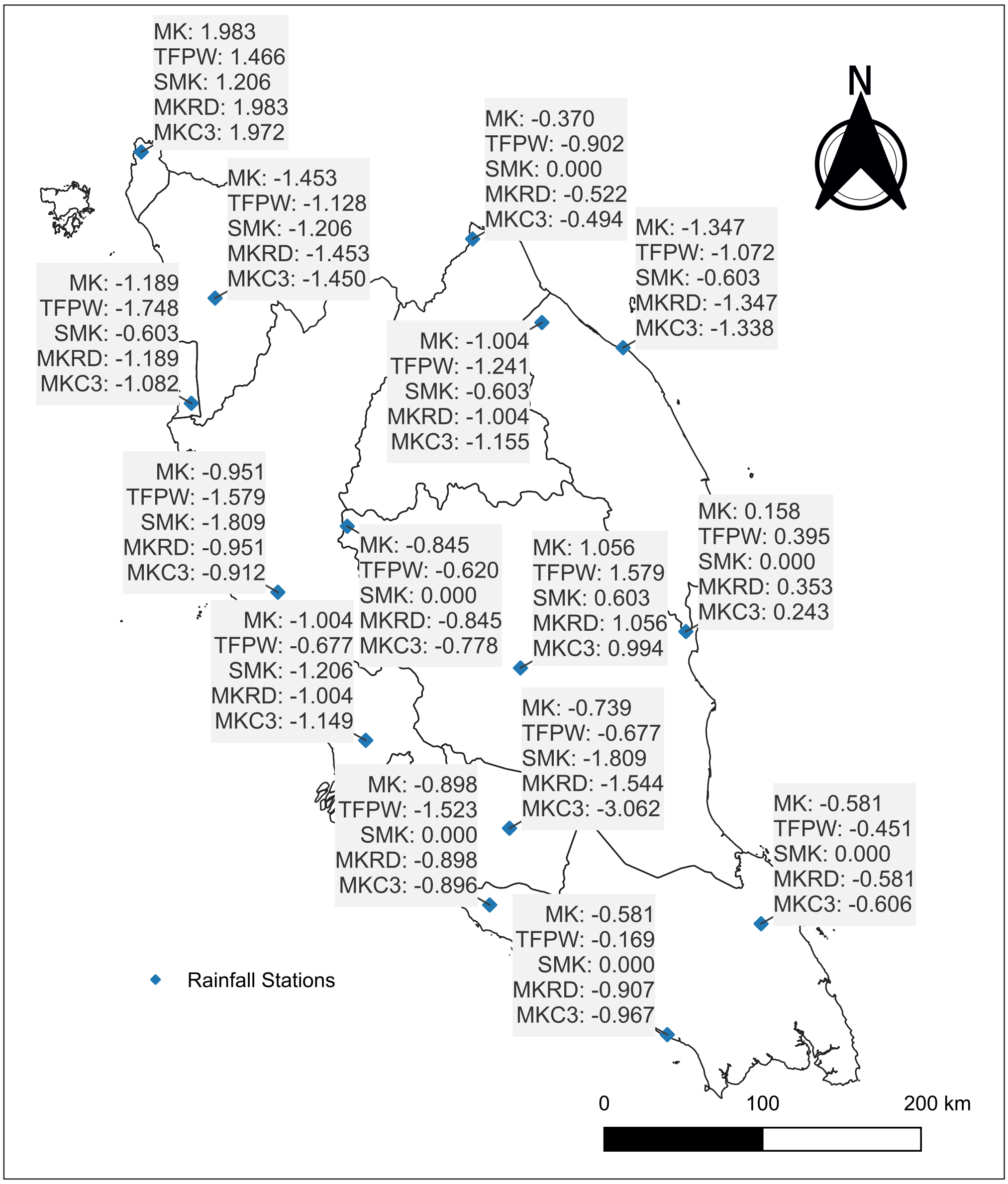

Fig. 11 MK Z-scores of Rx5day for the SWM period. Map generated using QGIS Desktop version 3.44.1 (**?**).

Table 5 Trend analysis of daily rainfall by MK, TFPW, SMK, MKRD and MKC3

		MK		TFPW		SMK		MKRD		MKC3	
Season	Station	Z	p	Z_{TF}	p	Z_{S}	p	Z^{*}	p	Z'	p
Full	1632096	3.962*	0.000	1.631	0.103	3.993*	0.000	1.978*	0.048	1.965*	0.049
	2237164	3.666*	0.000	2.085*	0.037	3.682*	0.000	1.388	0.165	1.380	0.168
	2723075	1.661	0.097	1.168	0.243	1.547	0.122	0.880	0.379	0.865	0.387
	3022001	3.007*	0.003	1.362	0.173	3.003*	0.003	1.424	0.155	1.391	0.164
	3214057	2.435*	0.015	2.150*	0.032	2.328*	0.020	1.057	0.291	1.052	0.293
	3723077	2.563*	0.010	0.948	0.343	2.502*	0.012	1.056	0.291	1.056	0.291
	3933001	2.211*	0.027	0.556	0.578	2.196*	0.028	0.738	0.460	0.706	0.480
	4109095	2.275*	0.023	0.927	0.354	2.353*	0.019	0.709	0.479	0.676	0.499
	4313033	7.112*	0.000	2.024*	0.043	7.111*	0.000	1.344	0.179	1.343	0.179
	5205052	1.831	0.067	1.157	0.247	1.781	0.075	1.034	0.301	1.029	0.304
	5529027	3.389*	0.001	0.326	0.744	3.314*	0.001	0.980	0.327	0.931	0.352
	5624001	2.333*	0.020	1.561	0.118	2.325*	0.020	1.272	0.204	1.267	0.205
	5806066	2.184*	0.029	0.409	0.682	2.173*	0.030	0.883	0.377	0.875	0.382
	6120014	5.566*	0.000	1.552	0.121	5.412*	0.000	1.809	0.070	1.771	0.076
	6602002	5.125*	0.000	2.489*	0.013	5.112*	0.000	2.032*	0.042	1.980*	0.048
NEM	1632096	5.207*	0.000	2.168*	0.030	5.083*	0.000	2.901*	0.004	2.831*	0.005
	2237164	4.065*	0.000	2.170*	0.030	3.856*	0.000	1.893	0.058	1.851	0.064
	2723075	1.766	0.077	1.036	0.300	1.677	0.094	1.076	0.282	1.047	0.295
	3022001	3.077*	0.002	1.163	0.245	2.980*	0.003	2.294*	0.022	2.006*	0.045
	3214057	2.122*	0.034	1.405	0.160	1.858	0.063	1.405	0.160	1.386	0.166
	3723077	4.330*	0.000	3.801*	0.000	4.238*	0.000	2.635*	0.008	2.615*	0.009
	3933001	2.507*	0.012	0.798	0.425	2.418*	0.016	1.184	0.236	1.127	0.260
	4109095	2.258*	0.024	0.699	0.485	2.104*	0.035	1.332	0.183	1.218	0.223
	4313033	4.505*	0.000	0.757	0.449	4.450*	0.000	1.316	0.188	1.314	0.189
	5205052	3.907*	0.000	2.646*	0.008	3.972*	0.000	1.573	0.116	1.574	0.115
	5529027	3.362*	0.001	0.325	0.745	3.374*	0.001	1.362	0.173	1.285	0.199
	5624001	2.876*	0.004	1.747	0.081	2.611*	0.009	1.686	0.092	1.667	0.095
	5806066	2.523*	0.012	1.029	0.303	2.470*	0.014	1.035	0.301	1.007	0.314
	6120014	4.488*	0.000	3.083*	0.002	4.352*	0.000	2.828*	0.005	2.740*	0.006
	6602002	5.791*	0.000	2.522*	0.012	5.868*	0.000	1.946	0.052	1.924	0.054
SWM	1632096	0.553	0.580	-0.161	0.872	0.588	0.557	0.395	0.693	0.390	0.696
	2237164	0.965	0.334	0.470	0.638	0.966	0.334	0.773	0.440	0.760	0.447
	2723075	0.787	0.431	0.660	0.509	0.803	0.422	0.455	0.649	0.449	0.653
	3022001	1.201	0.230	0.918	0.359	1.147	0.251	1.020	0.308	1.027	0.305
	3214057	2.447*	0.014	2.242*	0.025	2.451*	0.014	1.303	0.192	1.296	0.195
	3723077	-1.191	0.233	-1.193	0.233	-1.065	0.287	-0.872	0.383	-0.892	0.372
	3933001	-0.734	0.463	-0.941	0.347	-0.605	0.545	-0.655	0.513	-0.660	0.509
	4109095	-0.285	0.776	-0.061	0.951	-0.363	0.716	-0.167	0.867	-0.168	0.867
	4313033	5.583*	0.000	1.944	0.052	5.599*	0.000	1.668	0.095	1.665	0.096
	5205052	-0.924	0.356	-0.704	0.481	-1.090	0.276	-0.748	0.454	-0.726	0.468
	5529027	0.813	0.416	0.952	0.341	0.912	0.362	0.619	0.536	0.618	0.537
	5624001	-0.109	0.913	0.053	0.958	-0.102	0.919	-0.111	0.912	-0.110	0.912
	5806066	-0.774	0.439	-0.697	0.486	-0.914	0.361	-0.624	0.532	-0.625	0.532
	6120014	0.957	0.339	0.576	0.564	0.769	0.442	0.770	0.441	0.762	0.446
	6602002	1.612	0.107	0.817	0.414	1.761	0.078	1.674	0.094	1.624	0.104

*Statistically significant increasing trend

**Statistically significant decreasing trend

Table 6 Trend analysis of Rx1day by MK, TFPW, SMK, MKRD and MKC3

		MK		TFPW		SMK		MKRD		MKC3	
Season	Station	Z	p	Z_{TF}	p	Z_{S}	p	Z^{*}	p	Z'	p
Full	1632096	0.557	0.577	0.432	0.666	0.119	0.905	0.455	0.649	0.454	0.650
	2237164	0.530	0.596	0.432	0.666	-0.120	0.905	0.530	0.596	0.516	0.606
	2723075	-0.958	0.338	-0.863	0.388	-1.311	0.190	-1.378	0.168	-1.319	0.187
	3022001	1.624	0.104	1.328	0.184	1.668	0.095	2.965*	0.003	2.629*	0.009
	3214057	-0.774	0.439	-0.708	0.479	-0.596	0.551	-0.835	0.404	-0.802	0.423
	3723077	1.423	0.155	1.151	0.250	1.430	0.153	1.423	0.155	1.403	0.161
	3933001	0.395	0.693	0.542	0.588	0.119	0.905	0.347	0.728	0.357	0.721
	4109095	0.162	0.871	0.166	0.868	0.000	1.000	0.126	0.899	0.129	0.897
	4313033	1.773	0.076	1.738	0.082	2.107*	0.035	1.130	0.259	1.143	0.253
	5205052	0.184	0.854	-0.133	0.894	0.000	1.000	0.184	0.854	0.183	0.854
	5529027	1.071	0.284	0.885	0.376	0.477	0.634	0.708	0.479	0.723	0.470
	5624001	0.422	0.673	0.299	0.765	0.119	0.905	0.422	0.673	0.507	0.612
	5806066	1.283	0.200	0.996	0.319	1.311	0.190	1.283	0.200	1.265	0.206
	6120014	1.504	0.132	1.195	0.232	1.430	0.153	1.504	0.132	1.708	0.088
	6602002	0.000	1.000	0.033	0.974	0.238	0.812	0.000	1.000	0.000	1.000
NEM	1632096	0.113	0.910	0.000	1.000	0.949	0.343	0.113	0.910	0.121	0.904
	2237164	-0.338	0.735	-0.332	0.740	0.316	0.752	-0.338	0.735	-0.352	0.725
	2723075	0.310	0.756	0.211	0.833	0.316	0.752	0.310	0.756	0.306	0.759
	3022001	1.269	0.204	0.876	0.381	0.949	0.343	1.269	0.204	1.276	0.202
	3214057	-0.395	0.693	-0.332	0.740	-0.949	0.343	-0.563	0.574	-0.797	0.425
	3723077	2.030*	0.042	2.265*	0.024	1.581	0.114	2.030*	0.042	2.002*	0.045
	3933001	0.451	0.652	0.815	0.415	0.000	1.000	0.541	0.588	0.539	0.590
	4109095	0.508	0.612	0.815	0.415	1.581	0.114	0.975	0.330	0.790	0.429
	4313033	1.055	0.291	0.453	0.651	0.667	0.505	1.055	0.291	1.062	0.288
	5205052	1.610	0.107	1.117	0.264	0.949	0.343	1.610	0.107	1.618	0.106
	5529027	1.354	0.176	1.419	0.156	0.949	0.343	1.625	0.104	2.041*	0.041
	5624001	1.015	0.310	0.453	0.651	0.949	0.343	1.015	0.310	1.280	0.201
	5806066	1.128	0.259	0.574	0.566	0.949	0.343	2.369*	0.018	2.396*	0.017
	6120014	1.297	0.195	1.178	0.239	0.949	0.343	1.297	0.195	1.276	0.202
	6602002	1.128	0.259	1.540	0.124	1.581	0.114	1.128	0.259	1.064	0.287
SWM	1632096	-1.056	0.291	-0.846	0.398	-0.603	0.546	-1.056	0.291	-1.013	0.311
	2237164	0.026	0.979	-0.677	0.499	-0.316	0.752	0.026	0.979	0.028	0.978
	2723075	-1.268	0.205	-0.733	0.463	-2.412**	0.016	-1.611	0.107	-2.957**	0.003
	3022001	0.581	0.561	1.072	0.284	0.000	1.000	0.478	0.633	0.446	0.656
	3214057	-0.608	0.543	-1.184	0.236	-1.206	0.228	-0.608	0.543	-0.617	0.537
	3723077	-0.106	0.916	0.056	0.955	0.000	1.000	-0.106	0.916	-0.097	0.923
	3933001	-0.502	0.616	-1.241	0.215	0.000	1.000	-0.599	0.549	-0.555	0.579
	4109095	-0.370	0.712	-0.677	0.499	0.000	1.000	-0.528	0.597	-0.630	0.529
	4313033	2.328*	0.020	1.974*	0.048	1.206	0.228	2.328*	0.020	2.997*	0.003
	5205052	-0.555	0.579	-0.846	0.398	-0.603	0.546	-0.555	0.579	-0.577	0.564
	5529027	0.000	1.000	-0.169	0.866	0.000	1.000	0.000	1.000	0.000	1.000
	5624001	-0.343	0.731	-0.790	0.430	0.000	1.000	-0.639	0.523	-1.430	0.153
	5806066	-0.739	0.460	-0.395	0.693	0.000	1.000	-0.739	0.460	-0.672	0.501
	6120014	0.792	0.428	0.451	0.652	0.000	1.000	1.448	0.148	1.021	0.307
	6602002	-0.475	0.635	-0.113	0.910	0.000	1.000	-0.541	0.589	-0.482	0.630

*Statistically significant increasing trend

**Statistically significant decreasing trend

Table 7 Trend analysis of Rx5day by MK, TFPW, SMK, MKRD and MKC3

		MK		TFPW		SMK		MKRD		MKC3	
Season	Station	Z	p	Z_{TF}	p	Z_{S}	p	Z^{*}	p	Z'	p
Full	1632096	1.142	0.254	1.273	0.203	1.311	0.190	0.922	0.357	0.934	0.350
	2237164	0.785	0.433	0.531	0.595	1.073	0.283	0.810	0.418	0.795	0.427
	2723075	-0.254	0.799	-0.509	0.611	-0.953	0.340	-0.337	0.736	-0.337	0.736
	3022001	1.786	0.074	1.527	0.127	1.311	0.190	4.065*	0.000	3.783*	0.000
	3214057	-1.185	0.236	-1.184	0.236	-0.834	0.404	-0.949	0.343	-0.882	0.378
	3723077	0.839	0.402	0.498	0.618	0.060	0.952	0.731	0.465	0.777	0.437
	3933001	-0.027	0.978	0.177	0.859	-0.834	0.404	-0.017	0.986	-0.017	0.986
	4109095	0.216	0.829	0.022	0.982	0.358	0.721	0.171	0.865	0.170	0.865
	4313033	1.312	0.190	1.085	0.278	1.144	0.253	0.920	0.358	0.921	0.357
	5205052	-0.541	0.588	-0.786	0.432	-0.657	0.511	-0.541	0.588	-0.526	0.599
	5529027	0.200	0.841	0.089	0.929	0.477	0.634	0.130	0.897	0.132	0.895
	5624001	1.104	0.270	0.819	0.413	0.837	0.403	1.104	0.270	1.156	0.248
	5806066	0.384	0.701	0.266	0.791	0.179	0.858	0.477	0.633	0.437	0.662
	6120014	1.623	0.104	1.240	0.215	1.788	0.074	1.623	0.104	1.779	0.075
	6602002	0.660	0.509	0.498	0.618	-0.238	0.812	1.751	0.080	2.043*	0.041
NEM	1632096	0.790	0.430	0.574	0.566	0.316	0.752	0.790	0.430	0.783	0.434
	2237164	0.846	0.398	0.513	0.608	1.581	0.114	0.846	0.398	0.824	0.410
	2723075	0.282	0.778	-0.091	0.928	-0.316	0.752	0.282	0.778	0.268	0.789
	3022001	1.466	0.143	0.996	0.319	0.316	0.752	1.466	0.143	1.428	0.153
	3214057	-0.338	0.735	-0.151	0.880	0.000	1.000	-0.338	0.735	-0.411	0.681
	3723077	1.635	0.102	1.359	0.174	0.000	1.000	1.635	0.102	1.786	0.074
	3933001	0.508	0.612	0.996	0.319	0.000	1.000	0.508	0.612	0.509	0.611
	4109095	1.072	0.284	0.936	0.349	0.949	0.343	1.874	0.061	1.654	0.098
	4313033	0.029	0.977	-0.423	0.672	0.667	0.505	0.029	0.977	0.027	0.979
	5205052	1.495	0.135	1.117	0.264	0.949	0.343	2.470*	0.014	2.872*	0.004
	5529027	1.466	0.143	1.480	0.139	0.949	0.343	1.793	0.073	1.918	0.055
	5624001	1.072	0.284	0.634	0.526	1.581	0.114	1.309	0.191	1.471	0.141
	5806066	1.187	0.235	0.755	0.450	0.667	0.505	2.393*	0.017	2.118*	0.034
	6120014	0.959	0.338	0.574	0.566	0.949	0.343	0.959	0.338	0.936	0.349
	6602002	1.466	0.143	1.600	0.110	0.949	0.343	1.466	0.143	1.437	0.151
SWM	1632096	-0.581	0.561	-0.169	0.866	0.000	1.000	-0.907	0.365	-0.967	0.334
	2237164	-0.898	0.369	-1.523	0.128	0.000	1.000	-0.898	0.369	-0.896	0.370
	2723075	-0.739	0.460	-0.677	0.499	-1.809	0.070	-1.544	0.123	-3.062**	0.002
	3022001	1.056	0.291	1.579	0.114	0.603	0.546	1.056	0.291	0.994	0.320
	3214057	-0.951	0.342	-1.579	0.114	-1.809	0.070	-0.951	0.342	-0.912	0.362
	3723077	-0.845	0.398	-0.620	0.535	0.000	1.000	-0.845	0.398	-0.778	0.437
	3933001	-1.004	0.316	-1.241	0.215	-0.603	0.546	-1.004	0.316	-1.155	0.248
	4109095	-0.370	0.712	-0.902	0.367	0.000	1.000	-0.522	0.601	-0.494	0.621
	4313033	1.983*	0.047	1.466	0.143	1.206	0.228	1.983*	0.047	1.972*	0.049
	5205052	-1.189	0.234	-1.748	0.080	-0.603	0.546	-1.189	0.234	-1.082	0.279
	5529027	-1.347	0.178	-1.072	0.284	-0.603	0.546	-1.347	0.178	-1.338	0.181
	5624001	-1.004	0.316	-0.677	0.499	-1.206	0.228	-1.004	0.316	-1.149	0.251
	5806066	-1.453	0.146	-1.128	0.259	-1.206	0.228	-1.453	0.146	-1.450	0.147
	6120014	0.158	0.874	0.395	0.693	0.000	1.000	0.353	0.724	0.243	0.808
	6602002	-0.581	0.561	-0.451	0.652	0.000	1.000	-0.581	0.561	-0.606	0.544

*Statistically significant increasing trend

**Statistically significant decreasing trend

Table 8 Number of significant and insignificant increasing and decreasing trends for daily rainfall based on MK, TFPW, SMK, MKRD, and MKC3.

Season	Method	Inc. (sig.)	Dec. (sig.)	Inc. (insig.)	Dec. (insig.)	No trend
Full	MK	13	0	2	0	0
	TFPW	4	0	11	0	0
	SMK	13	0	2	0	0
	MKRD	2	0	13	0	0
	MKC3	2	0	13	0	0
NEM	MK	14	0	1	0	0
	TFPW	6	0	9	0	0
	SMK	13	0	2	0	0
	MKRD	4	0	11	0	0
	MKC3	4	0	11	0	0
SWM	MK	2	0	7	6	0
	TFPW	1	0	8	6	0
	SMK	2	0	7	6	0
	MKRD	0	0	9	6	0
	MKC3	0	0	9	6	0

Table 9 Number of significant and insignificant increasing and decreasing trends for Rx1day based on MK, TFPW, SMK, MKRD, and MKC3.

Season	Method	Inc. (sig.)	Dec. (sig.)	Inc. (insig.)	Dec. (insig.)	No trend
Full	MK	0	0	12	2	1
	TFPW	0	0	12	3	0
	SMK	1	0	9	3	2
	MKRD	1	0	11	2	1
	MKC3	1	0	11	2	1
NEM	MK	1	0	13	2	0
	TFPW	1	0	11	2	1
	SMK	0	0	13	1	1
	MKRD	2	0	11	2	0
	MKC3	3	0	10	2	0
SWM	MK	1	0	3	10	1
	TFPW	1	0	3	11	0
	SMK	0	1	1	4	9
	MKRD	1	0	3	10	1
	MKC3	1	1	3	9	1

Table 10 Number of significant and insignificant increasing and decreasing trends for Rx5day based on MK, TFPW, SMK, MKRD, and MKC3.

Season	Method	Inc. (sig.)	Dec. (sig.)	Inc. (insig.)	Dec. (insig.)	No trend
Full	MK	0	0	11	4	0
	TFPW	0	0	12	3	0
	SMK	0	0	10	5	0
	MKRD	1	0	10	4	0
	MKC3	2	0	9	4	0
NEM	MK	0	0	14	1	0
	TFPW	0	0	12	3	0
	SMK	0	0	11	1	3
	MKRD	2	0	12	1	0
	MKC3	2	0	12	1	0
SWM	MK	1	0	2	12	0
	TFPW	0	0	3	12	0
	SMK	0	0	2	7	6
	MKRD	1	0	2	12	0
	MKC3	1	1	2	11	0

5 Conclusion

This study examined the performance of MKC3 in terms of type I and type II error rates in comparison with MK, TFPW, SMK, and MKRD. A case study was conducted to analyze trends in daily rainfall, Rx1day, and Rx5day data from 15 stations across Peninsular Malaysia.

Overall, the simulation study revealed that no single MK modification performs best in all situations. The original MK remains reliable for large samples and white noise data. TFPW performs moderately in reducing type I errors but is robust in handling type II errors when trends and dependencies coexist. SMK effectively reduces type I errors for small samples and moderately autocorrelated data but tends to lose power and overcorrect in small sample sizes. MKRD is robust for strongly autocorrelated and sinusoidal data, while MKC3 performs well in bilinear and nonlinear models, especially in detecting weaker trends.

The trend analysis of daily rainfall, Rx1day, and Rx5day revealed different outcomes for MK, TFPW, SMK, MKRD, and MKC3. However, rainfall during the NEM generally shows an increasing pattern, while the SWM exhibits the opposite trend, particularly in the northern and east coast regions. It is also worth noting that the NEM season shows several insignificant increasing trends for Rx1day and Rx5day. Although these trends are not statistically significant across the various MK tests, they should still be given attention by the authorities, similar to the declining trends observed during the SWM season. This may lead to a rainfall discrepancy that warrants consideration by the relevant agencies.

Moreover, the Z-score statistics of Rx1day and Rx5day indicate that SMK is not suitable for small sample sizes due to insufficient information on the number of ties between seasonal pairs, resulting in a high occurrence of zero Z-score values. This finding is in line with the simulation results of SMK, which show an overcorrection effect in small sample size.

A limitation of this study is that MKC3 represents a relatively recent modification that introduces a higher-order term in the variance correction of the MK test. To the best of the authors' knowledge, variance correction in existing MK modifications has thus far been confined to second-order statistics, primarily correlation, as demonstrated in the works of **?** and **?**. The findings of this study are expected to promote further exploration and refinement of higher-order statistical integration in MK modifications within hydro-climatic applications.

Future research could also be extended to incorporate comprehensive EDA procedures, including but not limited to the TSE, ACF, and bispectrum analysis, prior to determining the most appropriate MK variant for trend detection. Ideally, such an approach should be demonstrated in the case study, which is left for future work, as the present study primarily focuses on the comparative performance of the various MK variants.

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Competing interests

No conflict interest

Ethics approval and consent to participate

Not applicable

Consent for publication

Not applicable

Data availability

Yes, the datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

Materials availability

Not applicable

Author contribution

Yick Jing Then: Conceptualization, Methodology, Software, Formal Analysis, Writing - Original Draft.
Syafrina Abdul Halim: Supervision, Validation, Resources, Writing - Review Editing, Funding Acquisition.

Appendix A Details of Simulated Data

The details of the simulated data used to evaluate the type I and type II error rates of the MK, MKRD, and the proposed MKC3 are provided in the following subsections.

A.1 White Noise

$$y_t = w_t \tag{A1}$$

where w_t is normally distributed with mean 0 and variance 1.

A.2 Autoregressive Model AR(1)

$$y_t = \phi y_{t-1} + w_t, \quad \phi = 0.3,\ 0.6,\ 0.9 \tag{A2}$$

A.3 Nonlinear Autoregressive

$$y_t = 0.5y_{t-1} + 0.05y_{t-1}^2 + w_t \tag{A3}$$

A.4 Bilinear Model

$$y_t = 0.9y_{t-1} + 0.5w_{t-1}y_{t-1} + w_t \tag{A4}$$

A.5 Sinusoidal Model

$$y_t = \sin\left(\frac{2\pi t}{50}\right) + w_t \tag{A5}$$

A.6 Chaotic

$$y_t = \sin\left(y_{t-1}\right) + w_t \tag{A6}$$

A.7 Deterministic Linear Trend

$$y_t = \frac{\beta^*\sigma}{n}t + w_t, \quad \beta^* = 0.25,\ 0.5,\ 0.75,\ 1 \tag{A7}$$

where $\frac{\beta^*\sigma}{n}$ is the standardized linear trend provided by **?**.

A.8 Autoregressive with Trend

$$y_t = 0.9y_{t-1} + \frac{\beta^*\sigma}{n}t + w_t, \quad \beta^* = 0,5 \tag{A8}$$

A.9 Sinusoidal with Trend

$$y_t = \sin\left(\frac{2\pi t}{50}\right) + \frac{\beta^*\sigma}{n}t + w_t, \quad \beta^* = 0.5 \tag{A9}$$