



UNIVERSITI PUTRA MALAYSIA

**NON-LINEAR ANALYSIS OF REINFORCED AND
PRE-STRESSED CONCRETE BEAMS**

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**NON-LINEAR ANALYSIS OF REINFORCED AND PRE-STRESSED
CONCRETE BEAMS**

By

AHMED M. M. HAMED

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in
Fulfillment of the Requirement for the Degree of Master of Science**

December 2002



DEDICATION

*To My family,
I dedicate this work*



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement of the degree of Master of Science

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Faculty: Engineering

Reinforced/pre-stressed (RC/PC) concrete is one of the most commonly used construction materials. This composite material demonstrates a highly non-linear behaviour caused by, cracking, crushing, aggregate interlock, bond slip, dowel action, shrinkage, creep, etc. Because the behaviour of reinforced/pre-stressed concrete involves so many non-linear phenomena interacting with one other, the formulation of rational analytical procedures to describe this behaviour is very difficult.

Since the advent of the computer, powerful methods of analysis such as the finite element method and stiffness matrix method have been implemented to study and develop analytical solutions for the non-linear phenomena. However, the success of such analysis depends on a thorough understanding and modeling of the composite material behaviour.



This study describes the development of a non-linear computer code used to predict the non-linear response of reinforced/pre-stressed concrete beams that are subjected to a combined axial force and bending moment. The study discusses a new simplified approach, whereby the non-linear response is captured via a series of sequentially linear steps. The finite element approach with the stiffness matrix method were used to model the beam element structure and generate its stiffness matrix.

A monotonic empirical model for the concrete's stress-strain curve has been implemented to predict both ascending and descending parts of the curve. An idealized bi-linear elasto-plastic in tension and compression model has been assumed for reinforcing and pre-stressing steel.

Concrete and reinforcing/pre-stressing steel are represented by separate algorithms, which when combined together, describe the behavior of the composite reinforced/pre-stressed concrete material.

The Newton-Raphson concept has been implemented in the iteration technique to satisfy equilibrium and compatibility conditions.

Finally, correlation studies between analytical and experimental results have been carried out with the objective to establish the validity of the proposed model. The verification of the program started by comparing the predictions of the non-linear response for a reinforced concrete beam with that of the results which have been obtained experimentally, and by using the finite element method for the same beam. The comparison was made for the failure load, yielding load and load-deflection

relation. Further more, a 6.0m reinforced concrete beam has been investigated in details as a simply supported and as a fixed supported beam. 2.5m prestressed concrete beam also investigated in details.

The comparison study shows that the predictions from the developed program correlate well with the other results. The investigated beam show that the program results are in complete agreement with the assumptions and models considered to develop the code. The results clearly showed the magnitude and locations of cracking, yielding, ultimate load, failure load and ductility requirements. The program may be used as a tool to do parametric study on the behaviour of reinforced and prestressed concrete structures.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

**ANALISIS TAK LELURUS RASUK KONKRIT TETULANG DAN PRA-
TEGASAN**

Oleh

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Konkrit bertetulang/konkrit prategasan adalah antara bahan pembinaan yang paling lazim digunakan. Bahan rencam ini memperlihatkan ciri tak lurus yang paling tinggi disebabkan retakan, penghancuran, panca agregat, gelincir ikatan, aksi dowel, pengecutan, rayap dan sebagainya. Formulasi prosedur beranalisis yang rasional untuk menghuraikan sifat ini adalah amat sukar disebabkan sifat konkrit bertetulang/konkrit prategasan mungkin melibatkan begitu banyak fenomena tak lurus yang saling bertindak.

Semenjak wujudnya komputer, pelbagai kaedah analisis yang hebat seperti kaedah unsur terhingga dan kaedah matriks kekukuhan telah dilaksanakan untuk mengkaji dan membina penyelesaian beranalisis bagi fenomena tak lurus. Walau bagaimanapun kejayaan analisis sedemikian bergantung pada pemahaman yang mendalam dan pemodelan kelakuan bahan rencam.

Kajian ini memerihalkan pembinaan kod komputer tak lurus yang digunakan untuk meramal respons tak lurus apabila rasuk konkrit bertetulang/konkrit prategasan dikenakan gabungan daya paksian dan momen lentur. Kajian ini membincangkan pendekatan baru yang dipermudahkan, yang melibatkan pemerolehan respons tak lurus melalui satu siri langkah lurus yang berjjukan. Pendekatan kaedah unsur terhingga dengan kaedah matriks kekukuhan telah digunakan untuk memodelkan unsure-unsur struktur rasuk dan menjana matriks kekukuhannya.

Model empirik ekanada bagi lengkung tegasan-tekanan konkrit telah dilaksanakan untuk meramal bahagian menaik dan menurun lengkung tersebut. Model dwilelurus terunggul yang bersifat anjal-plastik dari segi tegangan dan mampatan telah diandaikan untuk penetulang dan pemprategasan besi.

Konkrit dan besi bertetulang/besi prategasan diwakili oleh algoritma yang berbeza, yang apabila digabungkan dapat menerangkan kelakuan bahan konkrit rencam bertetulang/prategasan. Konsep Newton-raphson telah dilaksanakan dalam teknik lelaran untuk menentukan nilai terikan pada gentian atas dan bawah pada keratan rentas.

Akhir sekali, kajian korelasi antara keputusan beranalisis dan eksperimen, telah dijalankan yang bertujuan memastikan kesahan model yang dicadangkan. Pengesahan rancangan bermula dengan membandingkan ramalan respons tak lurus untuk rasuk konkrit bertetulang dengan keputusan daripada eksperimen, dan dengan menggunakan kaedah unsur terhingga bagi rasuk yang sama. Perbandingan telah dibuat berdasarkan

beban kegagalan, beban pengalihan dan hubungan beban-pesongan. Sebagai tambahan sebatang rasuk berukuran 6.0 m telah dikaji secara terperinci sebagai rasuk sokongan mudah dan rasuk sokongan tetap. Kajian perbandingan menunjukkan bahawa ramalan daripada rancangan yang dibina adalah berkorelasi dengan keputusan lain. Kajian terhadap rasuk konkrit berukuran 6.0 m menunjukkan bahawa keputusan komputer bersetuju sepenuhnya dengan anggapan dan model yang telah diambil kira apabila membina kod. Keputusan menunjukkan dengan jelas magnitud dan lokasi retakan, pengalihan, beban muktamad, beban kegagalan, keperluan kemuluran, bahagian melembut dan sebagainya. Komputer kod yang dimajukan boleh digunakan sebagai alat untuk membuat kajian parameter keatas kelakuan struktur konkrit tetulang dan pra-tegasan.

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NOTATIONS

a_1, a_2, a_3 and a_4	Polynomial constants
A_c, B_c, C_c and D_c	Constants to develop concrete stress-strain curve
A_c	Effective cross section area of concrete
A_{ci}	Area of concrete layer
A_{eff}	Effective axial stiffness
A_p	Effective cross section area pre-stressing steel
A_{pi}	Area pre-stressing steel elements in the layer
A_{ps}	Pre-stressing steel
A_s	Effective cross section area of reinforcing steel
A_{sc}	Reinforcing steel in compression zone
A_{si}	Area of reinforcement in i^{th} layer
A_{st}	Reinforcing steel in tension zone
b	Width of section
$\{d\}$	Nodal displacements
d_i	The distance of i^{th} layer of reinforcement from the top, compression, fiber
$\{d\}_i$	Converged deflection for i^{th} load step
$\{d\}^{Jt}$	Deflection in the current iteration
$\{d\}^{Jt-1}$	Deflection in the previous iteration
$\{D\}_{total}$	Accumulated deflection in the element
EA	Axial stiffness
E_c	Concrete modulus of elasticity in Mpa