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PARALLEL BOUNDARY INTEGRAL METHOD APPLIED TO CAVITATION BUBBLE DYNAMICS ON SHARED MEMORY COMPUTER SYSTEM

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By

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PARALLEL BOUNDARY INTEGRAL METHOD APPLIED TO CAVITATION BUBBLE DYNAMICS ON SHARED MEMORY COMPUTER SYSTEM

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The boundary integral method is a well-established technique for the solution of problem in engineering and applied science. This technique mainly involves with the solution of an integral equation, which applies to the boundary of a domain. Consequently, much smaller systems of equations are solved, which in turns will improve in a computing effort. However, for complex geometry, as in the three-dimensional case, dense meshes are required so that very large system of equations still exists. This is true for the solution of the bubble dynamics problems. The problems have a complicated geometry where the demand for computational time is high.

This thesis presents the implementations of new parallel and modified sequential algorithms for solving cavitation bubble dynamics problems on a shared memory multiprocessor computer system, the Sequent Symmetry 5000 SE30. The new parallel and modified sequential algorithms arising from the formulation on linear and quadratic elements are implemented. The implementation is applied to a 3D spherical bubble with constant potential in an infinite medium. Based on the numerical results, the algorithms for both linear and quadratic elements are compared. Overall, the comparison shows that quadratic element is less efficient then linear element. The numerical integration formulae used is the Gauss quadrature rules with 4, 6 and 8 Gauss points. However using different Gauss points only slightly effect the performance of the solution.

The other bubble dynamics problem implemented is a single spherical cavitation bubble growing and collapsing in an infinite medium near a rigid boundary. Both the new parallel and modified sequential algorithms are implemented to see the suitability of integral formulation of bubble dynamics problems for parallel implementation on shared memory multiprocessor computer systems.

This research is of great importance in the study of cavitation damage due to the bubble collapsing near a rigid boundary.

Abstrak dissertasi yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KAEDAH SELARI KAMIRAN SEMPADAN YANG DIAPLIKASIKAN KE ATAS GELEMBUNG KAVITASI DINAMIK PADA SISTEM KOMPUTER INGATAN SEPUNYA

Oleh

ROZITA BINTI JOHARI

April 2001

Pengerusi: **Profesor Madya Md Yazid bin Mohd Saman, Ph.D.**

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Kaedah kamiran sempadan adalah teknik yang telah dikenalpasti mantap untuk menyelesaikan masalah kejuruteraan dan aplikasi sains. Teknik ini melibatkan penyelesaian persamaan kamiran yang diaplikasikan keatas sempadan sesuatu domain. Justeru itu, satu sistem persamaan yang lebih kecil akan diselesaikan. Ini mengekonomikan kebolehupayaaan pengkomputeran. Walau bagaimanpun, untuk geometri yang lebih kompleks seperti kes tiga-dimensi, jejaring yang lebih tumpat diperlukan. Jadi sistem persamaan yang lebih besar masih wujud. Ini ternyata benar bila menyelesaikan masalah gelembung dinamik. Masalah ini terdiri daripada geometri yang lebih kompleks di mana ia memerlukan masa pengkomputeran yang lama.

Tesis ini menerangkan tentang pengimplementasian algoritma selari yang terbaru dan algoritma berjujukan yang diubahsuai untuk menyelesaikan masalah kavitasi gelembung dinamik pada sistem multipemproses ingatan sepunya Sequent Symmetry 5000 SE30. Algoritma selari yang terbaru dan algoritma berjujukan yang diubahsuai hasil daripada

formulasi untuk unsur kuadratik dan linear telah diimplementasikan. Pengimplementasian ini dilakukan ke atas gelembung sfera 3D dengan keupayaan malar dalam bahantara tak terhingga. Berasaskan kepada keputusan berangka, algoritma untuk kedua-dua unsur linear dan kuadratik dibandingkan. Pada keseluruhannya, hasil perbandingan menunjukkan yang unsur kuadratik kurang cekap daripada unsur linear. Formula kamiran berangka menggunakan peraturan kuadratur Gauss dengan 4,6 dan 8 titik Gauss. Walau bagaimanapun, penggunaan titik Gauss yang berbeza kurang menjejaskan prestasi pengiraan.

Satu lagi masalah gelembung dinamik yang diimplimentasikan adalah kavitasi gelembung sfera yang membesar dan mengecil dalam bahantara tak terhingga berhampiran dengan sempadan tegar. Algoritma selari terbaru dan algoritma berjujukan yang diubahsuai diimplementasikan untuk melihat kesesuaian formula kamiran untuk masalah buih dinamik apabila diimplementasikan secara selari pada sistem multipemproses ingatan sepunya.

Penyelidikan ini amat penting dari segi pengetahuan tentang kemusnahan kavitasi disebabkan oleh gelembung yang pecah berhampiran sempadan tegar.

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I certify that an Examination Committee met on 11th April 2001, to conduct the final examination of Rozita binti Johari on her Doctor of Philosophy thesis entitled "Parallel Boundary Integral Method Applied to Cavitation Bubble Dynamics on Shared Memory Computer System" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

ROZITA BINTI JOHARI

Date: 13/6/2001

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LIST OF ABBREVIATIONS

BEM	Boundary Element Method
BIM	Boundary Integral Method
CBNRB	Cavitation Bubble Near Rigid Boundary
DAP	Distributed Array Processor
GE	Gauss Elimination
MIMD	Multiple Instruction Multiple Data
MISD	Multiple Instruction Single Data
PE	Processing Element
PVM	Parallel Virtual Machine
SIMD	Single Instruction Multiple Data
SISD	Single Instruction Single Data
SMP	Shared Memory Processor

CHAPTER 1

INTRODUCTION

Over the past few years, the rapid availability of faster and cheaper processors has led to the development of a variety of parallel processing machines in which the processors are linked together in some way. With the availability of such systems, it has become possible to design software to exploit the advantages of the architecture (Almasi and Gottlieb, 1994).

Image processing, artificial intelligence, robotic, speech recognition, numerical modeling and simulation of scientific and engineering problems are some of the application areas that demand faster processing devices. Serial machines have been pushed to their limits in such applications; parallel machines however have been very successful when applied to these problems.

Users of parallel computing systems tend to be those with large mathematical problems to solve, with the demand for power reflecting a desire to obtain results faster and more accurate. Unfortunately, many of the existing algorithms were developed with a uniprocessor in mind, and the transition from a serial to a parallel environment is therefore, not straightforward.

The increasing speed and expanded storage capacity of modern high-performance computers, together with new advanced numerical methods and programming technique, have greatly improved the ability to solve complex engineering and scientific problems. Usually these problems involve the numerical solution of partial differential equations, with large numbers of degrees of freedom, and their solutions require high computing cost. A lot of work has been done on partial differential equation formulations of boundary value problems using finite elements or finite differences, see for example the review paper by Ortega and Voight (1985).

The solution of an integral equation, which applies only to the boundary of a domain, is known as boundary integral method (BIM). It is also referred to as the boundary element method (BEM). The BIM is a powerful numerical method and has been extensively used for many years solving different engineering problems (Kosztin and Schulten, 1997, Banerjee, 1994). The BIM is a technique which often presents important advantages over domain type solutions since it provides a great economy in computational efforts by discretizing only the boundary of the domains. Consequently, much smaller systems of equations are to be solved. However for complex geometry, as in the (3D) three-dimensional case, dense meshes are required so that quite large system of equations still remains, which make the solving steps slow.

The principal advantage of such a reformulation is that the dimensionality of the problem is reduced by one. For example, (2D) two-dimensional partial differential equation is replaced by a (1D) one-dimensional problem. It involves discretization into line segments on the boundary, in contrast to finite element and finite difference procedures, which require meshes over the plane domain area within the boundary. For problems involving an infinite domain, the boundary integral formulation is particularly advantageous because the behavior at infinity is usually automatically included without having to discretized an artificial 'remote' boundary as with other methods.

Davies (1995) identified three phases in BIM.

(i) The matrix set-up phase

(ii) The solution of linear equation phase

(iii) The recovery phase.

All the three phases exhibit a parallelism, which may be mapped onto a suitable parallel architecture.

1.1 Survey on Parallel Boundary Integral Method

The development of parallel computers has received considerable attention by users of BIM. Simkin (1982) had noted that parallel computer should provide a suitable environment for integral formulations of boundary-value problems, but he did not describe an implementation. Symm (1984) described the first parallel implementation of boundary integral method. Symm's implementation comprised of an indirect approach with constant elements for the solution of the Dirichlet problem in a circle on the ICL distributed array processor (DAP). DAP is a type of machine exhibits features which are typical of the SIMD class of architectures.

The parallel implementation of any particular problem requires that a suitable mapping of the problem onto the parallel architecture. It is often the case that the parallelism in a problem that is not easily identified with a particular parallel architecture and it may well require a considerable amount of ingenuity on behalf of the user to exploit it. In some circumstances, however, the parallelism inherent in the problem is easily identified with that of the parallel architecture and this is particularly true of integral formulations.

Various authors have described certain aspects of the parallel computation of BIM. Earlier, most researchers concentrated on the linear equation phase. For example, the method of substructures in elastostatic provides a coarse-grained parallelism which has been exploited using a vector processor (Bozek et al., 1983; Kline et al., 1985; Kane et al., 1990). Calitz and du Toit (1988) use an integrated phase in an axisymmetric electromagnetic problem.

Kim and Amann (1992), using the method of asynchronous iterations give parallel solution of equations in the area of micro hydrodynamics. Guru Prasad et al. (1992) considers a variety of equation solvers including preconditioned conjugate gradient methods.

Davies (1988a, 1988b, 1989, 1996, 1997) describes a complete fine-grained implementations, in which all phases exploit the parallelism. Variety of linear and quadratic element of potential problems are implemented on the ICL DAP. A coarse-grained implementation of the potential problem on network of transputers was considered by Davies (1991). Effendi et al. (1992) implemented the solution of problems in quantum chromodynamics on the QCDPAX machine, which is a parallel purpose-built architecture. Drake and Gray (1989) also considered a coarse-grained