Jurnal Inovasi Teknologi Pendidikan
Volume 11, No. 4, December 2024 (409-423)
Online: http://journal.uny.ac.id/index.php/jitp

Factor analysis of algebraic thinking skills: A case study on developing area model algebra worksheet based on PhET Interactive Simulation

Giyanti [1], Adika Artasari [1], Rina Oktaviyanthi [1] *, Sharifah Kartini Said Husain [2]
[1] Universitas Serang Raya, Indonesia.
[2] Universiti Putra Malaysia, Malaysia.
* Corresponding Author. E-mail: rinaokta@unsera.ac.id

ARTICLE INFO

Article History
Received:
24 November 2024;
Revised:
28 November 2024;
Accepted:
29 November 2024;
Available online:
31 December 2024.

Keywords
Algebraic thinking skills; Area model algebra; Factor analysis; Latent structure; PhET-based interactive simulation

ABSTRACT

Algebraic thinking is a vital skill in mathematics education, enabling students to generalize patterns, decompose expressions, and apply mathematical models in real-world contexts. However, students often struggle to connect abstract algebraic concepts to practical, real-world problems, which limits their ability to apply these skills effectively. This study aims to uncover the latent structures underlying students' algebraic thinking skills through Exploratory Factor Analysis (EFA). Data were collected from 60 junior high school students in Serang, Banten, who completed worksheets assessing five indicators of algebraic thinking: X1 (Generalization – Decomposing an expression), X2 (Generalization – Using area model), X3 (Transformational – Representing multiplication problem), X4 (Transformational – Strategies for multi-digit numbers), and X5 (Meta-global level – Using area model in real-world contexts), alongside algebraic thinking ability scores (Y). Using varimax rotation, the analysis identified two significant factors. The first, "Generalization and Area Model Application Capability," explained 31.118% of the variance, with high loadings for X2 (0.701) and X3 (0.724). The second, "Transformational Strategies in Multi-digit Numbers," accounted for 20.543% of the variance, with strong loadings for X1 (0.923) and X4 (0.631). Together, these factors explained 51.661% of the total variance. These findings underscore the importance of enhancing generalization skills through area models, including their application to real-world problems and strengthening transformational strategies for multi-digit operations. Incorporating interactive tools like PhET simulations may further support these cognitive processes. Future research should explore classroom implementation and its impact on students' long-term outcomes.

This is an open access article under the CC-BY-SA license.

How to cite:
Giyanti et.al., (2024). Factor analysis of algebraic thinking skills: A case study on developing area model algebra worksheet based on PhET Interactive Simulation. *Jurnal Inovasi Teknologi Pendidikan, 11*(4), 409-423. https://doi.org/10.21831/jitp.v11i4.79362

INTRODUCTION

In mathematics education, students at the junior high school and senior high school levels are expected to master algebraic thinking as a key competency, as it forms the foundation for more advanced mathematical reasoning and problem-solving (Newton et al., 2020; Kieran, 2022). Ideally, students should have a deep understanding of algebraic concepts and be able to apply mathematical strategies to solve complex problems. Mastery of algebra includes the ability to recognize patterns, make generalizations, and understand symbolic representations and relationships between variables (Unal et al., 2023). Students with strong algebraic thinking skills can use their knowledge to solve various types of problems, from simple to those requiring higher-order thinking and creative problem-solving. If students lack strong algebraic thinking skills, they will face various academic and practical challenges (Sibgatullin et al., 2022). Academically, students struggling with algebra may find it difficult to grasp other subjects that require algebraic understanding, such as physics, chemistry, and economics (Turşucu et al., 2020). The inability to understand and use algebraic concepts can also hinder their problem-solving and critical-thinking skills, which are essential in various fields. Practically, lacking algebraic thinking skills can limit students' job opportunities, especially in professions requiring strong mathematical and analytical abilities (Huincahue et al., 2021). Additionally, algebraic thinking is crucial in everyday life, such as managing personal finances, understanding statistical data, and making informed decisions. Therefore, it is essential to develop and strengthen students' algebraic thinking skills through innovative and effective teaching approaches.

In reality, many students struggle to understand and apply algebraic concepts. Common errors include misinterpreting algebraic expressions, difficulties in generalizing patterns, and challenges in translating words into algebraic expressions or vice versa (Chea & Baba, 2021). Students also often struggle to develop and justify methods for using area models to determine the products of monomials and binomials, as well as to recognize that area represents the product of two numbers and is additive. These difficulties reflect a lack of deep understanding and fundamental skills necessary for mastering algebra (Osei & Agyei, 2024). Traditional, non-interactive teaching methods often leave students feeling bored and unmotivated (Hemmati et al., 2024). Limited learning aids and a lack of innovative teaching methods also hinder achieving the expected algebraic competencies. Many students can only memorize formulas without understanding the underlying concepts (Thompson & Harel, 2021). This lack of algebraic thinking skills results in students being unprepared for academic challenges in other subjects requiring algebraic understanding, such as physics and chemistry, and limits their job opportunities in fields requiring strong mathematical and analytical skills (Mathaba et al., 2024). This indicates a gap between curriculum expectations and classroom reality. Observations from a preliminary study conducted over two consecutive academic years with junior high school students in Serang, Banten, revealed that 65% of students in the target group struggled to generalize patterns or decompose algebraic expressions, while 58% faced significant difficulties in applying area models or translating word problems into algebraic forms. These findings support the notion that students in the study context experience similar challenges. Students do not receive learning experiences that support the development of critical and analytical thinking skills.

There is a significant gap between the ideal and actual conditions in algebra learning. Ideally, algebra teaching should involve the use of interactive technology and innovative teaching methods that support deep understanding and practical application of algebraic concepts (Villa-Ochoa & Suárez-Téllez, 2021). However, in reality, many schools still rely on traditional teaching methods that are less engaging and ineffective in developing students' algebraic thinking skills. Despite many studies showing the effectiveness of interactive simulations in mathematics learning, their implementation in the field remains limited. This is due to a lack of resources, teacher training, and institutional support needed to integrate interactive technology into the curriculum (Akram et al., 2022). The gap is also evident in the quality and content of worksheets used in algebra teaching. Existing worksheets often do not meet specific indicators necessary to comprehensively develop students' algebraic thinking skills (Uyen et al., 2021). Important indicators such as generalization through expression decomposition, the use of area models, representation of multiplication problems, strategies for multi-digit numbers, and the application of area models in real-world contexts are often not considered in worksheet development. As a result, students do not receive adequate learning experiences to build the critical and analytical thinking skills needed in algebra. To bridge this gap, there must be systematic efforts to develop and implement worksheets that meet all these important indicators. Additionally, the use of interactive simulations can be an effective solution to create a more engaging and profound learning environment (Campos et al., 2020). By visualizing abstract concepts and increasing student engagement, these simulations can help students better understand

and apply algebraic concepts (Ziatdinov & Valles, 2022). Furthermore, adequate teacher training and institutional support are crucial to ensure that interactive technology and innovative teaching methods can be effectively integrated into everyday teaching.

To bridge the gap between the ideal and actual conditions in algebra learning, the development of worksheets based on PhET interactive simulations that meet algebraic thinking skills indicators can be an extremely effective solution. PhET interactive simulations allow students to visualize algebraic concepts and conduct virtual experiments, supporting deep conceptual understanding (Chinaka, 2021; Oktaviyanthi & Sholahudin, 2023). These simulations provide a dynamic learning environment where students can actively explore abstract concepts such as algebraic expression decomposition and the use of area models, helping them develop transformational skills, integrate strategies for solving multi-digit problems, and apply area models in real-world contexts (Perkins, 2020). The use of factor analysis in this research helps rationally identify and confirm the key indicators that worksheets must meet to develop students' algebraic thinking skills (Oktaviyanthi & Agus, 2023). Factor analysis allows for the exploration of latent structures within the data and identifies the main factors that significantly contribute to students' algebraic thinking skills (Pitta-Pantazi et al., 2020). Using the results of this factor analysis, we can determine which worksheet items and indicators specifically aid in optimizing students' algebraic thinking abilities.

Recent research indicates that interactive simulations in mathematics education, particularly PhET-based ones, have great potential to enhance students' algebraic thinking skills (Engelbrecht & Borba, 2024; Llorella et al., 2024). These simulations enable students to visualize and interact with abstract concepts, thereby increasing their engagement and motivation to learn (Huang et al., 2022). Previous studies have shown that students who use interactive simulations demonstrate significant improvements in conceptual understanding and problem-solving skills compared to those who learn through conventional methods (Araiza-Alba et al., 2021). Moreover, the development of worksheets based on PhET interactive simulations can be an effective solution to help students better develop critical and analytical thinking skills, as well as improve their conceptual understanding and ability to apply algebraic concepts in various contexts (Agus & Oktaviyanthi, 2023). Furthermore, the use of factor analysis in this research provides deeper insights into the latent structure of students' algebraic thinking skills and helps identify the main dimensions of algebraic thinking measured through five specific indicators (Bianchi, 2020; Zhdanov et al., 2023). By understanding the complex structure of algebraic thinking data, more effective and focused learning strategies can be designed, allowing researchers to develop better assessment tools and educational interventions (Suherman & Vidákovich, 2022; Durkin et al., 2023). This research adds a new dimension to the development of mathematics education by integrating PhET-based interactive simulations into specially designed worksheets to enhance students' algebraic thinking skills. The novelty of this research lies in its comprehensive and data-driven approach to designing worksheets, which focuses not only on conceptual understanding but also on practical application through the use of interactive technology. Leveraging factor analysis, this research provides deep insights into the latent structure of algebraic thinking skills, aiding in designing more targeted and effective educational interventions.

METHOD

This study employed a quantitative approach using Exploratory Factor Analysis (EFA) with a primary focus on factor analysis aimed at identifying the latent structure of students' algebraic thinking skills based on five skill indicators measured through worksheet items (X1 to X5) and the algebraic thinking ability scores of the students (Y). This approach provides deep insights into the dimensions underlying algebraic thinking skills and how these variables are interrelated. The worksheet item indicators are detailed in Table 1.

Table 1. Description of Variables from Worksheet Items

No.	Worksheet Item	Variable Description	Variable
1	Generalization Indicator: Students can Decompose Algebraic Expressions, Including those Containing Variables	Generalization - Decomposing an Expression	X1

No.	Worksheet Item	Variable Description	Variable
2	Generalization Indicator: Students can Develop and Justify the Area Model Method to Determine Algebraic Multiplication Results	Generalization - Using an Area Model	X2
3	Transformational Indicator: Students can Represent a Multiplication Problem as the Area of a Rectangle	Transformational - Representing Multiplication Problem	X3
4	Transformational Indicator: Students can Develop and Justify Algebraic Multiplication Results as the Sum of Rectangular Areas	Transformational - Strategies for Multi-Digit Numbers	X4
5	Meta-Global Level Indicator: Students can use the Algebraic Area Model to Solve Real-World Problems	Meta-global level - using Area Model in Real-World Contexts	X5

Table 1 presents the five variables (X1 to X5) that represent different indicators of algebraic thinking skills, which were assessed through the worksheet items. This research model focuses on data analysis from 60 students who completed worksheets containing specific indicators of algebraic thinking skills. The study does not involve direct experimentation or intervention but centers on factor analysis to understand the internal structure of the data. The research model involves data collection encompassing scores on worksheet items (X1 to X5) and students' algebraic thinking ability scores (Y) for analysis using factor analysis techniques. Figure 1 is the methodology presented in a flowchart format, which visually outlines the key steps and processes involved in our research.

Figure 1 illustrates the research methodology flowchart, which outlines the systematic steps taken throughout the study. The study begins with data collection involving students' scores on five worksheet items (X1 to X5) and their algebraic thinking ability scores (Y). After data collection, the next step is data standardization to ensure uniformity in the analysis. Then, data suitability is tested using the Kaiser-Meyer-Olkin (KMO) and Bartlett's Test of Sphericity to determine if the data is appropriate for factor analysis (Steenkamp & Maydeu-Livares, 2023). If the data meets the criteria, factor analysis is conducted to identify the number of relevant factors and the factor loading patterns (Morin et al., 2020).

Start

↓

[Data Collection]

↓

[Data Standardization]

↓

[KMO & Bartlett's Test]

↓

[Data is Suitable?] ----> [No] ----> End

↓ Yes

[Factor Extraction via PCA]

↓

[Varimax Rotation for Interpretation]

↓

[Factor Interpretation & Naming]

↓

[Conclusion: Latent Structure of Algebraic Thinking]

↓

End

Figure 1. Research Methodology Flowchart

The main instrument in this study is the worksheet items that encompass algebraic thinking skill indicators. These worksheets consist of five key items that assess various aspects of algebraic thinking, such as generalization, transformation, and application of area models. Scores on these items, along with students' algebraic thinking ability scores, are used as data for factor analysis. The data collection technique involves gathering scores from 60 junior high school students in Serang, Banten, and processing the data for analysis. Before participation, all students were informed about the purpose of the study, their voluntary involvement, and their right to withdraw at any time. They were given the option to accept or decline participation, and consent was obtained from those who agreed to participate.

The worksheets were distributed during regular class sessions. Clear instructions were provided to the students on how to complete the tasks, and they were given sufficient time to answer the questions. Assistance was available for clarification during the task completion. The collected data includes numerical values from the worksheet items and algebraic thinking ability scores that reflect students' performance in various aspects of algebraic thinking skills.

The data analysis technique used is Exploratory Factor Analysis (EFA), aimed at identifying the latent structure behind the measured variables (Sürücü et al., 2022; Rogers, 2022). Here is the process in more detail:

1. Data Standardization ensures that all scores are comparable by eliminating any biases due to differences in measurement scales.
2. Suitability Testing checks whether the data meets the requirements for factor analysis. If the KMO value is high (> 0.6), and Bartlett's Test is significant, the data is suitable for factor extraction.
3. Principal Component Analysis (PCA) is used to extract the factors. Each factor explains a certain percentage of variance in the data. Factors with eigenvalues greater than 1 are considered significant and are retained.
4. Varimax Rotation simplifies the interpretation by maximizing the loadings of each variable on one factor, making the results more interpretable.
5. Interpretation of Factors is based on the factor loadings. For instance, a variable with a high loading on a factor indicates a strong relationship with that factor, helping to name and define each factor (e.g., 'Generalization Skills' or 'Transformational Strategies').

RESULTS AND DISCUSSION

Results

Data Suitability Testing

The suitability and adequacy of the data for factor analysis were assessed using the Kaiser-Meyer-Olkin (KMO) test and Bartlett's Test of Sphericity. The KMO value ranges from 0 to 1 with the following interpretation:

Table 2. KMO and Bartlett Test Decision Criteria

<table>
<tr><th rowspan="2">No.</th><th colspan="3">KMO Test</th></tr>
<tr><th>KMO Value</th><th>Value Interpretation</th><th>Decision</th></tr>
<tr><td>1</td><td>0.90 to 1.00</td><td>Very Good Adequate</td><td rowspan="6">• If KMO > 0.60, the Data is Adequate for Factor Analysis.
• If KMO < 0.60, the Data is Inadequate for Factor Analysis.</td></tr>
<tr><td>2</td><td>0.80 to 0.89</td><td>Good Adequate</td></tr>
<tr><td>3</td><td>0.70 to 0.79</td><td>Adequate</td></tr>
<tr><td>4</td><td>0.60 to 0.69</td><td>Marginal</td></tr>
<tr><td>5</td><td>0.50 to 0.59</td><td>Inadequate</td></tr>
<tr><td>6</td><td>< 0.50</td><td>Very Inadequate</td></tr>
<tr><th colspan="4">Bartlett Test</th></tr>
<tr><th colspan="4">Bartlett's Test Decision Criteria</th></tr>
<tr><td colspan="4">• If p-value < 0.05, the null hypothesis is Rejected, Indicating that the Correlation Matrix is not an Identity Matrix and the Data is Adequate for Factor Analysis.
• If p-value > 0.05, the null Hypothesis is not Rejected, Indicating that the Correlation Matrix is an Identity Matrix and the Data is Inadequate for Factor Analysis.</td></tr>
</table>

Table 2 presents the decision criteria for the KMO and Bartlett tests. It outlines the KMO value interpretation, indicating data adequacy for factor analysis, and provides the p-value criteria for Bartlett's test, which assesses whether the correlation matrix is suitable for analysis.

Using IBM SPSS Statistics version 21, the KMO and Bartlett's Test of Sphericity results are shown in Table 3.

Table 3. KMO and Bartlett's Test for 5 Variables

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.872
Bartlett's Test of Sphericity	Approx. Chi-Square	465.650
	df	10
	Sig.	.001

From Table 3, the KMO value is 0.872, which falls in the range of 0.80 to 0.89, interpreted as good. This indicates that the data is suitable for factor analysis. The significance value of Bartlett's Test is 0.001 < 0.05, leading to the conclusion that the correlation matrix is not an identity matrix, thus the data is adequate for factor analysis.

Next to the MSA test, it evaluates the adequacy of the sample for each variable individually. The decision criteria for MSA are as follows if MSA > 0.60, the variable is considered adequate for inclusion in the factor analysis, and if MSA < 0.60, the variable is considered inadequate for inclusion in the factor analysis and should be excluded.

Table 4. Measures of Sampling Adequacy

		X1	**X2**	**X3**	**X4**	**X5**
Anti-image Correlation	X1	.876[a]	-.260	-.525	-.343	-.093
	X2	-.260	.836[a]	-.482	-.247	-.547
	X3	-.525	-.482	.845[a]	.065	.270
	X4	-.343	-.247	.065	.942[a]	.046
	X5	-.093	-.547	.270	.046	.872[a]

a. Measures of Sampling Adequacy(MSA)

Based on Table 4, the MSA values for X1 (0.876), X2 (0.836), X3 (0.845), X4 (0.942), and X5 (0.872) are all greater than 0.60, indicating that the variables have adequate sample sizes and are suitable for inclusion in the factor analysis.

Factor Extraction (Including Number of Factors)

Factor extraction determines the number of factors that adequately explain the underlying data structure. This process involves several steps and decisions based on various criteria and statistical methods.

Kaiser Criterion (Eigenvalue > 1)

Factors with eigenvalues greater than 1 are considered significant and retained in the model. The SPSS output showing eigenvalues is presented in Table 5.

Table 5. Total Variance Explained

Component	**Initial Eigenvalues**			**Extraction Sums of Squared Loadings**			**Rotation Sums of Squared Loadings**		
	Total	**% of Variance**	**Cumulative %**	**Total**	**% of Variance**	**Cumulative %**	**Total**	**% of Variance**	**Cumulative %**
1	1.556	31.118	31.118	1.556	31.118	31.118	1.529	30.588	30.588
2	1.027	20.543	51.662	1.027	20.543	51.662	1.054	21.074	51.662
3	.925	18.503	70.165						
4	.812	16.248	86.413						
5	.679	13.587	100.000						

Extraction Method: Principal Component Analysis.

Table 5 outlines the eigenvalues and percentage of variance explained by each factor. Factor 1, with an eigenvalue of 1.556, explains 31.118% of the total variance, while Factor 2, with an eigenvalue of 1.027, accounts for 20.543% of the variance. Together, these two factors explain 51.661% of the variance, meaning that more than half of the variability in students' algebraic thinking skills can be explained by these two factors. The remaining factors are not retained, as their eigenvalues are below 1, indicating that they do not significantly contribute to explaining the variance.

Scree Plot

A scree plot displays the eigenvalues of factors in descending order. Significant factors typically lie before the "elbow" point (a sharp change) in the plot, while factors after the elbow point are considered insignificant as they explain little additional variance.

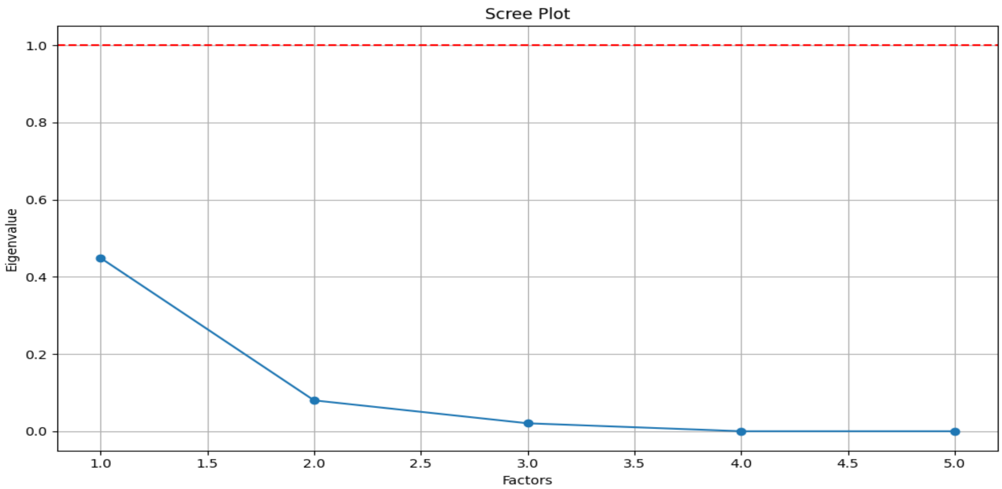

Figure 2. Scree Plot of Significant Factors

Figure 2 displays the Scree Plot, which helps to determine the number of significant factors to retain. The elbow joint is observed after Factor 2, indicating that only two factors should be retained for further analysis. This visual representation supports the decision made in Table 5 to retain two factors with eigenvalues greater than 1. Factors beyond this point (factors 3, 4, and 5) explain diminishing variance and are not considered significant for this study.

Communalities (> 0.5)

An additional criterion for determining significant factors is the communality values of variables. Communalities measure the proportion of variance in each variable explained by the extracted factors. Communalities > 0.5 indicate that the variable is well-explained by the extracted factors.

Table 6. Variable Communalities

	Initial	**Extraction**
X1	1.000	.867
X2	1.000	.794
X3	1.000	.725
X4	1.000	.646
X5	1.000	.454

Extraction Method: Principal Component Analysis.

Table 6 presents the commonalities for each variable, which indicate the proportion of variance in each variable that is explained by the retained factors. For instance, X1 has a commonality of 0.865, meaning that 86.5% of the variance in the ability to decompose expressions is explained by the two factors. Similarly, X2 (Generalization using the area model) has a commonality of 0.794, showing that the area model application skill is well-represented by the factors. Communalities greater than 0.5 suggest that the variables are well-explained by the factor model, indicating that the retained factors adequately capture the underlying structure of the data.

Based on the criteria and methods above, the factor extraction decision is to retain two factors: **Factor 1**: Combines X2 and X3, related to generalization and area model application capability. **Factor 2**: Combines X1 and X4, related to the decomposition of expression and transformational strategies in multi-digit numbers.

These factors sufficiently explain the data variance and provide a clear understanding of the latent structure of students' algebraic thinking skills measured by the provided worksheets. Communalities > 0.5 show that the variables in these factors are well-explained by the extracted factors.

Factor Rotation (Loading Factor)

After factor extraction, factor rotation is performed to enhance interpretability. The goal of factor rotation is to achieve a clearer loading pattern, where each variable has a high loading on one factor and low loadings on other factors, simplifying interpretation. Loading factors are evaluated to determine which variables have high loadings on each factor. Variables with loading factors > 0.5 on one factor are considered significant. The factor rotation in the SPSS output using varimax rotation is presented in Table 7.

Table 7. Varimax Factor Rotation

	Component	
	1	**2**
X1	.112	.923
X2	.701	-.048
X3	.724	-.029
X4	-.510	.631
X5	-.491	-.112

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 3 iterations.

Table 7 shows the results of the varimax rotation, which simplifies the interpretation of factor loadings. The table presents the loadings of each variable on the two identified factors. X2 (0.701) and X3 (0.724) load strongly on Factor 1, indicating that this factor represents the Generalization and Area Model Application Capability. On the other hand, X1 (0.923) and X4 (0.631) load heavily on Factor 2, representing Transformational Strategies in Multi-digit Numbers. The loadings make it clear which variables are most strongly associated with each factor, and how these variables contribute to the cognitive structures being analyzed in the study.

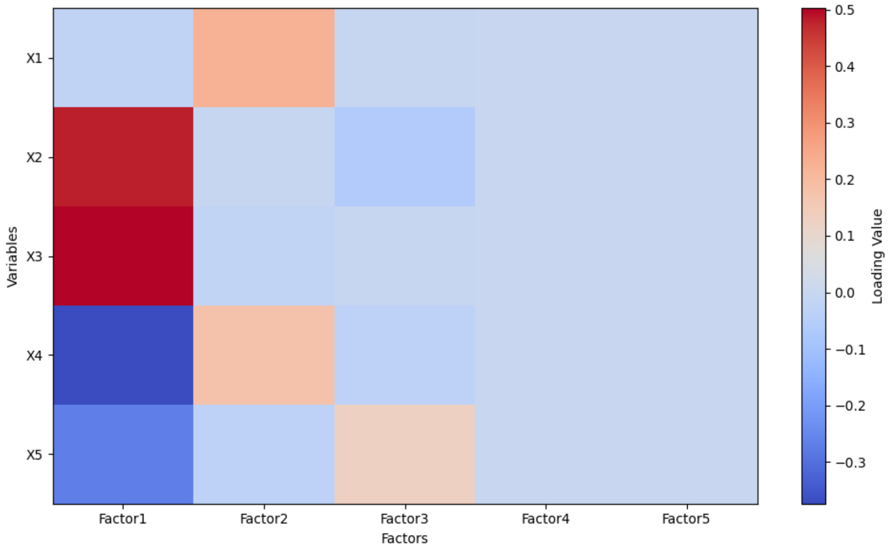

Figure 3. Factor Loadings Visualization

Figure 3 visually represents the factor loadings of the variables on the two identified factors. The darker areas in the plot correspond to higher loadings, highlighting the variables that contribute most to each factor. For example, X2 and X3 show strong associations with Factor 1, while X1 and X4 are closely linked to Factor 2. This visual aid helps to quickly interpret the relationships between variables and factors, making it easier to see how the underlying cognitive skills are grouped in the analysis.

Factor Interpretation and Naming

Based on the analysis and factor rotation results, the following is the interpretation and naming of the factors:

Factor 1: Generalization and Area Model Application Capability. This factor consists of X2 (Generalization - Using area model) with a loading of 0.701 and X3 (Transformational - Representing multiplication problem) with a loading of 0.724, explaining 31.118% of the variance. This factor reflects students' ability to use area models in generalization contexts and represent multiplication problems, indicating the application of area models in generalization and transformational skills.

Factor 2: Transformational Strategies in Multi-digit Numbers. This factor consists of X1 (Generalization - Decomposing an expression) with a loading of 0.923 and X4 (Transformational - Strategies for multi-digit numbers) with a loading of 0.631, explaining 20.543% of the variance. This factor reflects students' ability to decompose algebraic expressions and use strategies for multi-digit operations, indicating a focus on transformational strategies in multi-digit contexts.

To ensure that the formed factors have no further correlation among themselves, it is necessary to trace the values in the component transformation matrix presented in Table 8. If the correlation values for each factor in the main diagonal lie in the range of 0.8 to 0.9, it indicates that the factors are not further correlated and fall into the category of very strong correlation.

Table 8. Component Transformation Matrix

Component	**1**	**2**
1	.975	-.224
2	.224	.975

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

Table 8 provides the component transformation matrix, which helps to confirm the precision of the factor rotation. The values on the diagonal (both 0.975) indicate that the factors are well-formed and not further correlated. This confirms the independence of the two factors, reinforcing the validity of the factor rotation and the distinctiveness of the cognitive skills measured by each factor.

Discussion

Factor 1: Generalization and Area Model Application Capability

Factor 1 captures students’ abilities to generalize algebraic concepts using area models. This factor includes variables related to the use of area models to represent and solve algebraic problems, specifically focusing on how students can abstract general patterns from specific cases. The high loadings of X2 (Generalization - Using area model, 0.701) and X3 (Transformational - Representing multiplication problem, 0.724) suggest that students who perform well on these items demonstrate a strong capacity to visualize algebraic relationships spatially. The ability to apply area models plays a pivotal role in understanding more abstract mathematical concepts. Area models help students connect geometric representations to algebraic expressions, bridging concrete visual experiences with symbolic reasoning. This process of generalization is crucial in developing algebraic thinking, as it allows students to see the broader application of a mathematical principle across different contexts (Kieran, 2022; Ünal et al., 2023). For example, a student who can use an area model to represent multiplication problems is not only solving the immediate problem but also developing an understanding of how algebraic expressions can represent real-world quantities and relationships.

Research supports the idea that visual models, such as area models, are essential tools for helping students move from concrete arithmetic understanding to abstract algebraic reasoning.

Studies by Hawes et al., (2022) suggest that spatial reasoning, facilitated by visual tools like area models, enhances students' ability to generalize mathematical concepts, which is critical for success in algebra. Furthermore, the work of Ellis et al., (2022) emphasizes the importance of generalization in developing higher-order mathematical thinking, as it enables students to recognize patterns, make predictions, and apply mathematical reasoning across various problem types. Thus, the identification of this factor highlights a key cognitive skill—generalization through the application of area models—that is foundational to algebraic competence. Instructional strategies that emphasize the use of visual models can be effective in strengthening this ability, as they provide students with concrete ways to explore and manipulate algebraic expressions, making abstract concepts more accessible.

Factor 2: Transformational Strategies in Multi-digit Numbers

The second factor, which explains 20.543% of the variance, is defined by high loadings on X1 (Generalization - Decomposing an expression, 0.923) and X4 (Transformational - Strategies for multi-digit numbers, 0.631). This factor represents students' abilities to decompose algebraic expressions and apply transformational strategies for multi-digit operations. These skills are essential for developing algebraic thinking (Findell et al., 2001). Decomposing algebraic expressions requires breaking down complex expressions into simpler parts for further manipulation, a higher-order cognitive skill critical in algebra (Spiller et al., 2023). Likewise, transformational strategies enable students to solve large-scale problems by applying efficient methods such as factoring or breaking down numbers into smaller components. Mastery of these skills helps students approach algebraic problems more flexibly and efficiently.

The varimax rotation method used in the factor analysis resulted in two independent factors, confirming that the first factor is related to generalization and area model application, while the second focuses on transformational strategies for multi-digit numbers. The independence of these factors is confirmed by a main diagonal correlation of 0.975. According to West (2021), transformational strategies are crucial for a deeper understanding of algebraic concepts, and students who can apply these strategies are better prepared for more complex algebraic tasks. Similarly, Dröse & Prediger (2023) found that the ability to decompose multi-digit operations is closely linked to algebraic success. These findings highlight the importance of educational interventions that emphasize not just basic algebraic operations but also transformational strategies to build a strong foundation for advanced algebraic reasoning.

While the two factors identified explain a significant portion of the variance in students' algebraic thinking, accounting for 51.661% of the total variance, there remains a 48.39% unexplained variance. This suggests that other factors, not captured by the current model, could also play a role in shaping students' algebraic thinking abilities. Possible influences include students' prior mathematical knowledge, individual differences in cognitive ability, and instructional variables that were not explicitly measured in this study. For instance, factors such as the quality of teaching, classroom environment, or students' motivational levels may contribute to the unexplained variance. These elements could have an impact on student's performance but were not part of the scope of this analysis. Additionally, the relatively low loadings of some items, such as X5 (using area models in real-world contexts), indicate that these variables might require further refinement or additional factors to explain students' algebraic thinking more comprehensively (Krawitz et al., 2022). These results highlight the need for further research to investigate the full range of cognitive and external factors influencing algebraic thinking skills.

The findings from this study are consistent with previous research emphasizing the importance of area models in teaching mathematics to foster skills in generalization and multiplication problem representation (Goldin, 2020; Lischka & Stephens, 2020). In addition, the ability to decompose expressions and use strategies for multi-digit numbers is recognized as a key component in developing students' algebraic thinking (Ortiz-Laso & Diego-Mantecón, 2020; Dröse & Prediger, 2023). These results reinforce the critical role that visual models and transformational strategies play in enhancing students' conceptual understanding of algebra. To address the primary research question, which aimed to uncover the cognitive structures underlying students' algebraic thinking, Exploratory Factor Analysis (EFA) identified two significant factors. Factor 1, Generalization and Area Model Application Capability, and Factor 2, Transformational Strategies in Multi-digit

Numbers, both represent distinct cognitive dimensions that are fundamental to algebraic thinking. These findings suggest that effective algebra instruction should emphasize the use of visual tools like area models to aid generalization and encourage the use of transformational strategies to handle complex, multi-digit problems. Both factors are supported by strong loadings, indicating that these skills are crucial for the development of algebraic proficiency.

The study's identification of two key cognitive factors has important implications for teaching and learning algebra. Factor 1, which emphasizes generalization through area models, points to the necessity of incorporating visual and spatial reasoning tools in mathematics curricula. Generalization is crucial in algebra as it allows students to recognize patterns and make predictions (Ureña et al., 2024). Providing students with opportunities to engage with area models can facilitate their transition from concrete arithmetic to abstract algebraic reasoning (Alam & Mohanty, 2024). Factor 2, which highlights transformational strategies in multi-digit numbers, emphasizes the importance of teaching students how to decompose complex algebraic expressions and apply strategies to simplify them (Whitacre & Rumsey, 2020). These skills are essential for higher-order problem-solving in algebra (Ortiz-Laso & Diego-Mantecón, 2020). Instructional interventions should focus on explicit instruction in decomposition, factoring, and simplification techniques, which will equip students with the tools necessary to tackle more advanced algebraic tasks. By addressing both factors, educators can help students build a robust foundation in algebraic thinking, enhancing their problem-solving abilities and overall mathematical competency. Moreover, integrating interactive tools like PhET simulations could further support these cognitive processes by providing dynamic, hands-on experiences that reinforce both generalization and transformational strategies.

Several recommendations arise from this study. First, the mathematics curriculum should incorporate the use of area models and transformational strategies to support the development of students' algebraic thinking. Second, teacher training should be enhanced to adopt teaching methods that effectively utilize physical manipulatives and real-world contexts to help students grasp complex mathematical concepts. Furthermore, the development of instructional materials that emphasize the use of area models and transformational strategies should be prioritized to provide relevant practice and examples for students. Further research is needed to test the effectiveness of this approach and provide deeper insights into how students' algebraic thinking skills develop over time. This study reinforces previous findings, provides strong empirical evidence on the structure of students' algebraic thinking abilities, and adds to the existing literature in mathematics education.

CONCLUSION

This study offers a novel contribution to the field of algebra education by revealing the latent cognitive structures underlying students' algebraic thinking skills. Through factor analysis, two key factors were identified: Generalization and Area Model Application Capability and Transformational Strategies in Multi-digit Numbers. These findings provide a deeper understanding of how students approach and solve algebraic problems, demonstrating that both visual representations and transformational problem-solving strategies are critical to enhancing algebraic competency. The innovative use of PhET interactive simulations further strengthens these insights by offering a dynamic, hands-on learning environment that aligns with the push for technology integration in education. The simulations provide a unique opportunity for students to visualize abstract concepts, reinforcing the need for such tools in modern classrooms. The identification of these two cognitive factors, which explain over 50% of the variance in student responses, underscores the foundational role of visual models and transformational strategies in algebraic success. Students who can generalize through visual aids and apply transformational techniques to multi-digit problems exhibit stronger algebraic proficiency. These results confirm the critical nature of these skills, offering educators a clear path for enhancing their instructional methods. Several recommendations emerge from these findings. Teachers should prioritize the use of visual models, such as area models, to help students generalize algebraic concepts and connect geometric representations with algebraic reasoning. Strengthening students' transformational strategies, like decomposition and multi-digit problem-solving techniques, is equally important in improving their flexibility in tackling complex algebraic tasks. Additionally, integrating interactive simulations like PhET into classroom instruction can significantly enhance student engagement and deepen their understanding of abstract algebraic ideas. While the study has offered valuable insights, further research is needed to examine how students' algebraic thinking evolves, particularly their generalization and transformational abilities. Future studies should also assess the long-term impact of visual models and simulations across diverse student populations. Lastly, exploring how these cognitive structures transfer to other areas of mathematics or science could provide further insight into problem-solving skill development across disciplines. In conclusion, by focusing on generalization through visual tools and strengthening transformational strategies, educators can better support students in developing a deeper, more flexible understanding of algebra.

REFERENCES

Agus, R. N., & Oktaviyanthi, R. (2023). Worksheet graphing quadratics berbantuan Phet Simulation untuk optimalisasi mathematical visual thinking mahasiswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *12*(2), 2166-2180. http://dx.doi.org/10.24127/ajpm.v12i2.6908

Akram, H., Abdelrady, A. H., Al-Adwan, A. S., & Ramzan, M. (2022). Teachers' perceptions of technology integration in teaching-learning practices: A systematic review. *Frontiers in psychology*, *13*, 1-9. https://doi.org/10.3389/fpsyg.2022.920317

Alam, A., & Mohanty, A. (2024). Unveiling the complexities of 'abstract algebra' in University Mathematics Education (UME): Fostering 'conceptualization and understanding' through advanced pedagogical approaches. *Cogent Education*, *11*(1), 1-24. https://doi.org/10.1080/2331186X.2024.2355400

Araiza-Alba, P., Keane, T., Chen, W. S., & Kaufman, J. (2021). Immersive virtual reality as a tool to learn problem-solving skills. *Computers & Education*, *164*, 1-13. https://doi.org/10.1016/j.compedu.2020.104121

Bianchi, R. (2020). Do burnout and depressive symptoms form a single syndrome? Confirmatory factor analysis and exploratory structural equation modeling bifactor analysis. *Journal of Psychosomatic Research*, *131*, 1-7. https://doi.org/10.1016/j.jpsychores.2020.109954

Campos, N., Nogal, M., Caliz, C., & Juan, A. A. (2020). Simulation-based education involving online and on-campus models in different European universities. *International journal of educational technology in higher education*, *17*, 1-15. https://doi.org/10.1186/s41239-020-0181-y

Chea, S., & Baba, T. (2021). Analysing students' conceptions underlying the errors in algebraic expressions: A case study on Cambodian ninth grade students. *Journal of Science and Mathematics Education in Southeast Asia*, *44*, 22-40. https://myjms.mohe.gov.my/index.php/jsmesea/article/download/12533/8778/

Chinaka, T. W. (2021). The effect of PhET simulation vs. phenomenon-based experiential learning on students' integration of motion along two independent axes in projectile motion. *African Journal of Research in Mathematics, Science and Technology Education*, *25*(2), 185-196. https://doi.org/10.1080/18117295.2021.1969739

Dröse, J., & Prediger, S. (2023). Prospective teachers' diagnostic thinking on students' understanding of multi-digit multiplication: A content-related analysis on unpacking of knowledge elements. *Journal für Mathematik-Didaktik*, *44*(1), 1-28. https://doi.org/10.1007/s13138-022-00214-w

Durkin, K., Rittle-Johnson, B., Star, J. R., & Loehr, A. (2023). Comparing and discussing multiple strategies: An approach to improving algebra instruction. *The Journal of Experimental Education*, *91*(1), 1-19. https://doi.org/10.1080/00220973.2021.1903377

Ellis, A. B., Lockwood, E., Tillema, E., & Moore, K. (2022). Generalization across multiple mathematical domains: Relating, forming, and extending. *Cognition and Instruction*, *40*(3), 351-384. https://doi.org/10.1080/07370008.2021.2000989

Engelbrecht, J., & Borba, M. C. (2024). Recent developments in using digital technology in mathematics education. *ZDM–Mathematics Education*, *56*(2), 281-292. https://doi.org/10.1007/s11858-023-01530-2

Findell, B., Swafford, J., & Kilpatrick, J. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

Goldin, G. A. (2020). Mathematical representations. *Encyclopedia of mathematics education*, 566-572. https://doi.org/10.1007/978-94-007-4978-8_103

Hawes, Z. C., Gilligan-Lee, K. A., & Mix, K. S. (2022). Effects of spatial training on mathematics performance: A meta-analysis. *Developmental Psychology*, *58*(1), 112-137. https://doi.org/10.1037/dev0001281

Hemmati, R., Zarrinabadi, N., & Ebrahimi, H. (2024). “I can’t bear sitting in this class”: Exploring Iranian undergraduate female student experiences of their college classrooms. *Learning Environments Research*, *27*, 913-937. https://doi.org/0.1007/s10984-024-09510-3

Huang, Y. M., Silitonga, L. M., & Wu, T. T. (2022). Applying a business simulation game in a flipped classroom to enhance engagement, learning achievement, and higher-order thinking skills. *Computers & Education*, *183*, 104494. https://doi.org/10.1016/j.compedu.2022.104494

Huincahue, J., Borromeo-Ferri, R., Reyes-Santander, P., & Garrido-Véliz, V. (2021). Mathematical thinking styles the advantage of analytic thinkers when learning mathematics. *Education Sciences*, *11*(6), 1-15. https://doi.org/10.3390/educsci11060289

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: Background, overarching dimensions, and new directions. *ZDM–Mathematics Education*, *54*(6), 1131-1150. https://doi.org/10.1007/s11858-022-01435-6

Krawitz, J., Chang, Y. P., Yang, K. L., & Schukajlow, S. (2022). The role of reading comprehension in mathematical modelling: improving the construction of a real-world model and interest in Germany and Taiwan. *Educational Studies in Mathematics*, *109*(2), 337-359. https://doi.org/10.1007/s10649-021-10058-9

Lischka, A. E., & Stephens, D. C. (2020). The area model: Building mathematical connections. *Mathematics Teacher: Learning and Teaching PK-12*, *113*(3), 186-195. https://doi.org/10.5951/MTLT.2019.0115

Llorella, F., Cebrián, J. A., Corbi, A., & Pérez, A. M. (2024). Fostering scientific methods in simulations through symbolic regressions. *Physics Education*, *59*(4), 1-16. https://doi.org/10.1088/1361-6552/ad3cad

Mathaba, P. N., Bayaga, A., Tîrnovan, D., & Bossé, M. J. (2024). Error analysis in algebra learning: Exploring misconceptions and cognitive levels. *Journal on Mathematics Education*, *15*(2), 575-592. https://doi.org/10.22342/jme.v15i2.pp575-592

Morin, A. J. S., Myers, N. D., & Lee, S. (2020). Modern factor analytic techniques: Bifactor models, exploratory structural equation modeling (ESEM), and Bifactor-ESEM. In G. Tenenbaum, R. C. Eklund, & N. Boiangin (Eds.), *Handbook of sport psychology: Exercise, methodologies, & special topics* (4th ed., pp. 1044–1073). John Wiley & Sons, Inc.. https://doi.org/10.1002/9781119568124.ch51

Newton, K. J., Barbieri, C. A., & Booth, J. L. (2020). Key mathematical competencies from arithmetic to algebra. In *Oxford Research Encyclopedia of Education*. https://doi.org/10.1093/acrefore/9780190264093.013.956

Oktaviyanthi, R., & Agus, R. N. (2023). Evaluating graphing quadratic worksheet on visual thinking classification: A confirmatory analysis. *Infinity Journal*, *12*(2), 207-224. https://doi.org/10.22460/infinity.v12i2.p207-224

Oktaviyanthi, R., & Sholahudin, U. (2023). Phet assisted trigonometric worksheet for students' trigonometric adaptive thinking. *Mosharafa: Jurnal Pendidikan Matematika*, *12*(2), 229-242. https://doi.org/10.31980/mosharafa.v12i2.779

Ortiz-Laso, Z., & Diego-Mantecón, J. M. (2020). Strategies of pre-service early childhood teachers for solving multi-digit division problems. *Sustainability*, *12*(23), 1-20. https://doi.org/10.3390/su122310217

Osei, W., & Agyei, D. D. (2024). Analyses of algebra knowledge and difficulties among Ghanaian Junior High School learners: Insights from standard-based mathematics curriculum implementation. *Social Sciences & Humanities Open*, *10*, 1-12. https://doi.org/10.1016/j.ssaho.2024.101019

Perkins, K. (2020). Transforming STEM learning at scale: PhET interactive simulations. *Childhood Education*, *96*(4), 42-49. https://doi.org/10.1080/00094056.2020.1796451

Pitta-Pantazi, D., Chimoni, M., & Christou, C. (2020). Different types of algebraic thinking: an empirical study focusing on middle school students. *International Journal of Science and Mathematics Education*, *18*, 965-984. https://doi.org/10.1007/s10763-019-10003-6

Rogers, P. (2022). Best practices for your exploratory factor analysis: A factor tutorial. *Revista de Administração Contemporânea*, *26*(06), 1-17. https://doi.org/10.1590/1982-7849rac2022210085.en

Sibgatullin, I. R., Korzhuev, A. V., Khairullina, E. R., Sadykova, A. R., Baturina, R. V., & Chauzova, V. (2022). A Systematic review on algebraic thinking in education. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(1), 1-15. https://doi.org/10.29333/ejmste/11486

Spiller, J., Clayton, S., Cragg, L., Johnson, S., Simms, V., & Gilmore, C. (2023). Higher level domain specific skills in mathematics; The relationship between algebra, geometry, executive function skills and mathematics achievement. *Plos one*, *18*(11), 1-20. https://doi.org/10.1371/journal.pone.0291796

Suherman, S., & Vidákovich, T. (2022). Assessment of mathematical creative thinking: A systematic review. *Thinking Skills and Creativity*, *44*, 1-13. https://doi.org/10.1016/j.tsc.2022.101019

Sürücü, L., Yıkılmaz, İ., & Maşlakçı, A. (2022). Exploratory factor analysis (EFA) in quantitative researches and practical considerations. *Gümüşhane Üniversitesi Sağlık Bilimleri Dergisi*, *13*(2), 947-965. https://doi.org/10.31219/osf.io/fgd4e

Steenkamp, J. B. E., & Maydeu-Olivares, A. (2023). Unrestricted factor analysis: A powerful alternative to confirmatory factor analysis. *Journal of the Academy of Marketing Science*, *51*(1), 86-113. https://doi.org/10.1007/s11747-022-00888-1

Thompson, P. W., & Harel, G. (2021). Ideas foundational to calculus learning and their links to students' difficulties. *ZDM–Mathematics Education*, *53*(3), 507-519. https://doi.org/10.1007/s11858-021-01270-1

Turşucu, S., Spandaw, J., & de Vries, M. J. (2020). Search for symbol sense behavior: Students in upper secondary education solving algebraic physics problems. *Research in Science Education*, *50*(5), 2131-2157. https://doi.org/10.1007/s11165-018-9766-z

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and symbolic representations as components of algebraic reasoning. *Journal of Numerical Cognition*, *9*(2), 327-345. https://doi.org/10.5964/jnc.11151

Ureña, J., Ramírez, R., Molina, M., & Cañadas, M. C. (2024). Generalization: Strategies and representations used by sixth to eighth graders in a functional context. *Mathematics Education Research Journal*, *36*(3), 519-545. https://doi.org/10.1007/s13394-023-00458-w

Uyen, B. P., Tong, D. H., & Tram, N. T. B. (2021). Developing mathematical communication skills for students in grade 8 in teaching congruent triangle topics. *European Journal of Educational Research*, *10*(3), 1287-1302. https://doi.org/10.12973/eu-jer.10.3.1287

Villa-Ochoa, J. A., & Suárez-Téllez, L. (2021). Computer algebra systems and dynamic geometry for mathematical thinking. In *Handbook of cognitive mathematics* (pp. 1-27). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-44982-7_36-1

West, H. M. (2021). *Using Growth Patterns to simultaneously promote elementary students' multiplicative reasoning and early algebraic thinking: An examination of children's thinking, teachers' perspectives, and curriculum*. North Carolina State University. https://repository.lib.ncsu.edu/items/e92fa7ed-5b64-43e3-aa58-cdf733a8805a

Whitacre, I., & Rumsey, C. (2020). The roles of tools and models in a prospective elementary teachers' developing understanding of multidigit multiplication. *The Journal of Mathematical Behavior*, *60*, 1-18. https://doi.org/10.1016/j.jmathb.2020.100816

Zhdanov, S. P., Sadrieva, L. M., Astakhov, I. A., Sokolova, N. L., Grishnova, E. E., & Tararina, L. I. (2023). Psychometric properties of information and communication technology competencies scale: Latent profile analysis. *Online Journal of Communication and Media Technologies*, *13*(4), 1-13. https://doi.org/10.30935/ojcmt/13479

Ziatdinov, R., & Valles Jr, J. R. (2022). Synthesis of modeling, visualization, and programming in GeoGebra as an effective approach for teaching and learning STEM topics. *Mathematics*, *10*(3), 1-16. https://doi.org/10.3390/math10030398