NUMERICAL AND STATISTICAL ANALYSIS OF BOUNDARY LAYER FLOW AND HEAT TRANSFER IN HYBRID NANOFLUID OVER VARIOUS PERMEABLE SURFACES

By

RUSYA IRYANTI BINTI YAHAYA

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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DEDICATIONS

To my beloved family, thank you for everything!

To Kanak-Kanak Kolej, all the best!

To me, myself, and I, great work!

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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RUSYA IRYANTI BINTI YAHAYA

August 2024

Chairman : Norihan binti Md Arifin, PhD

Institute : Mathematical Research

This thesis presents the studies on hybrid nanofluid flow over various permeable surfaces and conditions. Five different flow problems consisting of unsteady hybrid nanofluid flow past a permeable Riga plate with thermal radiation and convective boundary condition, mixed convection hybrid nanofluid flow past a permeable non-isothermal cone and wedge with thermal radiation and convective boundary condition, hybrid nanofluid flow past a permeable biaxial stretching/shrinking surface with thermal radiation, oblique stagnation-point flow of hybrid nanofluid towards a permeable shrinking surface, and magnetohydrodynamics (MHD) stagnation-point flow of ternary hybrid nanofluid over a permeable radially shrinking disk with thermal radiation, viscous dissipation, and convective boundary condition are solved, analyzed, and discussed. The geometries and governing conditions of these flow problems are defined using partial differential equations and boundary conditions. Then, similarity transformation reduced these equations into non-linear ordinary differential equations before being solved numerically using the bvp4c solver in MATLAB. Multiple solutions are found within certain ranges of unsteadiness, mixed convection, and stretching/shrinking parameters. However, stability analysis confirms that only the first solution is stable while the others are unstable. The numerical results show that hybrid nanofluid and ternary hybrid nanofluid improve the physical quantities of interest, namely the local skin friction coefficient and Nusselt number. Nevertheless, in some cases where boundary suction is applied, hybrid nanofluids may have a lower local Nusselt number than nanofluids. Increasing the suction parameter can help compensate for this reduction in heat transfer performance. The imposition of thermal radiation and convective boundary condition also increases the local Nusselt number. Additionally, the assisting mixed convection flow exhibits a higher local skin friction coefficient and Nusselt number than the opposing flow. Finally, response surface methodology (RSM) is employed to determine the significance and optimal settings of the controlling parameters on the local Nusselt number. Generally, the highest value of the suction parameter maximizes the local Nusselt number and improves the heat transfer rate at the surface.

Keywords: Multiple solutions, nanofluid, RSM, suction

SDG: GOAL 9: Industry, Innovation and Infrastructure

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ANALISIS BERANGKA DAN BERSTATISTIK BAGI ALIRAN LAPISAN SEMPADAN DAN PEMINDAHAN HABA NANOBENDALIR HIBRID TERHADAP PELBAGAI PERMUKAAN TELAP

Oleh

RUSYA IRYANTI BINTI YAHAYA

Ogos 2024

Pengerusi : Norihan binti Md Arifin, PhD
Institut : Penyelidikan Matematik

Tesis ini membincangkan kajian aliran nanobendalir hibrid terhadap pelbagai permukaan telap dan keadaan. Lima masalah aliran berbeza yang terdiri daripada alir tak mantap nanobendalir hibrid terhadap plat telap Riga dengan sinaran terma dan syarat sempadan olakan, aliran nanobendalir hibrid terhadap kon dan baji yang telap dan tak sesuhu dengan olakan campuran, sinaran terma dan syarat sempadan olakan, aliran nanobendalir hibrid terhadap permukaan telap yang meregang/mengecut secara dwipaksi dengan sinaran terma, aliran titik genangan serong nanobendalir hibrid terhadap permukaan telap mengecut, dan aliran titik genangan magnetohidrodinamik (MHD) nanobendalir hibrid ternari terhadap permukaan cakera yang telap dan mengecut dengan sinaran terma, lesapan likat dan syarat sempadan olakan telah diselesaikan, dianalisis dan dibincangkan. Geometri dan keadaan yang mengawal masalah-masalah aliran ini diterjemahkan melalui persamaan pembezaan separa dan syarat sempadan. Setelah itu, penjelmaan keserupaan meringkaskan persamaan ini kepada persamaan pembezaan biasa tak linear sebelum diselesaikan menggunakan penyelesai bvp4c dalam MATLAB. Penyelesaian berganda diperoleh dalam julat tertentu parameter ketakmantapan, parameter olakan campuran, dan parameter meregang/mengecut.

Walau bagaimanapun, analisis kestabilan mengesahkan hanya penyelesaian pertama yang stabil, manakala penyelesaian lain adalah tidak stabil. Analisis berangka pula menunjukkan bahawa nanobendalir hibrid dan nanobendalir hibrid ternari meningkatkan pekali geseran dan nombor Nusselt setempat. Namun begitu, pengenaan sedutan pada permukaan boleh menyebabkan nanobendalir hibrid mempunyai nombor Nusselt setempat yang lebih rendah berbanding nanobendalir biasa. Kenaikan parameter sedutan dapat membantu mengimbangi penurunan prestasi pemindahan haba ini. Manakala, kehadiran sinaran terma dan syarat sempadan olakan juga meningkatkan nombor Nusselt setempat. Selain itu, aliran membantu dalam olakan campuran didapati menghasilkan nilai pekali geseran dan nombor Nusselt setempat yang lebih besar berbanding aliran berlawanan. Kemudian, kaedah tindak balas permukaan (RSM) telah diaplikasikan bagi menentukan kepentingan pelbagai parameter kawalan terhadap nombor Nusselt setempat dan tetapan optimum bagi parameter-parameter tersebut. Secara umum, nilai tertinggi parameter sedutan menghasilkan nombor Nusselt setempat yang maksimum dan meningkatkan kadar pemindahan haba di permukaan.

Kata Kunci: Nanobendalir, penyelesaian berganda, RSM, sedutan
SDG: MATLAMAT 9: Industri, Inovasi dan Infrastuktur

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In the name of Allah, the Most Gracious, the Most Merciful

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfillment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Norihan binti Md Arifin, PhD
Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Fadzilah binti Md Ali, PhD
Senior Lecturer
Faculty of Science
Universiti Putra Malaysia
(Member)

Siti Suzilliana Putri binti Mohamed Isa, PhD
Senior Lecturer
Centre for Foundation Studies in Science
Universiti Putra Malaysia
(Member)

ZALILAH MOHD SHARIFF, PhD
Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 13 January 2025

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LIST OF ABBREVIATIONS

MHD	Magnetohydrodynamics
RSM	Response surface methodology
bvp4c	Boundary value problem with fourth-order
a, c, M_0	Constants
A	Unsteadiness parameter
$A1$	Symbol for suction parameter in RSM
$A2$	Symbol for Biot number in RSM
$A3$	Symbol for radiation parameter in RSM
$A4$	Symbol for wall temperature parameter in RSM
$A5$	Symbol for nanoparticle volume fraction of Cu in RSM
$A6$	Symbol for nanoparticle volume fraction of Al_2O_3 in RSM
$A7$	Symbol for shrinking parameter in RSM
$A8$	Symbol for Eckert number in RSM
$A9$	Symbol for magnetic parameter in RSM
b	Parameter associated with the magnets and electrode width
B_0	Strength of magnetic field
Bi	Biot number
c_0	Intercept term
c_i	Linear term
c_{ii}	Quadratic term
c_{ij}	Two-factor bilinear term
C	Concentration

C^*	Center points
C_{fx}	Local skin friction coefficient
C_p	Heat capacity at constant pressure
d	Degree
d^*	Individual desirability function
D^*	Composite desirability
e_b	Blackbody emissive power
Ec	Eckert number
g	Gravitational acceleration
Gr	Grashof number
Gr_x	Local Grashof number
h	Heat transfer coefficient
h_f	Convective heat transfer coefficient
j_0	Current density applied in the electrodes
k	Thermal conductivity
k^*	Rosseland mean spectral absorption coefficient
L	Characteristic length
L^*	Lower limit
m	Dimension of vector
m^*	Number of response
$\tilde{M}$	Magnetization of permanent magnets
M	Magnetic parameter
n	Shape factor
N	Number of independent variables

Nu_x	Local Nusselt number
p	Pressure
Pr	Prandtl number
q	Width of electrodes
q_w	Surface heat flux
q_r	Radiative heat flux
r^*	Radius
R	Gas constant
R_m	Magnetic Reynolds number
Rd	Radiation parameter
Re	Reynolds number
Re_x	Local Reynolds number
Res	Response
s	Irrotational straining flow strength
s_1	Wall temperature parameter
S	Mass flux velocity parameter
t	Time
T	Temperature
T^*	Target
u, v, w	Velocity components in the $x-$, $y-$, and $z-$directions, respectively
U^*	Upper limit
X	Independent variables
Z	Modified Hartmann number

Greek symbols

α, κ	Constants
α^*	Distance from center points
β	Thermal expansion
δ	Boundary layer thickness
ϵ	Random experimental error
ε	Stretching/shrinking parameter
ρ	Density
μ	Dynamic viscosity
μ_0	Permeability of free space
ν	Kinematic viscosity
Ψ	Sphericity
σ	Electrical conductivity
σ^*	Stefan-Boltzmann constant
λ	Mixed convection parameter
ξ	Magnetic diffusivity
γ	Unknown eigenvalue
ϕ	Nanoparticle volume fraction
Θ	Half angle
τ	Dimensionless time variable
τ_w	Surface shear stress
η	Similarity variable

Subscript

bf	Base fluid

hnf	Hybrid nanofluid
$thnf$	Ternary hybrid nanofluid
nf	Nanofluid
$n1$	First nanoparticle
$n2$	Second nanoparticle
$n3$	Third nanoparticle
w	Condition at the wall or surface of the solid boundary
∞	Free stream condition

Superscript

$'$	Differentiation with respect to η

CHAPTER 1

INTRODUCTION

Fluid dynamics is an academic discipline that focuses on the study of fluid flow. This field of study incorporates applied mathematics, physical principles, and empirical results from experiments to design and solve fluid flow problems.

This chapter will highlight the background of the flow problems studied in this thesis. It will explain the problem statement, objectives, scope, and significance of the studies. Additionally, a concise overview and explanation of some of the terms used throughout this thesis will be given to enhance understanding of the flow problems studied.

1.1 Hybrid nanofluid

Nanofluid is a fluid with nanoparticles suspended in it. This fluid was first proposed by Choi and Eastman (1995) to resolve the drawbacks of conventional heat transfer fluids, such as water, ethylene glycol, toluene, engine oil, alcohol, and refrigerant, which have low thermal conductivity. The swift progress of industry and technology demands more efficient and cost-effective heat transfer fluids. Therefore, Maxwell (1873) suggested integrating solid particles into conventional heat transfer fluids. This approach effectively improved the thermal conductivity of the fluid but also caused sedimentation and clogging of flow passages. Masuda et al. (1993) encountered a similar issue while introducing the dispersion of ultra-fine particles, such as Al_2O_3, SiO_2, and TiO_2, into conventional fluids. Later, in 1995, Choi and Eastman (1995) from Argonne National Laboratory, United States, presented nanofluids composed of nanometer-sized particles with diameters of less than 100 nm, dispersed in a base fluid (i.e., conventional fluid). The nanoparticles have greater surface areas than the millimeter- and micrometer-sized particles used in prior investigations. Thus, using the nanoparticles is more advantageous because heat transfer primarily occurs at the particle surfaces. In addition, nanoparticles remain suspended in base fluids much longer than micrometer-sized particles and experience minimal sedimentation under static conditions (Puliti et al., 2011). Hence, nanofluids are more stable, have a lower probability of erosion, can reduce pumping power, and provide better heat transfer performance.

Nanoparticles can be prepared using various chemical and physical methods such as sol-gel synthesis, hydrothermal, electron beam lithography, solvothermal, coprecipitation, microemulsion, laser pyrolysis, micelle synthesis, flow injection, thermolysis, chemical vapor deposition, sonochemical, microwave-assisted, carbon arc, thermal decomposition, and gas-phase decomposition (Lenin et al., 2021). Appropriate methods can be selected based on the size and shape of nanoparticles required for a particular application. Common types of nanoparticles used in preparing nanofluids include metallic particles, metal oxides, carbon, and ferrites, as shown in Figure 1.1. Meanwhile, the base fluid is usually a non-dielectric liquid (e.g., water (H_2O), engine oil, and ethylene glycol ($C_2H_6O_2$)) or dielectric liquid (e.g., aliphatic liquids, silicone liquids, and fluorocarbons) (Bakthavatchalam et al., 2020).

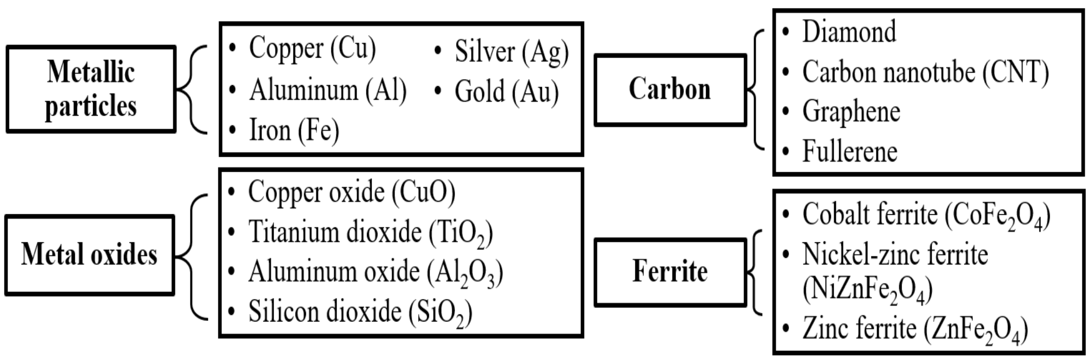


Figure 1.1: Examples of nanoparticles

Nevertheless, dispersing just one type of nanoparticle in a base fluid has disadvantages. Each type of nanoparticle has distinct properties and behavior. For example, metal oxide nanoparticles have low thermal conductivity and are chemically inert, while metallic nanoparticles exhibit high thermal conductivity and are chemically reactive.

Therefore, only one of these qualities, either high thermal conductivity or better stability, can be imparted to a nanofluid, depending on the type of nanoparticles dispersed. The evolution of industrial and engineering applications requires a trade-off between the various properties of nanofluids. The favorable features of different nanoparticles can be incorporated into a single fluid by introducing hybrid nanofluids and ternary hybrid nanofluids. A hybrid nanofluid has two distinct nanoparticles in a base fluid, while a ternary hybrid nanofluid comprises a suspension of three different nanoparticles in a base fluid (see Figure 1.2). For instance, metallic and metal oxide nanoparticles can be dispersed in a base fluid to create a hybrid nanofluid with excellent thermal conductivity and chemical inertness. Therefore, hybrid nanofluids as working fluids offer better thermophysical and rheological properties. Nonetheless, experimental and theoretical studies of hybrid nanofluids are still needed to understand the behavior of these fluids over various geometries and flow conditions.

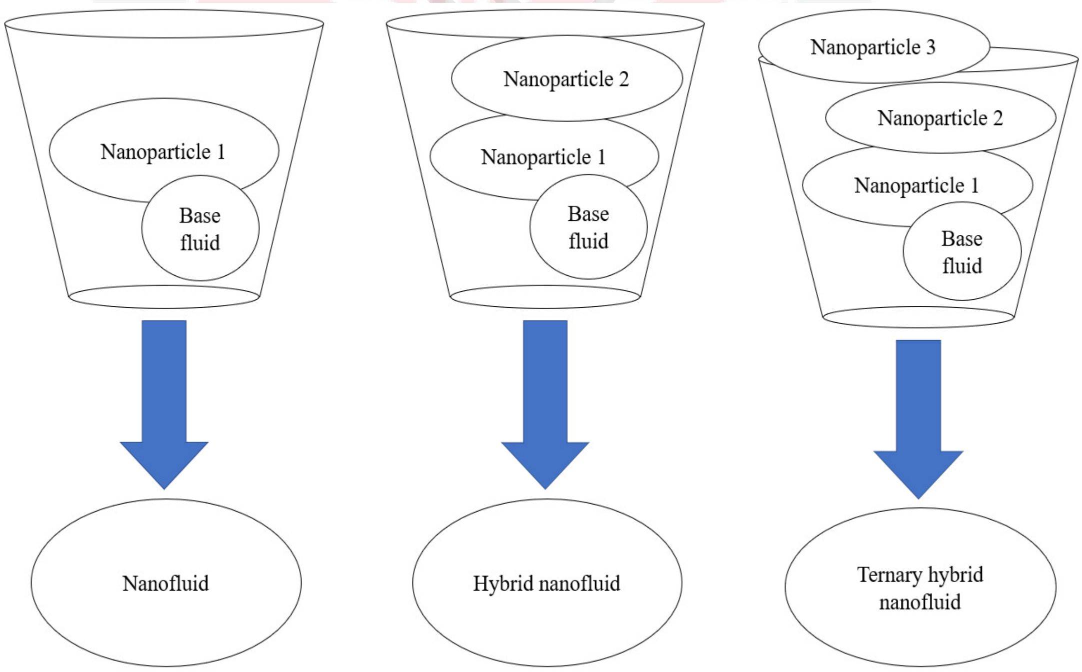


Figure 1.2: The illustration of nanofluid, hybrid nanofluid, and ternary hybrid nanofluid

In preparing nanofluids, hybrid nanofluids, and ternary hybrid nanofluids, one of the significant challenges is the agglomeration of nanoparticles in the base fluid. Nanoparticles agglomerate when the Van der Waals attractive force surpasses the electrostatic repulsive potential. Consequently, this can cause sedimentation, destabilization of nanofluids, and reduced thermal conductivity. Since the improvement of heat transfer characteristics of nanofluids depends on the synthesis and uniform dispersion of nanoparticles, numerous approaches have been developed to minimize nanoparticle agglomeration. Two preparation methods are commonly used: the single-step method and the two-step method.

The single-step method simultaneously synthesizes and disperses nanoparticles in a base fluid. This method avoids intermediate drying, storage, and transportation steps to reduce nanoparticle agglomeration. According to Chakraborty and Panigrahi (2020), the stability of nanofluids produced using the single-step method is better than those produced by the two-step method. Additionally, the single-step method prevents nanoparticle oxidation. However, this method is less economical as it is only compatible with low vapor pressure liquids (e.g., ethylene glycol).

Meanwhile, the two-step method is more economical for large-scale production. In this method, the nanoparticles are first synthesized, or commercially available nanoparticles are used and dispersed in a base fluid. The separation of synthesizing and dispersing processes raises the risk of nanoparticle agglomeration. Hence, techniques such as the addition of surfactants, intensive magnetic force agitation, ultrasonic agitation, homogenizing, ball milling, and high-shear mixing are frequently used to minimize nanoparticle agglomeration and improve the stability of nanofluids (Li et al., 2009b; Yu and Xie, 2012). However, the two-step method has been proven more favorable for oxide nanoparticles and unsuitable for metallic nanoparticles (Bakthavatchalam et al., 2020). Thus, a careful selection of preparation methods must be made to develop stable nanofluids with excellent heat transfer performance for various potential applications, as shown in Figure 1.3.

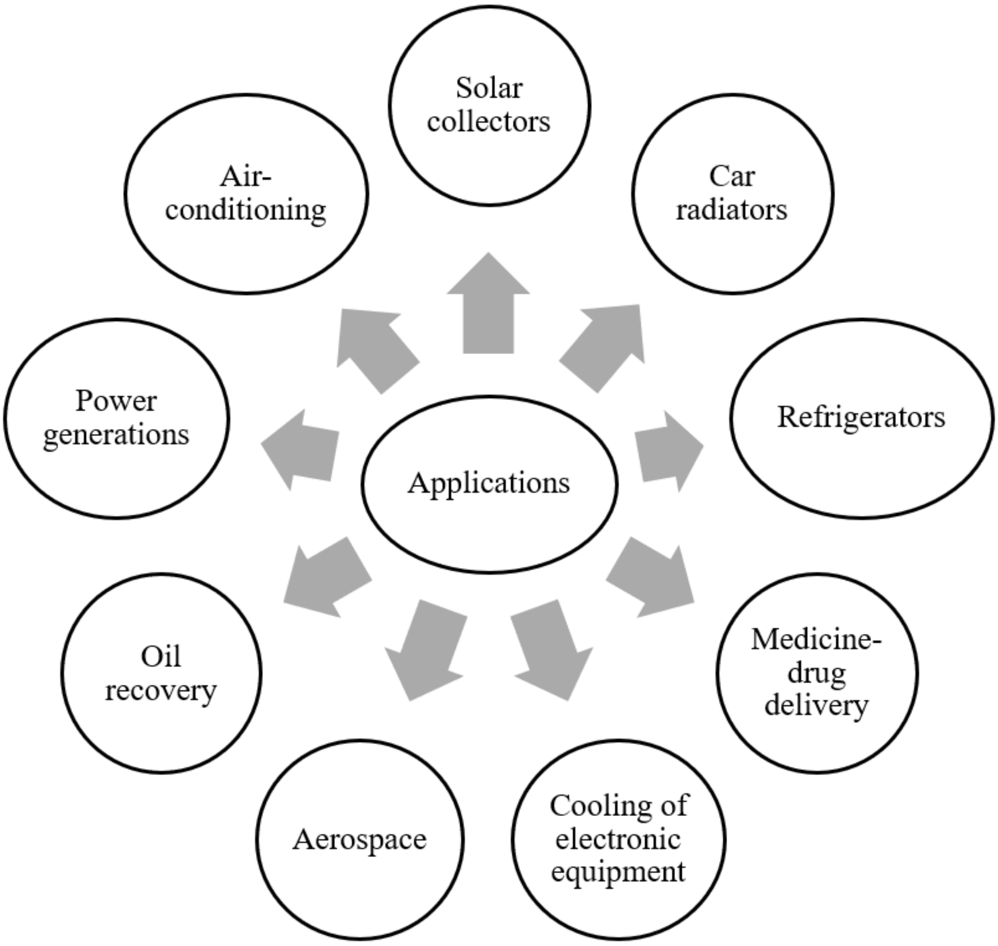


Figure 1.3: Potential applications of nanofluids, hybrid nanofluids, and ternary hybrid nanofluids

1.2 Types of fluid flow

A fluid, whether a liquid or a gas, has no definite shape and conforms to the shape of the container that holds it. Physically, a fluid is a substance that undergoes continuous deformation when subjected to a tangential force known as shear stress. The inability of a fluid to resist even the smallest magnitude of shear stress makes it continuously deform and flow.

1.2.1 Laminar and turbulent flows

Laminar flow, also known as streamline or viscous flow, occurs when a fluid moves smoothly in layers without disruptions such as eddies, swirls, or cross-currents. In laminar flow, fluid particles near a solid surface move in straight lines parallel to the surface. This flow usually occurs when the fluid has a high viscosity that impedes turbulent tendencies or moves slowly through relatively small flow passages. However, the laminar flow may transform into a turbulent flow when the fluid has low viscosity, high velocity, or moves through large passages (Streeter and Wylie, 1975). The transition from laminar to turbulent flow is illustrated in Figure 1.4.

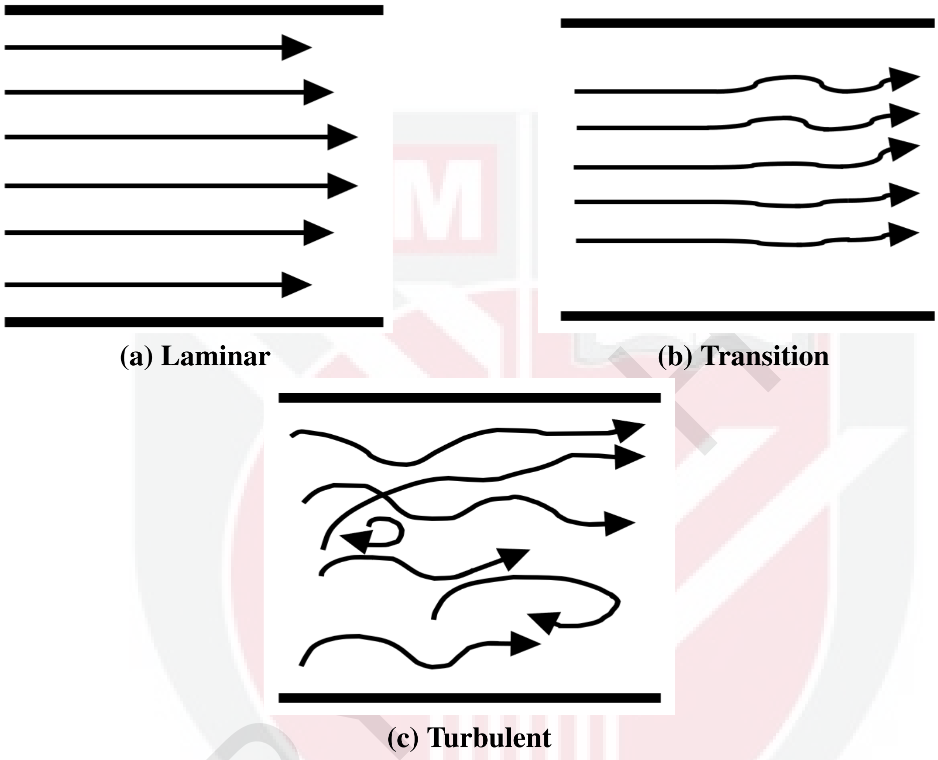


Figure 1.4: Illustration of the transition from laminar to turbulent

Turbulent flow involves the irregular motion of fluid particles with no definite frequency and observable pattern. Common phenomena with turbulent flow include the rise of smoke, waterfalls, blood flow in arteries, lava flow, atmospheric and ocean currents, and vehicle aerodynamics. A fluid in turbulent flow contains high kinetic energy, with its speed fluctuating in both magnitude and direction. Once this energy dissipates, the flow returns to a laminar state.

In 1883, Osborne Reynolds conducted the first experimental study of laminar and turbulent flows (Finnemore and Franzini, 2002). The existence of laminar and turbulent flows, separated by a transition phase, was observed in this study. The transition was assumed to be related to the ratio of inertial and viscous forces. This relationship was proposed by George Gabriel Stokes in 1851 and popularized by Osborne Reynolds to describe the transition phase, ultimately being dubbed the Reynolds number (Re) by Arnold Sommerfeld in 1908 (Rott, 1990). The Reynolds number is a dimensionless quantity that represents the relationship between the inertial and viscous forces:

$$Re = \frac{\rho u L}{\mu} = \frac{uL}{\nu},$$

where ρ is the density of the fluid, u is the flow velocity, L is the characteristic length, μ is the dynamic viscosity, and ν is the kinematic viscosity of the fluid. When the Reynolds number is small, the flow is considered laminar. As the Reynolds number surpasses a certain threshold value, the flow becomes semi-turbulent and enters the transition phase. Beyond this value, the flow becomes completely turbulent. The mean value of the Reynolds number in the transition phase is the critical Reynolds number.

1.2.2 Steady and unsteady flows

A steady flow occurs when all flow properties, such as pressure, velocity, temperature, and density, remain constant over time but may vary from point to point. For example,

$$\frac{\partial v}{\partial t} = 0, \quad \frac{\partial p}{\partial t} = 0,$$

where v is the velocity, t is time, and p is pressure. True steady flow is typically found only in laminar flow. However, a turbulent flow can be considered steady if the average rate of change of all properties remains constant, and this case is known as the mean steady flow.

The steady flow can be further classified as uniform or non-uniform. Uniform flow refers to a condition where the velocity is constant in magnitude and direction at every point in the fluid. Meanwhile, non-uniform flow occurs when the velocity varies at every point in the fluid. Real fluids or fluids flowing near a solid boundary generally exhibit non-uniform flow. Nevertheless, the flow can be considered uniform if the size and shape of the cross-section along the length of the solid boundary remain consistent and the average fluid velocity is constant.

In contrast, unsteady or time-dependent flow occurs when flow properties at a point change with time. Thus,

$$\frac{\partial v}{\partial t} \neq 0, \quad \frac{\partial p}{\partial t} \neq 0.$$

The unsteady flow may arise due to fluctuations in the surrounding fluid or voluntary motions of a body, and it can be observed in various devices such as marine propellers, hydrofoil flutters, rotor blades, and turbomachines (McCroskey, 1977). According to Finnemore and Franzini (2002), unsteady flow is a transient phenomenon that may eventually become steady or cease entirely. Additionally, this flow can include periodic motion such as beach waves, tidal motion, and other oscillations.

Steady, unsteady, uniform, and non-uniform flows can exist independently. Hence, this leads to four possible combinations, each accompanied by an example (Streeter and Wylie, 1975):

1. Steady, uniform flow: Flow through a long pipe at a constant rate.
2. Steady, non-uniform flow: Flow through an expanding tube at a constant rate.
3. Unsteady, uniform flow: Flow through a long pipe at a decreasing rate.
4. Unsteady, non-uniform flow: Flow through an expanding tube at an increasing rate.

1.2.3 Compressible and incompressible flows

Compressible flow is characterized by varying density from point to point ($\rho \neq$ constant). In contrast, incompressible flow refers to flow with a constant density ($\rho =$

constant). Generally, liquids are treated as incompressible, while gases are compressible. However, the compressibility effects in liquids can become substantial under high-pressure conditions. Conversely, the flow of gases with negligible heat transfer can be considered incompressible when the flow speed is low relative to the speed of sound (Fox et al., 2004). Common examples of compressible flow include high-pressure gas transmission in pipelines, compressed air systems in dental drills, and various sensing systems.

1.2.4 Single- and multi-dimensional flows

One-dimensional flow refers to a condition where the flow properties, such as velocity, vary with time and one spatial coordinate, such as x. Variations in flow properties perpendicular to the main flow direction are neglected. According to Streeter and Wylie (1975), the flow through a pipe can be described as one-dimensional. In steady, one-dimensional flow, the flow properties are only a function of one spatial coordinate; hence,

$$u = f(x), \quad v = 0, \quad w = 0,$$

where u, v, and w are velocity components in the $x-$, $y-$, and $z-$ directions, respectively.

Next, two-dimensional flow is characterized by variations in flow properties with time and two spatial coordinates, such as x and y. The fluid particles in a two-dimensional flow are assumed to move on parallel planes along identical routes within each plane, resulting in no changes in flow perpendicular to these planes. The variation of velocity for steady, two-dimensional flow is:

$$u = f(x, y), \quad v = g(x, y), \quad w = 0.$$

Meanwhile, three-dimensional flow is defined by the fluctuations of flow properties with time and three spatial coordinates, such as x, y, and z. The variation of flow properties, for example, velocity, in steady, three-dimensional flow can be represented by:

$$u = f(x, y, z), \quad v = g(x, y, z), \quad w = h(x, y, z).$$

1.2.5 Boundary layer flow

Boundary layer flow describes the fluid flow within a boundary layer. The notion of boundary layer was first introduced by Prandtl in 1904 during the Third International Congress of Mathematicians at Heidelberg (Acheson, 1990). This concept helps explain the relationship between ideal and real fluid flows. An ideal or inviscid fluid has zero viscosity (i.e., no internal resistance) and is incompressible. In contrast, a real or viscous fluid, such as air, kerosene, and honey, has viscosity. Although ideal fluids do not exist in reality, fluids are often modeled as ideal to approximate the behavior of real fluids.

A boundary layer refers to a narrow region of fluid located near a solid boundary, as shown in Figure 1.5. When a real fluid with a velocity of u_e flows past a solid boundary, the fluid particles adhere to the solid surface due to the no-slip condition and the frictional force between the fluid and the solid surface. Consequently, these fluid particles acquire the same velocity as the solid boundary. If the solid boundary is stationary, the attached fluid particles will have zero velocity. The viscosity of the fluid, or its resistance to flow, then causes the deceleration of fluid particles near the solid boundary upon colliding with the stationary fluid particles. As a result, the fluid velocity decreases until some distance from the solid boundary, at which the viscosity effect becomes less prominent. The fluid velocity then approaches the initial velocity in the mainstream (i.e., the free stream velocity) asymptotically. The region where the velocity gradient of the fluid ranges from zero to 99% of the free stream velocity is called the boundary layer. Within this boundary layer, the fluid may exhibit either laminar or turbulent flow. The region beyond the boundary layer, where the effect of viscosity is negligible and the fluid behaves as an ideal fluid, is known as the free stream region.

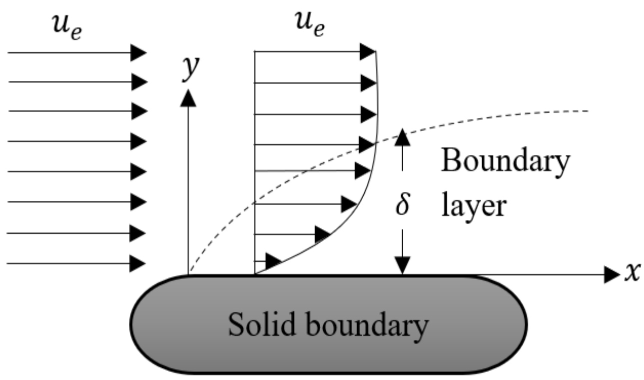


Figure 1.5: Formation of boundary layer

Next, the formation of a thermal boundary layer is depicted in Figure 1.6. In this case, the real fluid is assumed to have an initial temperature of T_∞ and flows past a solid boundary with a temperature of $T_w > T_\infty$. Due to the temperature difference, heat transmission occurs between the solid surface and the surrounding fluid, leading to a gradual increase in the temperature of the fluid layers. However, the amount of heat transferred diminishes with increasing distance from the solid surface. As a result, the fluid temperature remains constant after some distance from the solid boundary and equals the free stream temperature (i.e., T_∞). Therefore, the region between the solid surface and the point at which the fluid reaches 99% of the free stream temperature is known as the thermal boundary layer.

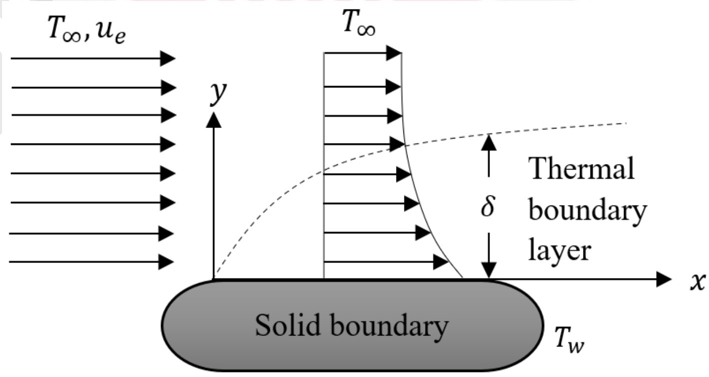


Figure 1.6: Formation of thermal boundary layer

Most heat and mass transfers occur within the boundary layer. The thickness of the boundary layer (δ) influences the heat and mass transfer rates between the solid boundary and the surrounding fluid. A thinner boundary layer enhances heat and mass transfer rates. According to Schlichting and Gersten (2017), fluids with a high Reynolds number or low viscosity produce a thinner boundary layer.

The stability of a boundary layer, the transition point from laminar to turbulent, and the separation point are mainly influenced by the adverse pressure gradient. As the pressure increases while moving downstream, it acts against the flow direction and produces a retarding force towards the fluid flow. Due to viscosity, fluid elements lose momentum or kinetic energy to overcome this adverse pressure gradient. Consequently, the fluid velocity near the solid surface decreases, and the boundary layer thickens. As the velocity gradient approaches zero, the boundary layer is forced to detach or separate from the solid surface, causing a reverse flow (see Figure 1.7). Generally, boundary layer separation begins at the point on the solid surface where the flow is strongly pushed back, causing the velocity gradient in the $y-$direction $\left(\left(\frac{\partial u}{\partial y}\right)_w\right)$ and the wall shear stress (τ_w) to become zero:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y}\right)_w = 0.$$

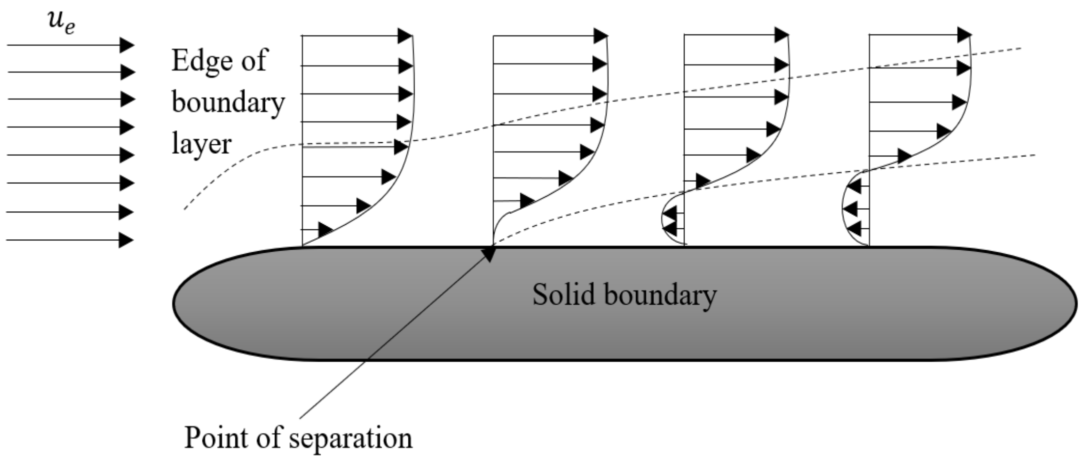


Figure 1.7: Boundary layer separation

1.2.6 Stagnation-point flow

Stagnation point flow refers to the fluid flow near a stagnation point. The stagnation point exists on the surface of a solid boundary and denotes the point at which the stream of a fluid attaches to and separates from the solid boundary. As stated by Wang (2008), the stagnation region exhibits the highest pressure, heat transmission, and mass deposition rates. The depiction of the stagnation-point flow is presented in Figure 1.8. When a real fluid with a velocity of u_e encounters a stationary solid boundary and strikes orthogonally at a stagnation point, the fluid stream halts, reducing its velocity to zero. The fluid then separates along the stagnation streamline and flows towards the upper and lower regions of the solid boundary. Understanding the stagnation-point flow is beneficial for designing thrust bearings and radial diffusers, reducing drag, optimizing transpiration cooling, and enhancing thermal oil recovery (Merkin et al., 2022).

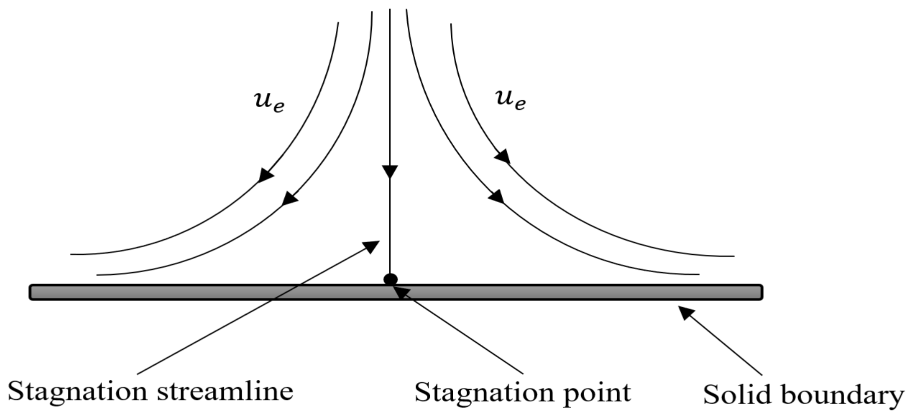


Figure 1.8: Formation of stagnation point flow

In cases where the fluid strikes the stagnation point at an arbitrary angle, the flow is known as the oblique stagnation-point flow (see Figure 1.9). According to Tamada (1979), the oblique stagnation-point flow comprises two components: an irrotational stagnation-point flow perpendicular to the solid surface and a shear flow parallel to the surface. Typically, this flow emerges during the enhancement of heat and mass transfers using fluid jets. The fluid may impact a surface obliquely due to the geometric configuration of the solid boundary or the blockage of the nozzle (Wang, 1985). Besides that, the oblique stagnation-point flow can occur during the reattachment of viscous flow to a solid boundary (Tamada, 1979). Furthermore, fluid flow can stagnate at the interface of two immiscible fluids (Tilley and Weidman, 1998).

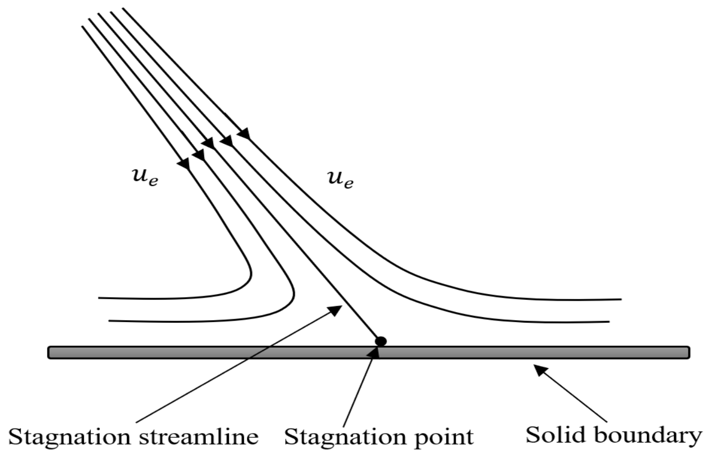


Figure 1.9: Formation of oblique stagnation-point flow

1.3 Types of effects

Several phenomena, such as thermal radiation, mixed convection, viscous dissipation, and magnetic field, can accompany fluid flow. These phenomena can be treated as effects within the fluid flow. Mathematical descriptions of these effects can be achieved by incorporating specific terms into the governing equations or boundary conditions of the flow problem.

1.3.1 Thermal radiation

Thermal radiation is a mode of heat transfer involving the emission of electromagnetic radiation from a heated object in all directions through an unoccupied gap. This emission arises from the molecular and atomic agitation generated by the internal energy of matter (Siegel and Howell, 1992). All matter at temperatures above absolute zero emits electromagnetic radiation. Thus, thermal radiation is prevalent in most practical situations and applications.

The applications of thermal radiation range from heat transfer in furnaces and combustion chambers to energy transmission from nuclear explosions. According to Siegel and Howell (1992), thermal radiation becomes more prominent in conditions with large temperature differences. Several space exploration devices, such as rocket nozzles, nuclear power plants for space applications, and gaseous-core nuclear rockets, are specifically engineered to function at high temperatures for optimal thermal efficiency (Siegel and Howell, 1992). Therefore, thermal radiation must be considered when calculating the thermal effects of these devices. Additionally, thermal radiation can transfer heat through a vacuum without requiring a medium. Other applications of thermal radiation include astrophysical flows, electric power generation, and cooling of nuclear reactors (Kejela et al., 2021). In some instances, thermal radiation modifies temperature distributions and affects conduction, free convection, or forced convection (Siegel and Howell, 1992). For example, thermal radiation in the boundary layer flow of gases can influence convective heat transfer. Therefore, it is important to consider radiative heat transfer in various flow problems.

However, studying radiative heat transfer between a solid surface and a fluid faces two significant challenges (Aboeldahab and El Gendy, 2002). The first difficulty is predicting the absorption of radiation by the fluid. In a system containing radiating fluid, radiation is absorbed and emitted not only at the boundary but also within the interior of the system. Hence, it makes prediction a difficult task. Secondly, the absorption coefficients of absorbing-emitting fluids are highly influenced by wavelength. Consequently, computing radiative flux will require complex integration with respect to wavelength and other independent variables.

Several researchers have developed simplified models with lower computational costs but remain adequately accurate for practical applications. One of these simplifications is the Rosseland approximation, where the net radiative heat flux (q_r) is expressed as follows (Rosseland, 1931; Magyari and Pantokratoras, 2011b):

$$q_r = -\frac{4}{3k^*}\nabla(e_b), \tag{1.1}$$

with k^* as the Rosseland mean spectral absorption coefficient. This approximation applies to an optically thick medium where radiation exchange occurs only between neighboring volume elements (Kataria and Mittal, 2015). Hence, the thermal radiation in an optically thick medium can be described as a diffusion process. According to Rohsenow et al. (1998), the calculation of thermal radiation at the macroscopic level is based on the Stefan-Boltzmann law, which relates the energy flux emitted by a blackbody to the fourth power of the absolute temperature:

$$e_b = \sigma^* T^4,$$

with e_b as the blackbody emissive power, T is the absolute temperature, and $\sigma^* = 5.669 \cdot 10^{-8}$ $\mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant. A blackbody is an ideal surface that absorbs all incident radiation, regardless of wavelength and direction, and is also an ideal emitter with radiation emitted independent of direction. The blackbody emissive power is the rate at which energy is released per unit area.

For a plane boundary layer flow over a hot surface, Equation (1.1) is simplified to the following form:

$$q_r = -\frac{16\sigma^*}{3k^*} T^3 \frac{dT}{dy}, \tag{1.2}$$

where y represents the coordinate of the region perpendicular to the surface. The Rosseland approximation in Equation (1.2) is sometimes called the diffusion approximation for the density of a radiation flux (Shvydkii et al., 2018). This approximation has been used to model processes such as glass cooling, ceramic manufacturing, viscous electrically conducting incompressible flows, and convection–radiation heat transfer (Malek et al., 2021).

1.3.2 Mixed convection

Convection is a heat transfer process in which heat is transferred from one region to another through the movement of fluids. Free or natural convection arises when fluid movement is generated by buoyancy forces resulting from differences in density. These differences may occur due to gradients in temperature, concentration, or composition (Rohsenow et al., 1998). For instance, when a fluid is heated from below, the hot fluid molecules become less dense and rise, while the denser cold fluid molecules sink, resulting in the bulk movement of fluid and free convection. The Grashof number (Gr) is a dimensionless number related to free convection heat transfer. It describes the relationship between buoyancy forces that facilitate free convection and the resisting viscous forces (Rohsenow et al., 1998):

$$Gr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2},$$

where β is the thermal expansion coefficient and g is the gravitational acceleration. Here, the term $g\beta(T_w - T_\infty)$ represents the buoyancy force. The convective mass transfer can be represented by $g\beta(C_w - C_\infty)$ where C_w is the concentration at the solid surface and C_∞ is the free stream concentration. In this case, the natural convection occurs due to concentration gradients.

Meanwhile, forced convection arises when an external source, such as a fan, pump, or compressor, drives fluid movement. For example, a small fan is installed in the chassis to dissipate heat from electronic components in a computer. Generally, forced convection can be expressed using Newton's Law of Cooling:

$$\frac{dT}{dt} = q_{\text{conv}} = h(T_w - T_\infty).$$

Newton's Law of Cooling states that the rate of heat exchange between a solid boundary and its surroundings (q_{conv}) is proportional to the difference in temperature between the solid surface and the surroundings. The constant of proportionality in this relationship is known as the heat transfer coefficient (h). It is significantly influenced by fluid properties, the shape and roughness of the solid surface, and the type of fluid flow (i.e., laminar or turbulent). In the case of a no-slip boundary condition, heat transfer between the solid surface and the adjacent stationary fluid layer occurs purely by conduction.

Hence,

$$
\begin{aligned}
q_{\text{cond}} = -k_{\text{fluid}} \frac{\partial T}{\partial y}\bigg|_{y=0} &= q_{\text{conv}}, \\
-k_{\text{fluid}} \frac{\partial T}{\partial y}\bigg|_{y=0} &= h(T_w - T_\infty),
\end{aligned} \tag{1.3}
$$

where k is the thermal conductivity of the fluid. Equation (1.3) is defined at the boundary condition and named the convective boundary condition. The convective boundary condition is useful for various engineering and industrial applications, including transpiration cooling, material drying, and laser pulse heating (Ramreddy et al., 2015).

Mixed convection describes the combination of free and forced convection. Meanwhile, mixed convection flow is characterized by the significant influence of forced flow on free convection or the buoyancy force in forced convection. This flow type becomes prominent when there is a substantial difference in temperature and/or a low forced flow velocity (Bachok et al., 2013). Various applications of mixed convection flows include electronic devices, nuclear reactors, solar collectors, and heat exchangers (Bachok et al., 2013). Mixed convection flow can occur in either assisting or opposing flows. According to Joye and Wojnovich (1996), the buoyancy force in free convection is a gravity effect that consistently acts in a vertical direction, whereas the direction of the forced flow is random. When the buoyancy and forced motions are in the same direction, the flow is referred to as assisting or aiding. In this case, free convection complements forced convection and enhances heat transfer. In contrast, when buoyancy and forced motions are in opposite directions, the flow is termed the opposing flow. Here, the free convection resists the forced convection and diminishes heat transfer.

In solving the laminar, two-dimensional mixed convection flow over a flat surface, the following dimensionless mixed convection parameter (λ) is encountered:

$$
\lambda = \frac{Gr_x}{Re_x^2}.
$$

Here, Gr_x is the local Grashof number and Re_x is the local Reynolds number. The mixed convection parameter represents the ratio of buoyancy forces to inertial forces in the boundary layer. Hence, it provides a measure of the influence of free convection on fluid flow compared to forced convection. Different values of λ denote different situations, such that (Pop and Ingham, 2001):

- When $\lambda = \frac{Gr_x}{Re_x^2} \to 0$, forced convection is more dominant than free convection.
- When $\lambda = \frac{Gr_x}{Re_x^2} \to \infty$, free convection is more dominant than forced convection.

1.3.3 Viscous dissipation

Viscous dissipation, or frictional heating, is an irreversible process in which the kinetic energy of a fluid is converted into thermal energy due to the work done against viscous forces. This dissipation acts as an energy source and becomes significant in high-velocity fluid flows, highly viscous flows, flows through microchannels, and in fluids with a moderate Prandtl number and moderate velocities with low wall heat fluxes (Morini, 2008). According to Desale and Pradhan (2015), the impact of high-velocity flow on heat transfer can be observed in various practical applications, such as heat transfer around gas turbine blades and rocket engines.

Generally, viscous dissipation is characterized by the dimensionless Eckert number (Ec), which expresses the ratio of kinetic energy to the boundary layer enthalpy difference (Desale and Pradhan, 2015):

$$Ec = \frac{u^2}{C_p \Delta T}.$$

Here, C_p is the heat capacity at constant pressure and ΔT is the temperature difference between the solid surface and the surroundings. If viscous dissipation is neglected in the fluid flow, the Eckert number is zero.

1.3.4 Magnetohydrodynamics (MHD)

Magnetohydrodynamics (MHD) , also known as magnetofluid dynamics or hydromagnetics, is the study of the flow of an electrically conducting fluid in the presence of a magnetic field (Roberts, 1967). Electrically conducting fluids include plasmas, liquid metals, and salt water (electrolytes). Research on MHD flow was initiated in the late 1930s or early 1940s. The discovery of the Alfvén wave by Hannes Alfvén in 1942, which later received the Nobel Prize in 1970, played a crucial role in developing magnetohydrodynamics. This discovery demonstrated the ability of electromagnetic waves to travel through conducting fluids (Cramer, 2001). Applications of MHD include thermonuclear fusion, electromagnetic pumps, and electromagnetic stirring.

The fundamental concept of MHD is that magnetic fields can generate electric currents within a flowing conducting fluid, creating forces on the fluid and modifying the magnetic field itself (Sheikholeslami and Ganji, 2016). In MHD, the flow of a conducting fluid across a magnetic field generates a potential difference, which induces the flow of electric currents. These induced electric currents give rise to a secondary magnetic field called the induced magnetic field (Davidson, 2001). The induced magnetic field counteracts the externally applied magnetic field, effectively preventing the magnetic field lines from entering the conducting fluid. Conversely, when the magnetic field enters the conducting fluid, the induced magnetic field strengthens the applied magnetic field (Sheikholeslami and Ganji, 2016). Consequently, this leads to the fluid appearing to 'drag' the magnetic field lines along with it. Additionally, the interaction generates a Lorentz force that affects the relative movement of the magnetic field and the fluid.

Meanwhile, the magnetic Reynolds number (R_m) is a significant dimensionless parameter in MHD. It can be defined as follows:

$$R_m = \frac{uL}{\xi} = uL\mu_0\sigma,$$

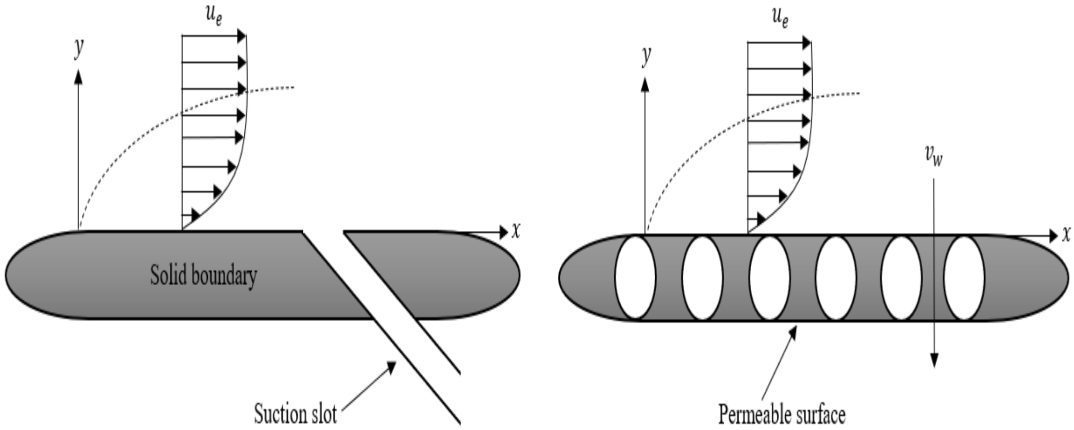
where $\xi = \frac{1}{\mu_0 \sigma}$ is the magnetic diffusivity with μ_0 as the permeability of free space and σ as the electrical conductivity. The magnetic Reynolds number estimates the relative effects of magnetic induction on magnetic diffusion. When R_m is very large, the diffusion of the magnetic field can be neglected. The magnetic field lines become 'frozen' into the conductive fluid and move along with the fluid. In contrast, when R_m is very small, the influence of magnetic induction can be neglected as magnetic diffusion dominates.

1.3.5 Permeable surface

A permeable surface allows fluids to penetrate or pass through it. Fluid flow over a permeable surface is frequently observed in various engineering applications, such as the design of thrust bearings, radial diffusers, thermal oil recovery systems, food processing, cooling of turbine blades and nuclear reactors, electronic cooling, filtration process, and extraction of geothermal energy (Hajmohammadi et al., 2015). When analyzing boundary layer flow, the term "permeable surface" is synonymous with the suction and injection of fluids.

Suction and injection were first introduced by Prandtl in 1904 as methods to stabilize laminar flow against disturbances and to prevent or delay boundary layer separation (Preston, 1946; Halima et al., 2023). Suction involves the removal of fluid from a system, while injection supplies fluid into a system. The injection or blowing of high-energy fluid particles into the boundary layer can delay boundary layer separation. Meanwhile, suction removes the low-energy part of the boundary layer (i.e., the portion with the lowest velocity) for a more filled velocity profile that reduces the thickening of the boundary layer.

As illustrated in Figure 1.10, two different kinds of boundary layer suction are commonly applied: slot suction and continuous suction (Delery, 1985). In slot suction, the boundary layer is removed through several slots. Meanwhile, continuous suction assumes that the wall or solid surface is permeable. Hence, the vertical velocity at the surface boundary condition is not zero and can have either a negative or positive value to indicate the presence of suction or injection effects, respectively.



(a) **(b)**

Figure 1.10: Illustration of (a) slot suction and (b) continuous suction

1.4 Stability analysis

Stability analysis is a technique used to assess the stability of solutions to a given problem. A boundary value problem, representing a fluid flow, may possess no solution, a single solution, or multiple solutions. When multiple solutions exist, stability analysis is conducted to determine the stability and significance of each solution. Unlike unstable solutions, stable solutions are physically relevant and realizable in practice.

Stability analysis was first performed by Wilks and Bramley (1981). In this investigation, dual solutions were obtained and categorized as upper-branch and lower-branch solutions. Stability analysis was then conducted by treating the problem as unsteady or time-dependent. A linear eigenvalue problem with γ as the unknown eigenvalue was introduced, and the smallest value of γ was determined through numerical computation. The findings indicated that the upper-branch and lower-branch solutions had positive and negative values of γ, respectively. It was concluded that the upper-branch solutions were stable with initial decay of disturbance, while the lower-branch solutions were unstable due to initial growth of disturbance.

Thus, conducting stability analysis is crucial, particularly for flow problems with multiple solutions. This approach helps determine the stability of solutions, which is essential for addressing flow problems effectively.

1.5 Response surface methodology (RSM)

Response surface methodology (RSM), or the Box-Wilson methodology, is an experimental design-based approach incorporating statistical and mathematical techniques for developing, improving, and optimizing processes (Box and Wilson, 1992). According to Myers et al. (2009), the RSM is particularly significant when multiple input variables or controlling parameters can impact a performance measure or process. This statistical analysis offers several advantages of reducing experimental cost, minimizing variability around a target when bringing the performance value to the target value, and ensuring the optimal conditions discovered through simulation can be replicated in actual applications (Han et al., 2015). In subsequent paragraphs, the performance measure is referred to as the response, while the input variables are the independent parameters. The relationship between the response and the independent parameters can be described by a low-degree polynomial model (Khuri and Mukhopadhyay, 2010):

$$Res = \boldsymbol{f}(\boldsymbol{X})^{\boldsymbol{T}}\boldsymbol{c} + \epsilon,$$

where Res is the response, $\boldsymbol{X}^{\boldsymbol{T}} = \begin{bmatrix} X_1 & X_2 & X_3 & \dots & X_N \end{bmatrix}$ is the independent parameters, N is the number of independent parameters, $\boldsymbol{f}(\boldsymbol{X})$ is a vector function of m elements that consists of powers and cross-products of powers of $X_1, X_2, X_3, \dots X_N$ up to a certain degree of d, $\boldsymbol{c}$ is a vector of m unknown constant coefficients, and ϵ is a random experimental error assumed to have a zero mean. When $d = 2$, the dimension of the vectors become $m = 1+2N+[N(N-1)]/2$. Meanwhile, $\boldsymbol{f}(\boldsymbol{X})^{\boldsymbol{T}} = \begin{bmatrix} 1 & X_1 & X_2 & \dots & X_N & X_1^2 & X_2^2 & \dots & X_N^2 & X_1X_2 & X_1X_3 & \dots & X_{N-1}X_N \end{bmatrix}$ and

$\boldsymbol{c}^{\boldsymbol{T}} = \begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_N & c_{11} & c_{22} & \dots & c_{NN} & c_{12} & c_{13} & \dots & c_{N-1,N} \end{bmatrix}$. Thus, the following second-order model is obtained (Myers et al., 2009):

$$Res = c_0 + \sum_{i=1}^{N} c_i X_i + \sum_{i=1}^{N} c_{ii} X_i^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij} X_i X_j, \quad (1.4)$$

where c_0 is the intercept term, and c_i, c_{ii}, and c_{ij} are the linear, quadratic, and two-factor bilinear terms, respectively. This model can predict the response values for given settings of independent parameters (controlling parameters). Besides that, the significance of the independent parameters can be determined through hypothesis testing, and the optimum settings of the independent parameters that produce the maximum or minimum response within a specific region of interest can be identified.

In response surface methodology, a series of experiments with an appropriate experimental design must be carried out to measure the response values for specific configurations of the independent parameters. One of the most popular designs for fitting the second-order model is the central composite design (CCD). In this design, the number of experimental runs can be calculated using the formula of $2^N + 2N + C^*$ where 2^N is the factorial points, $2N$ is the axial points, and C^* is the center points. The factorial points are significant for estimating linear and interaction terms (i.e., the two-factor bilinear term). Generally, these points are the vertices of an N-dimensional cube representing the experimental domain, with each vertex indicating a specific combination of the high (1) and low (-1) levels of the independent parameters. Then, the axial points are located at the center of each face of the cube, at a distance α^* from the center point, and are significant for estimating the quadratic terms. Meanwhile, the center points are the replicates at the center of the cube that provide an internal estimate of error and contribute to the estimation of the quadratic terms. In general, statistical analysis utilizing RSM involves the following steps:

1. Designation of independent parameters and their ranges.

2. Selection of experimental design.

3. Prediction and validation of model equation.

4. Determination of optimal point.

Each step will be explained in the Methodology section.

In fluid dynamics, RSM is commonly employed to predict the relationship between a system's input and output variables and to optimize the system. Several researchers have utilized the RSM to enhance the significance and novelty of numerical investigations. Integrating numerical and statistical investigations provides a better understanding of the results obtained. In addition, most studies of heating or cooling problems aim to identify optimal settings for improved heat transfer performance. Hence, the RSM can be used for heat transfer optimization, where the local Nusselt number serves as the response. The RSM is executed using a powerful statistical software called Minitab. A comprehensive theoretical explanation of the RSM methodology has been put forth by Myers et al. (2009) and Khuri and Mukhopadhyay (2010).

1.6 Problem statement

The idea of hybridizing different nanoparticles arises from the attempt to find efficient working fluids for heat transfer applications. Nanofluids were initially proposed as potential substitutes for conventional heat transfer fluids. Various theoretical and experimental studies have demonstrated that nanofluids possess better thermophysical properties than conventional fluids. Subsequently, a more advanced type of nanofluid, termed hybrid nanofluid, was developed and is believed to have better rheological and thermophysical properties than nanofluids and conventional fluids. Then, the successful dispersion of three dissimilar nanoparticles in a conventional fluid led to the synthesis of ternary hybrid nanofluids. These newly introduced heat transfer fluids are expected to outperform conventional fluids in various applications. However, comprehensive studies are necessary to investigate the flow and thermal behaviors of hybrid nanofluids in various geometries and conditions, as well as the mixing ratio, nanoparticle combinations, stability, and mechanisms contributing to enhanced heat transfer performance.

Nevertheless, most fluid flow problems are too complex and costly to investigate experimentally. Therefore, theoretical or numerical investigations provide a faster and more economical alternative by translating flow problems into mathematical formulations. These problems can be analyzed and solved using various numerical methods and built-in solvers in mathematical software.

In theoretical investigations, various hybrid nanofluid flow problems can be mathematically described using governing partial differential equations and boundary conditions. Different geometries (e.g., cone, sheet, and wedge) and conditions (e.g., thermal radiation, viscous dissipation, suction, magnetic field, and convective boundary condition) are introduced as effects within the fluid flow by adding related terms to the governing equations and boundary conditions. Each effect becomes a controlling parameter with a distinct correlation to the flow and heat transfer characteristics. However, the flow configurations explored by researchers may be limited to certain combinations of these effects; for example, a hybrid nanofluid flow over a cone is considered with the effects of magnetic field and suction, while other effects may be overlooked or excluded for future research. The present study aims to examine the neglected effects on different fluid flow problems, intending to address existing literature gaps and improve previous studies.

Additionally, the numerical computation of the governing equations and boundary conditions may yield multiple solutions. However, it has been noted that some studies discussed multiple solutions without thoroughly analyzing the stability and significance of each solution. Hence, it can be challenging to verify the realizability of the solutions. Therefore, stability analysis is important to assess the stability and significance of solutions.

Since hybrid nanofluids have been developed to serve as superior heat transfer fluids (heating or cooling fluids), it is important to examine the relationship between the controlling parameters and their role in influencing heat transfer performance. The heat transfer performance can be measured using the local Nusselt number related to the heat transfer rate at the solid surface. However, more than numerical investigations are required to elucidate the relationship between controlling parameters and the local Nusselt number. This relationship can be better visualized and described through statistical investigation using RSM. Additionally, the RSM can be utilized to determine the optimal conditions for achieving maximum heat transfer rate.

Accordingly, the research questions related to the problem statement are as follows:

- What are the effects of controlling parameters on the flow and heat transfer of hybrid nanofluid in different flow geometries and conditions?
- Do the studied flow problems yield multiple solutions, and among these solutions, which are stable and significant for real-world applications?
- Which controlling parameters are significant to the heat transfer performance of hybrid nanofluid in the studied flow problems?
- What is the optimal setting for the controlling parameters to achieve the maximum heat transfer rate in the studied flow problems?

1.7 Objectives

The main objectives are to

i. formulate the mathematical model of the following flow problems:

   (a) Unsteady mixed convection hybrid nanofluid flow past a permeable Riga plate with thermal radiation and convective boundary condition.

   (b) Mixed convection hybrid nanofluid flow past a permeable non-isothermal cone and wedge with thermal radiation and convective boundary condition.

   (c) Hybrid nanofluid flow past a permeable biaxial stretching/shrinking surface with thermal radiation effect.

(d) Oblique stagnation-point flow of hybrid nanofluid towards a shrinking surface with suction.

(e) MHD stagnation-point flow of ternary hybrid nanofluid over a permeable radially shrinking disk with thermal radiation, viscous dissipation, and convective boundary condition.

ii. conduct stability analysis on multiple solutions in determining the stable and significant solution.

iii. analyze the effects of various controlling parameters on thermal and rheological behaviors of hybrid nanofluid.

iv. utilize the response surface methodology (RSM) in identifying the significant controlling parameters influencing the Nusselt number and the optimal conditions for maximum heat transfer rate.

1.8 Scope

The scope of the studies presented in this thesis is limited to the laminar, incompressible, two-dimensional, and three-dimensional boundary layer flow of hybrid nanofluid and ternary hybrid nanofluid. All flow problems are modeled based on the single-phase nanofluid model proposed by Tiwari and Das (2007). Various geometries are considered, including sheet, cone, wedge, Riga plate, and disk with permeable surfaces to allow suction. Additionally, a no-slip condition is assumed on the solid surface.

The thesis focuses on two types of working fluids: Al_2O_3-Cu/water hybrid nanofluid and Al_2O_3-TiO_2-Cu/water ternary hybrid nanofluid. Water is chosen as the base fluid due to its widespread availability, excellent dispersion stability for nanoparticles, high heat capacity, and low viscosity (Panduro et al., 2022). Meanwhile, the combination of nanoparticles is selected based on the complementary qualities of each nanoparticle. Each nanoparticle has the following qualities (Ukueje et al., 2022; Mohammed Zayan et al., 2023):

- Cu: Offers high thermal conductivity and corrosion resistance, making it an ideal metal for various heat transfer applications, but lacks chemical inertness.
- Al_2O_3: Exhibits good chemical inertness, is not susceptible to surface oxidation, is simpler to combine into liquid due to its hydrophilic surface properties, is a low-cost metal oxide that allows for large-scale production, is economically accessible, but possesses low thermal conductivity.
- TiO_2: Adapts well to high-pressure applications with varying concentrations and is widely used as a thermal conductivity enhancer in various applications, such as refrigerant, conduction enhancers, convective heat transfer, and antifogging coatings.

Incorporating a small amount of Cu nanoparticles with Al_2O_3 and TiO_2 nanoparticles can significantly improve the thermal properties. Furthermore, Al_2O_3-Cu/water hybrid nanofluid and Al_2O_3-TiO_2-Cu/water ternary hybrid nanofluid are commonly used in numerical investigations (see Chapter 2). Various experimental studies on these hybrid nanofluids have also been extensively discussed by Suresh et al. (2011), Suresh et al. (2012), Selvakumar and Suresh (2012), Siddiqui et al. (2019), Çolak et al. (2020), Ma et al. (2021), Xuan et al. (2021), and Marulasiddeshi et al. (2022). Hence, these studies affirm that the chosen fluids are feasible in real-life applications and can be employed for numerical investigations.

Additionally, it is assumed that the nanoparticles are spherical in shape, have a uniform size, and are in thermal equilibrium. The base fluid and the suspended nanoparticles are also in thermal equilibrium. Meanwhile, the correlations for thermophysical properties (e.g., density, viscosity, heat capacity, thermal conductivity, and electrical conductivity) of hybrid nanofluid and ternary hybrid nanofluid are based on Takabi et al. (2016) and Jakeer et al. (2023), respectively. For the response surface methodology (RSM), the experimental design is based on the face-centered central composite design.

1.9 Significance of the study

Research on the flow and thermal behaviors of hybrid nanofluids in various flow geometries and conditions is necessary due to the rapid progress in technology and industries. The present research involves formulating, solving, analyzing, and discussing mathematical models for various flow problems that may arise in real-life applications. Mathematical simplifications of these flow problems provide essential insights into the underlying flow processes before addressing the corresponding actual flow phenomena.

Furthermore, other researchers have not yet investigated the flow problems discussed in this thesis. Therefore, valuable information on the behavior of hybrid nanofluid and ternary hybrid nanofluid in different geometries and conditions can be gained and shared for future research. The implementation of stability analysis in these studies helps assess the practical applicability of the numerical solutions and facilitates the analysis and discussion of the results. The behavior of hybrid nanofluid and ternary hybrid nanofluid in the studied flow problems can be approximated using the results of the stable solution. Thus, this can assist other researchers in predicting the anticipated outcome if the studies are extended or replicated experimentally for real-life applications.

Meanwhile, incorporating statistical investigation into the current studies will provide a better understanding of the relationship between different controlling parameters and the local Nusselt number. This approach can also identify the significant parameters and the optimal settings for maximizing the local Nusselt number. Hence, valuable insights can be obtained regarding the heat transfer performance of hybrid nanofluid and ternary hybrid nanofluid in specific flow problems.

1.10 Thesis outline

This thesis comprises a total of nine chapters. Chapter 1 provides a brief introduction to the terminology employed in this thesis. It includes a historical description and definition of certain types of fluid, fluid flow, permeable surface, types of effects, stability analysis, and response surface methodology. At the end of this chapter, the problem statement, objectives, scope, and significance of the study are presented to help readers gain a better understanding of the thesis.

Meanwhile, Chapter 2 consists of literature reviews related to the studied problems. This chapter summarizes and discusses past studies conducted by other researchers on nanofluids, hybrid nanofluids, and ternary hybrid nanofluids. Then, Chapter 3 explains the general mathematical formulation of the studied problems and the numerical methods used to solve these problems. Additionally, this chapter describes the steps involved in stability analysis and response surface methodology.

Next, Chapters 4 to 8 present an elaborate discussion of the five flow problems listed in the objectives. Chapter 4 discusses the unsteady mixed convection hybrid nanofluid flow past a permeable Riga plate with radiation and convective boundary condition. Chapter 5 examines the mixed convection hybrid nanofluid flow past a non-isothermal cone and wedge with radiation and convective boundary condition. Then, Chapter 6 addresses the hybrid nanofluid flow past a biaxial stretching/shrinking permeable surface with radiation effect. Chapter 7 analyzes the oblique stagnation-point flow of hybrid nanofluid towards a permeable shrinking surface. Finally, Chapter 8 scrutinizes the MHD stagnation-point flow of ternary hybrid nanofluid over a permeable radially shrinking disk. These chapters generally comprise an introduction section, followed by sections for problem formulation, stability analysis, response surface methodology, results and discussion, and conclusions. The thesis concludes with Chapter 9, which includes recommendations for possible future work.

REFERENCES

Aamar Abbasi, W. F. and Mabood, F. (2022). Mixed convective flow and heat transfer of hybrid nanofluid impinging obliquely on a vertical cylinder. *International Journal of Ambient Energy*, 43(1):4343–4355.

Aboeldahab, E. M. and El Gendy, M. S. (2002). Radiation effect on mhd free-convective flow of a gas past a semi-infinite vertical plate with variable thermophysical properties for high-temperature differences. *Canadian Journal of Physics*, 80(12):1609–1619.

Acheson, D. J. (1990). *Elementary Fluid Dynamics*. Comparative Pathobiology - Studies in the Postmodern Theory of Education. Clarendon Press, Oxford.

Ahmad, I., Zan-Ul-Abadin, Q., Faisal, M., Loganathan, K., Javed, T., and Chaudhary, D. K. (2022). Prescribed thermal activity in the radiative bidirectional flow of magnetized hybrid nanofluid: Keller-box approach. *Journal of Nanomaterials*, 2022(1):5531041.

Aladdin, N. A. L., Bachok, N., and Anuar, N. S. (2020). Mhd stagnation point flow in nanofluid over shrinking surface using buongiorno's model: A stability analysis. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 76(3):12–24.

Alhadri, M., Raza, J., Yashkun, U., Lund, L. A., Maatki, C., Khan, S. U., and Kolsi, L. (2022). Response surface methodology (rsm) and artificial neural network (ann) simulations for thermal flow hybrid nanofluid flow with darcy-forchheimer effects. *Journal of the Indian Chemical Society*, 99(8):100607.

Alipour, N., Jafari, B., and Hosseinzadeh, K. (2023). Thermal analysis and optimization approach for ternary nanofluid flow in a novel porous cavity by considering nanoparticle shape factor. *Heliyon*, 9(12):e22257.

Alraddadi, I., Ayub, A., Hussain, S. M., Khan, U., Hussain Shah, S. Z., and Hassan, A. M. (2023). The significance of ternary hybrid cross bio-nanofluid model in expanding/contracting cylinder with inclined magnetic field. *Frontiers in Materials*, 10.

Aly, E. H. and Pop, I. (2019). Mhd flow and heat transfer over a permeable stretching/shrinking sheet in a hybrid nanofluid with a convective boundary condition. *International Journal of Numerical Methods for Heat & Fluid Flow*, 29(9):3012–3038.

Ananth Subray, P. V., Hanumagowda, B. N., Varma, S. V. K., and Hatami, M. (2022). The impacts of shape factor and heat transfer on two-phase flow of nano and hybrid nanofluid in a saturated porous medium. *Scientific Reports*, 12(1):21864.

Anderson, J. D. (1995). *Computational Fluid Dynamics: The Basics with Applications*. McGraw-Hill international editions. McGraw-Hill, New York City, New York, U.S.

Asghar, A., Lund, L. A., Shah, Z., Vrinceanu, N., Deebani, W., and Shutaywi, M. (2022a). Effect of thermal radiation on three-dimensional magnetized rotating flow of a hybrid nanofluid. *Nanomaterials*, 12(9):1566.

Asghar, A., Ying, T. Y., and Zaimi, W. M. K. A. W. (2022b). Two-dimensional mixed convection and radiative al2o3-cu/h2o hybrid nanofluid flow over a vertical exponentially shrinking sheet with partial slip conditions. *CFD Letters*, 14(3):22–38.

Awaludin, I. S., Ishak, A., and Pop, I. (2018). On the stability of mhd boundary layer flow over a stretching/shrinking wedge. *Scientific Reports*, 8(1):13622.

Awaludin, I. S., Weidman, P. D., and Ishak, A. (2016). Stability analysis of stagnation-point flow over a stretching/shrinking sheet. *AIP Advances*, 6(4).

Awan, A. U., Ali, B., Shah, S. A. A., Oreijah, M., Guedri, K., and Eldin, S. M. (2023). Numerical analysis of heat transfer in ellis hybrid nanofluid flow subject to a stretching cylinder. *Case Studies in Thermal Engineering*, 49:103222.

Bachok, N., Ishak, A., and Pop, I. (2013). Mixed convection boundary layer flow over a moving vertical flat plate in an external fluid flow with viscous dissipation effect. *Plos One*, 8(4):e60766.

Bakthavatchalam, B., Habib, K., Saidur, R., Saha, B. B., and Irshad, K. (2020). Comprehensive study on nanofluid and ionanofluid for heat transfer enhancement: A review on current and future perspective. *Journal of Molecular Liquids*, 305:112787.

Batchelor, G. K. (1977). The effect of brownian motion on the bulk stress in a suspension of spherical particles. *Journal of Fluid Mechanics*, 83(1):97–117.

Batchelor, G. K. (2000). *An introduction to fluid dynamics*. Cambridge university press, Cambridge, 2 edition.

Benkhada, M., Bensouici, F., and Boufendi, T. (2021). Parametric study of nanoparticles effects on convective heat transfer of nanofluids in a heated horizontal annulus. *Journal of Nano Research*, 70:81–100.

Berrehal, H., Dinarvand, S., and Khan, I. (2022). Mass-based hybrid nanofluid model for entropy generation analysis of flow upon a convectively-warmed moving wedge. *Chinese Journal of Physics*, 77:2603–2616.

Bhatti, M. M., Abbas, T., and Rashidi, M. M. (2016). Effects of thermal radiation and electromagnetohydrodynamics on viscous nanofluid through a riga plate. *Multidiscipline Modeling in Materials and Structures*, 12(4):605–618.

Blasius, H. (1908). Grenzschichten in flüssigkeiten mit kleiner reibung. *Zeitschrift für Mathematik und Physik*, 56:1–37.

Boroomandpour, A., Toghraie, D., and Hashemian, M. (2020). A comprehensive experimental investigation of thermal conductivity of a ternary hybrid nanofluid containing mwcnts- titania-zinc oxide/water-ethylene glycol (80:20) as well as binary and mono nanofluids. *Synthetic Metals*, 268:116501.

Box, G. E. P. and Wilson, K. B. (1992). On the experimental attainment of optimum conditions. In Kotz, S. and Johnson, N. L., editors, *Breakthroughs in Statistics: Methodology and Distribution*, pages 270–310. Springer New York, New York, U.S.

Brinkman, H. C. (1952). The viscosity of concentrated suspensions and solutions. *The Journal of Chemical Physics*, 20(4):571–571.

Buongiorno, J. (2005). Convective transport in nanofluids. *Journal of Heat Transfer*, 128(3):240–250.

Chakraborty, S. and Panigrahi, P. K. (2020). Stability of nanofluid: A review. *Applied Thermal Engineering*, 174:115259.

Choi, S. U. S. and Eastman, J. A. (1995). Enhancing thermal conductivity of fluids with nanoparticles. Technical report, Argonne National Lab.(ANL), Argonne, IL (United States).

Çolak, A. B., Yildiz, O., Bayrak, M., Celen, A., Dalkiliç, A. S., and Wongwises, S. (2020). Experimental study on the specific heat capacity measurement of water-based al2o3-cu hybrid nanofluid by using differential thermal analysis method. *Current Nanoscience*, 16(6):912–928.

Cramer, N. F. (2001). *The Physics of Alfven Waves*. Wiley-VCH, Germany.

Das, S. K., Choi, S. U. S., and Patel, H. E. (2006). Heat transfer in nanofluids—a review. *Heat Transfer Engineering*, 27(10):3–19.

Datta, S., Bandyopadhyay, A., and Pal, P. K. (2008). Solving multi-criteria optimization problem in submerged arc welding consuming a mixture of fresh flux and fused slag. *The International Journal of Advanced Manufacturing Technology*, 35:935–942.

Davidson, P. A. (2001). *An Introduction to Magnetohydrodynamics*. Cambridge Texts in Applied Mathematics. Cambridge University Press, Cambridge.

Dawar, A., Shah, Z., Tassaddiq, A., Kumam, P., Islam, S., and Khan, W. (2021). A convective flow of williamson nanofluid through cone and wedge with non-isothermal and non-isosolutal conditions: A revised buongiorno model. *Case Studies in Thermal Engineering*, 24:100869.

Delery, J. M. (1985). Shock wave/turbulent boundary layer interaction and its control. *Progress in Aerospace Sciences*, 22(4):209–280.

Derringer, G. and Suich, R. (1980). Simultaneous optimization of several response variables. *Journal of Quality Technology*, 12(4):214–219.

Desale, S. and Pradhan, V. H. (2015). Numerical solution of boundary layer flow equation with viscous dissipation effect along a flat plate with variable temperature. *Procedia Engineering*, 127:846–853. International Conference on Computational Heat and Mass Transfer (ICCHMT) - 2015.

Devi, S. P. A. and Devi, S. S. U. (2016). Numerical investigation of hydromagnetic hybrid cu–al2o3/water nanofluid flow over a permeable stretching sheet with suction. *International Journal of Nonlinear Sciences and Numerical Simulation*, 17(5):249–257.

Devi, S. U. and Devi, S. P. A. (2017). Heat transfer enhancement of cu-al2o3/water hybrid nanofluid flow over a stretching sheet. *Journal of the Nigerian Mathematical Society*, 36(2):419–433.

Dezfulizadeh, A., Aghaei, A., Joshaghani, A. H., and Najafizadeh, M. M. (2021). An experimental study on dynamic viscosity and thermal conductivity of water-cu-sio2-mwcnt ternary hybrid nanofluid and the development of practical correlations. *Powder Technology*, 389:215–234.

Dorrepaal, J. M. (1986). An exact solution of the navier-stokes equation which describes non-orthogonal stagnation-point flow in two dimensions. *Journal of Fluid Mechanics*, 163:141–147.

Drazin, P. G. and Riley, N. (2006). *The Navier-Stokes equations: a classification of flows and exact solutions*, volume 334 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge.

Dzulkifli, N. F., Bachok, N., Yacob, N. A., Arifin, N. M., Rosali, H., and Pop, I. (2022). Thermal radiation on mixed convection heat and mass transfer over a vertical permeable stretching/shrinking sheet with soret and dufour effects. *Journal of Engineering Mathematics*, 132:3.

Einstein, A. (1906). A new determination of molecular dimensions. *Annalen der Physik*, 19:289–306.

El-Zahar, E. R., Rashad, A. M., Saad, W., and Seddek, L. F. (2020). Magneto-hybrid nanofluids flow via mixed convection past a radiative circular cylinder. *Scientific Reports*, 10(1):10494.

Esfe, M. H., Ardeshiri, E. M., and Toghraie, D. (2022). Experimental study and sensitivity analysis of a new generation of special ternary hybrid nanofluids (thnfs) and investigation of factors affecting its thermal conductivity. *Case Studies in Thermal Engineering*, 34:101940.

Fadodun, O. G., Amosun, A. A., Ogundeji, J. A., and Olaloye, D. O. (2019). Numerical investigation of themal efficiency and pumping power of al2o3/h2o nanofluid in pipe using response surface methodology. *Journal of Nanofluids*, 8(7):1566–1576.

Fagbade, A. I. and Omowaye, A. J. (2016). Influence of thermal radiation on free convective heat and mass transfer past an isothermal vertical oscillating porous plate in the presence of chemical reaction and heat generation-absorption. *Boundary Value Problems*, 2016:97.

Faisal, M., Mabood, F., Kenneth Asogwa, K., and Badruddin, I. A. (2023). Bidirectional radiative transport of magnetic maxwell nanofluid mobilized by arrhenius energy and prescribed thermal/concentration conditions: Significance of ludwig-soret and pedesis effects. *Ain Shams Engineering Journal*, 14(4):101933.

Finnemore, E. J. and Franzini, J. B. (2002). *Fluid Mechanics with Engineering Applications*. McGraw-Hill, New York City, New York, U.S., 10 edition.

Fox, R. W., McDonald, A. T., and Pritchard, P. J. (2004). *Introduction to Fluid Mechanics*. John Wiley & Sons, Hoboken, New Jersey, 6 edition.

Gailitis, A. and Lielausis, O. (1961). On a possibility to reduce the hydrodynamic resistance of a plate in a electrolyte. *Applied Magnetohydrodynamics*, 12:143–146.

Ganesh, N. V., Al-Mdallal, Q. M., Al Fahel, S., and Dadoa, S. (2019). Riga – plate flow of γ al2o3-water/ethylene glycol with effective prandtl number impacts. *Heliyon*, 5(5):e01651.

Ghaffari, A., Javed, T., and Labropulu, F. (2017). Oblique stagnation point flow of a non-newtonian nanofluid over stretching surface with radiation: a numerical study. *Thermal Science*, 21(5):2139–2153.

Grinberg, E. (1961). On determination of properties of some potential fields. *Applied Magnetohydrodynamics*, 12:147–154.

Groşan, T. and Pop, I. (2020). Flow and heat transfer over a permeable biaxial stretching/shrinking sheet in a nanofluid. *Neural Computing and Applications*, 32:4575–4582.

Guedri, K., Khan, A., Sene, N., Raizah, Z., Saeed, A., Galal, A. M., et al. (2022). Thermal flow for radiative ternary hybrid nanofluid over nonlinear stretching sheet subject to darcy–forchheimer phenomenon. *Mathematical Problems in Engineering*, 2022(1):3429439.

Gupta, G. and Rana, P. (2022). Comparative study on rosseland's heat flux on three-dimensional mhd stagnation-point multiple slip flow of ternary hybrid nanofluid over a stretchable rotating disk. *Mathematics*, 10(18):3342.

Hajmohammadi, M. R., Maleki, H., Lorenzini, G., and Nourazar, S. S. (2015). Effects of cu and ag nano-particles on flow and heat transfer from permeable surfaces. *Advanced Powder Technology*, 26(1):193–199.

Halima, U., Dogondaji, A. M., and Sammani, A. (2023). Effects of injection/suction on unsteady mhd natural convective radiative flow of heat mass transfer in a plumb frequency. *Saudi Journal of Engineering and Technology*, 8(7):171–180.

Hamilton, R. L. and Crosser, O. K. (1962). Thermal conductivity of heterogeneous two-component systems. *Industrial & Engineering chemistry fundamentals*, 1(3):187–191.

Han, H. Z., Li, B. X., Hao, W., and Wei, S. (2015). Multi-objective shape optimization of double pipe heat exchanger with inner corrugated tube using rsm method. *International Journal of Thermal Sciences*, 90:173–186.

Hanif, H., Khan, I., and Shafie, S. (2020). Heat transfer exaggeration and entropy analysis in magneto-hybrid nanofluid flow over a vertical cone: a numerical study. *Journal of Thermal Analysis and Calorimetry*, 141(5):2001–2017.

Harris, S. D., Ingham, D. B., and Pop, I. (2009). Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: brinkman model with slip. *Transport in Porous Media*, 77(2):267–285.

Hassanien, I. A., Abdullah, A. A., and Gorla, R. S. R. (1998). Flow and heat transfer in a power-law fluid over a nonisothermal stretching sheet. *Mathematical and computer modelling*, 28(9):105–116.

Hebbale, A. M. and Srinath, M. S. (2017). Taguchi analysis on erosive wear behavior of cobalt based microwave cladding on stainless steel aisi-420. *Measurement*, 99:98–107.

Hussain, S., Ali, A., Rasheed, K., Pasha, A. A., Algarni, S., Alqahtani, T., and Irshad, K. (2023). Application of response surface methodology to optimize mhd nanofluid flow over a rotating disk with thermal radiation and joule heating. *Case Studies in Thermal Engineering*, 52:103715.

Hussain, Z., Aljuaydi, F., Ayaz, M., and Islam, S. (2024). Significance of slips and convective conditions towards the non-newtonian hybrid nanofluid flow over a bi-directional stretching surface. *International Journal of Thermofluids*, 21:100537.

Ibrahim, W. and Anbessa, T. (2019). Mixed convection flow of nanofluid with hall and ion-slip effects using spectral relaxation method. *Journal of the Egyptian Mathematical Society*, 27(1):52.

Ibrahim, W. and Gizewu, T. (2023). Stability analysis of dual solutions for mixed convection and thermal radiation with hybrid nanofluid flow past shrinking/stretching curved surface. *Scientific Reports*, 13(1):21676.

Iranian, D., Sudarmozhi, K., Khan, I., and Mohamed, A. (2023). Significance of heat generation and impact of suction/injection on maxwell fluid over a horizontal plate by the influence of radiation. *International Journal of Thermofluids*, 20:100396.

Ishtiaq, B., Nadeem, S., and Abbas, N. (2022). Theoretical study of two-dimensional unsteady maxwell fluid flow over a vertical riga plate under radiation effects. *Scientia Iranica*, 29(6):3072–3083.

Jaafar, A., Jamaludin, A., Mohd Nasir, N. A. A., Nazar, R., and Pop, I. (2023). Mhd opposing flow of cu-tio2 hybrid nanofluid under an exponentially stretching/shrinking surface embedded in porous media with heat source and slip impacts. *Results in Engineering*, 17:101005.

Jaafar, A., Waini, I., Jamaludin, A., Nazar, R., and Pop, I. (2022). Mhd flow and heat transfer of a hybrid nanofluid past a nonlinear surface stretching/shrinking with effects of thermal radiation and suction. *Chinese Journal of Physics*, 79:13–27.

Jafar, K., Ishak, A., and Nazar, R. (2011). Magnetohydrodynamic stagnation point flow with a convective surface boundary condition. *Zeitschrift für Naturforschung A*, 66(8–9):495–499.

Jahan, S., Ferdows, M., Shamshuddin, M., and Zaimi, K. (2021). Radiative mixed convection flow over a moving needle saturated with non-isothermal hybrid nanofluid. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 88(1):81–93.

Jakeer, S., Reddy, S. R. R., Rashad, A. M., Lakshmi Rupa, M., and Manjula, C. (2023). Nonlinear analysis of darcy-forchheimer flow in emhd ternary hybrid nanofluid (cu-cnt-ti/water) with radiation effect. *Forces in Mechanics*, 10:100177.

Jamrus, F. N., Waini, I., Khan, U., and Ishak, A. (2024). Effects of magnetohydrodynamics and velocity slip on mixed convective flow of thermally stratified ternary hybrid nanofluid over a stretching/shrinking sheet. *Case Studies in Thermal Engineering*, 55:104161.

Jana, S., Salehi-Khojin, A., and Zhong, W. H. (2007). Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives. *Thermochimica Acta*, 462(1–2):45–55.

Japili, N., Rosali, H., and Bachok, N. (2022). Slip effect on stagnation point flow and heat transfer over a shrinking/stretching sheet in a porous medium with suction/injection. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 90(2):73–89.

Joye, D. D. and Wojnovich, M. J. (1996). Aiding and opposing mixed-convection heat transfer in a vertical tube: loss of boundary condition at different grashof numbers. *International Journal of Heat and Fluid Flow*, 17(5):468–473.

Jusoh, R., Nazar, R., and Pop, I. (2019). Magnetohydrodynamic boundary layer flow and heat transfer of nanofluids past a bidirectional exponential permeable stretching/shrinking sheet with viscous dissipation effect. *Journal of Heat Transfer*, 141(1):012406.

Kata, S., Ganganapalli, S., and Kuppalapalle, V. (2022). Transport of thermal energy in the magnetohydrodynamic oblique stagnation point flow in a hybrid nanofluid with nanoparticle shape effect. *Heat Transfer*, 51(5):4331–4348.

Kataria, H. R. and Mittal, A. S. (2015). Mathematical model for velocity and temperature of gravity-driven convective optically thick nanofluid flow past an oscillating vertical plate in presence of magnetic field and radiation. *Journal of the Nigerian Mathematical Society*, 34(3):303–317.

Kejela, S. B., Daba, M., and Girum, A. (2021). Investigation of effects of thermal radiation, magnetic field, eckert number, and thermal slip on mhd hiemenz flow by optimal homotopy asymptotic method. *Journal of Mathematics*, 2021(1):5590657.

Khan, I., Rahman, A. U., Dawar, A., Islam, S., and Zaman, A. (2023a). Second-order slip flow of a magnetohydrodynamic hybrid nanofluid past a bi-directional stretching surface with thermal convective and zero mass flux conditions. *Advances in Mechanical Engineering*, 15(2):16878132221149894.

Khan, M. R. (2020). Numerical analysis of oblique stagnation point flow of nanofluid over a curved stretching/shrinking surface. *Physica Scripta*, 95(10):105704.

Khan, M. R., Pan, K., Khan, A. U., and Nadeem, S. (2020). Dual solutions for mixed convection flow of sio2-al2o3/water hybrid nanofluid near the stagnation point over a curved surface. *Physica A: Statistical Mechanics and its Applications*, 547:123959.

Khan, M. S., Mei, S., Shabnam, Fernandez-Gamiz, U., Noeiaghdam, S., Shah, S. A., and Khan, A. (2022). Numerical analysis of unsteady hybrid nanofluid flow comprising cnts-ferrousoxide/water with variable magnetic field. *Nanomaterials*, 12(2):180.

Khan, U., Mahmood, Z., Eldin, S. M., Makhdoum, B. M., Fadhl, B. M., and Alshehri, A. (2023b). Mathematical analysis of heat and mass transfer on unsteady stagnation point flow of riga plate with binary chemical reaction and thermal radiation effects. *Heliyon*, 9(3):e14472.

Khan, W. A. and Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer*, 53(11):2477–2483.

Khashi'ie, N. S., Arifin, N. M., Nazar, R., Hafidzuddin, E. H., Wahi, N., and Pop, I. (2020). Magnetohydrodynamics (mhd) axisymmetric flow and heat transfer of a hybrid nanofluid past a radially permeable stretching/shrinking sheet with joule heating. *Chinese Journal of Physics*, 64:251–263.

Khashi'ie, N. S., Arifin, N. M., Sheremet, M., and Pop, I. (2021). Shape factor effect of radiative cu–al2o3/h2o hybrid nanofluid flow towards an emhd plate. *Case Studies in Thermal Engineering*, 26:101199.

Khashi'ie, N. S., Hamzah, K. B., Waini, I., Zainal, N. A., Nordin, S. K. S., Kasim, A. R. M., et al. (2024). Response surface methodology of the unsteady axisymmetric magnetic hybrid nanofluid flow subject to a shrinking disk. *Journal of Advanced Research in Applied Mechanics*, 112(1):137–148.

Khashi'ie, N. S., Roşca, N. C., Roşca, A. V., and Pop, I. (2023). Dual solutions on mhd radiative three-dimensional bidirectional nanofluid flow over a non-linearly permeable shrinking sheet. *Alexandria Engineering Journal*, 71:401–411.

Khashi'ie, N. S., Arifin, N. M., Wahid, N. S., and Pop, I. (2023). Insight into unsteady separated stagnation point flow of hybrid nanofluids subjected to an electro-magnetohydrodynamics riga plate. *Magnetochemistry*, 9(2):46.

Khashi'ie, N. S., Md Arifin, N., and Pop, I. (2020). Mixed convective stagnation point flow towards a vertical riga plate in hybrid cu-al2o3/water nanofluid. *Mathematics*, 8(6):912.

Khashi'ie, N. S., Waini, I., Mukhtar, M. F., Zainal, N. A., Hamzah, K. B., Arifin, N. M., and Pop, I. (2022). Response surface methodology (rsm) on the hybrid nanofluid flow subject to a vertical and permeable wedge. *Nanomaterials*, 12(22):4016.

Khuri, A. I. and Mukhopadhyay, S. (2010). Response surface methodology. *WIREs Computational Statistics*, 2(2):128–149.

Kitetu, V. M., Onyango, T., Kwanza, J. K., and Mutua, N. M. (2019). Analysis of the effect of stretching parameter and time parameter on mhd nanofluid flow in the presence of suction. *Global Journal of Pure and Applied Mathematics*, 15:637–648.

Kolsi, L., Abbasi, A., Alqsair, U. F., Farooq, W., Omri, M., and Khan, S. U. (2021). Thermal enhancement of ethylene glycol base material with hybrid nanofluid for oblique stagnation point slip flow. *Case Studies in Thermal Engineering*, 28:101468.

Kundu, P. K., Cohen, I. M., and Dowling, D. R. (2012). Chapter 8 - laminar flow. In *Fluid Mechanics (Fifth Edition)*, pages 309–359. Academic Press, Boston, 5 edition.

Kuznetsov, A. V. and Nield, D. A. (2010). Natural convective boundary-layer flow of a nanofluid past a vertical plate. *International Journal of Thermal Sciences*, 49(2):243–247.

Kuznetsov, A. V. and Nield, D. A. (2013). The cheng–minkowycz problem for natural convective boundary layer flow in a porous medium saturated by a nanofluid: A revised model. *International Journal of Heat and Mass Transfer*, 65:682–685.

Labropulu, F., Dorrepaal, J. M., and Chandna, O. P. (1996). Oblique flow impinging on a wall with suction or blowing. *Acta Mechanica*, 115(1):15–25.

Lawal, D., Bin Ali, A., and Mohammed, A. S. (2016). Tribological investigations of carbon nanotube-reinforced polymer (uhmwpe) nanocomposites using taguchi methodology. *Journal of Applied Polymer Science*, 133(40):44018.

Lenin, R., Joy, P. A., and Bera, C. (2021). A review of the recent progress on thermal conductivity of nanofluid. *Journal of Molecular Liquids*, 338:116929.

Li, D., Labropulu, F., and Pop, I. (2009a). Oblique stagnation-point flow of a viscoelastic fluid with heat transfer. *International Journal of Non-Linear Mechanics*, 44(10):1024–1030.

Li, S., Shahmir, N., Ramzan, M., Ahammad, N. A., Alhuthali, A. M. S., Saleel, C. A., and Kadry, S. (2024). Thermal inspection of hybrid nanofluid flows over a stretched cylinder at an oblique stagnation point with variable characteristics. *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik*, 104(7):e202300837.

Li, X., Khan, A. U., Khan, M. R., Nadeem, S., and Khan, S. U. (2019). Oblique stagnation point flow of nanofluids over stretching/shrinking sheet with cattaneo–christov heat flux model: Existence of dual solution. *Symmetry*, 11(9):1070.

Li, Y., Zhou, J., Tung, S., Schneider, E., and Xi, S. (2009b). A review on development of nanofluid preparation and characterization. *Powder Technology*, 196(2):89–101.

Lok, Y. Y., Ishak, A., and Pop, I. (2018). Oblique stagnation slip flow of a micropolar fluid towards a stretching/shrinking surface: A stability analysis. *Chinese Journal of Physics*, 56(6):3062–3072.

Lok, Y. Y., Merkin, J. H., and Pop, I. (2015). Mhd oblique stagnation-point flow towards a stretching/shrinking surface. *Meccanica*, 50(12):2949–2961.

Ma, M. Y., Yu, L. Z., Zhou, H. L., Pei, T. Y., and Hua, W. (2021). Particle size-dependent rheological behavior and mechanism of al2o3-cu/w hybrid nanofluids. *Journal of Molecular Liquids*, 335:116297.

Madhukesh, J. K., Sarris, I. E., Prasannakumara, B. C., and Abdulrahman, A. (2023). Investigation of thermal performance of ternary hybrid nanofluid flow in a permeable inclined cylinder/plate. *Energies*, 16(6):2630.

Magyari, E. and Pantokratoras, A. (2011a). Aiding and opposing mixed convection flows over the riga-plate. *Communications in Nonlinear Science and Numerical Simulation*, 16(8):3158–3167.

Magyari, E. and Pantokratoras, A. (2011b). Note on the effect of thermal radiation in the linearized rosseland approximation on the heat transfer characteristics of various boundary layer flows. *International Communications in Heat and Mass Transfer*, 38(5):554–556.

Mahanthesh, B., Shehzad, S. A., Mackolil, J., and Shashikumar, N. S. (2021). Heat transfer optimization of hybrid nanomaterial using modified buongiorno model: A sensitivity analysis. *International Journal of Heat and Mass Transfer*, 171:121081.

Mahapatra, T. R., Nandy, S. K., and Gupta, A. S. (2010). Dual solution of mhd stagnation-point flow towards a stretching surface. *Engineering*, 2(04):299–305.

Mahmood, Z., Eldin, S. M., Soliman, A. F., Assiri, T. A., Khan, U., and Mahmoud, S. R. (2023). Impact of an effective prandtl number model on the flow of nanofluids past an oblique stagnation point on a convective surface. *Heliyon*, 9(2):e13224.

Mahmood, Z., Iqbal, Z., Alyami, M. A., Alqahtani, B., Yassen, M. F., and Khan, U. (2022). Influence of suction and heat source on mhd stagnation point flow of ternary hybrid nanofluid over convectively heated stretching/shrinking cylinder. *Advances in Mechanical Engineering*, 14(9):16878132221126278.

Makhdoum, B. M., Mahmood, Z., Khan, U., Fadhl, B. M., Khan, I., and Eldin, S. M. (2023). Impact of suction with nanoparticles aggregation and joule heating on unsteady mhd stagnation point flow of nanofluids over horizontal cylinder. *Heliyon*, 9(4):e15012.

Malek, M., Izem, N., Mohamed, M. S., Seaid, M., and Wakrim, M. (2021). Numerical solution of rosseland model for transient thermal radiation in non-grey optically thick media using enriched basis functions. *Mathematics and Computers in Simulation*, 180:258–275.

Mandal, P. K., Seth, G. S., Sarkar, S., and Chamkha, A. (2021). A numerical simulation of mixed convective and arbitrarily oblique radiative stagnation point slip flow of a cnt-water mhd nanofluid. *Journal of Thermal Analysis and Calorimetry*, 143:1901–1916.

Manjunatha, S., Puneeth, V., Gireesha, B. J., and Chamkha, A. (2022). Theoretical study of convective heat transfer in ternary nanofluid flowing past a stretching sheet. *Journal of Applied and Computational Mechanics*, 8(4):1279–1286.

Maranna, T., Mahabaleshwar, U. S., Pérez, L. M., and Manca, O. (2023). Flow of viscoelastic ternary nanofluid over a shrinking porous medium with heat source/sink and radiation. *Thermal Science and Engineering Progress*, 40:101791.

Marulasiddeshi, H. B., Kanti, P. K., Jamei, M., Prakash, S. B., Sridhara, S. N., and Said, Z. (2022). Experimental study on the thermal properties of al2o3-cuo/water hybrid nanofluids: Development of an artificial intelligence model. *International Journal of Energy Research*, 46(15):21066–21083.

Masuda, H., Ebata, A., Teramae, K., and Hishinuma, N. (1993). Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles. *Netsu Bussei*, 7(4):227–233.

Maxwell, J. C. (1873). *A Treatise on Electricity and Magnetism*, volume 2. Clarendon Press, Oxford.

McCroskey, W. J. (1977). The 1976 freeman scholar lecture: some current research in unsteady fluid dynamics. *Journal of Fluids Engineering*, 99(1):8–39.

Mehmood, T., Ramzan, M., Howari, F., Kadry, S., and Chu, Y. M. (2021). Application of response surface methodology on the nanofluid flow over a rotating disk with autocatalytic chemical reaction and entropy generation optimization. *Scientific Reports*, 11(1):4021.

Merkin, J. H. (1986). On dual solutions occurring in mixed convection in a porous medium. *Journal of Engineering Mathematics*, 20(2):171–179.

Merkin, J. H., Pop, I., Lok, Y. Y., and Grosan, T. (2022). Chapter 5 - stretching/shrinking sheets in nanofluids and hybrid nanofluids. In Merkin, J. H., Pop, I., Lok, Y. Y., and Grosan, T., editors, *Similarity Solutions for the Boundary Layer Flow and Heat Transfer of Viscous Fluids, Nanofluids, Porous Media, and Micropolar Fluids*, pages 113–162. Academic Press.

Mia, M. (2018). Mathematical modeling and optimization of mql assisted end milling characteristics based on rsm and taguchi method. *Measurement*, 121:249–260.

Mohamed, M. K. A., Salleh, M. Z., Nazar, R., and Ishak, A. (2013). Numerical investigation of stagnation point flow over a stretching sheet with convective boundary conditions. *Boundary Value Problems*, 2013:4.

Mohammed Zayan, J., Rasheed, A. K., John, A., Faris, W. F., Aabid, A., Baig, M., and Alallam, B. (2023). Synthesis and characterization of novel ternary-hybrid nanoparticles as thermal additives. *Materials*, 16(1):173.

Mohyud-Din, S. T., Khan, U., Ahmed, N., and Rashidi, M. M. (2018). A study of heat and mass transfer on magnetohydrodynamic (mhd) flow of nanoparticles. *Propulsion and Power Research*, 7(1):72–77.

Morini, G. L. (2008). Viscous dissipation. In Li, D., editor, *Encyclopedia of Microfluidics and Nanofluidics*, pages 2155–2164. Springer US, Boston, MA.

Motahari, K. and Barati, S. (2019). Optimization of nusselt number of al2o3/water nanofluid using response surface methodology. *Iranian Journal of Chemistry and Chemical Engineering (IJCCE)*, 38(3):309–317.

Mousavi, M., Pouranfard, A., and Darvishi, P. (2024). Experimental study and modeling of thermal and rheological characteristics of water-based cuo/caco3/sio2 ternary hybrid nanofluid. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 686:133367.

Mousavi, S. M., Esmaeilzadeh, F., and Wang, X. P. (2019). Effects of temperature and particles volume concentration on the thermophysical properties and the rheological behavior of cuo/mgo/tio2 aqueous ternary hybrid nanofluid: Experimental investigation. *Journal of Thermal Analysis and Calorimetry*, 137:879–901.

Muhaimin, Kandasamy, R., and Khamis, A. B. (2008). Effects of heat and mass transfer on nonlinear mhd boundary layer flow over a shrinking sheet in the presence of suction. *Applied Mathematics and Mechanics*, 29:1309–1317.

Myers, R. H., Montgomery, D. C., and Anderson-Cook, C. M. (2009). *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. Wiley Series in Probability and Statistics. Wiley, Hoboken, New Jersey.

Nadeem, S., Khan, M. R., and Khan, A. U. (2019). Mhd oblique stagnation point flow of nanofluid over an oscillatory stretching/shrinking sheet: existence of dual solutions. *Physica Scripta*, 94(7):075204.

Nanda Kishore, P. V. R., Venkatachalapathy, S., Kalidoss, P., and Chaupal, P. (2023). Experimental investigation with ann modeling of thermal conductivity and viscosity of a ternary nanofluid at different mixing ratios and volume concentrations. *Journal of Molecular Liquids*, 383:122006.

Nayak, M. K., Shaw, S., Makinde, O. D., and Chamkha, A. J. (2018). Effects of homogenous–heterogeneous reactions on radiative nacl-cnp nanofluid flow past a convectively heated vertical riga plate. *Journal of Nanofluids*, 7(4):657–667.

Neethu, T. S., Areekara, S., Sabu, A. S., Mathew, A., and Anakha, K. K. (2022). Bioconvective electromagnetohydrodynamic hybrid nanoliquid flow over a stretching sheet with stratification effects: a four-factor response surface optimized model. *Waves in Random and Complex Media*, pages 1–26.

Oztop, H. F. and Abu-Nada, E. (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow*, 29(5):1326–1336.

Pak, B. C. and Cho, Y. I. (1998). Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. *Experimental Heat Transfer an International Journal*, 11(2):151–170.

Pakdemirli, M. and Yurusoy, M. (1998). Similarity transformations for partial differential equations. *SIAM Review*, 40(1):96–101.

Panda, S., Ontela, S., Mishra, S. R., and Pattnaik, P. K. (2023). Response surface methodology and sensitive analysis for optimizing heat transfer rate on the 3d hybrid nanofluid flow through permeable stretching surface. *Journal of Thermal Analysis and Calorimetry*, 148:7369–7382.

Panduro, E. A. C., Finotti, F., Largiller, G., and Lervåg, K. Y. (2022). A review of the use of nanofluids as heat-transfer fluids in parabolic-trough collectors. *Applied Thermal Engineering*, 211:118346.

Pantokratoras, A. and Magyari, E. (2009). Emhd free-convection boundary-layer flow from a riga-plate. *Journal of Engineering Mathematics*, 64:303–315.

Patil, P. M. and Kulkarni, M. (2022). Effects of surface roughness and thermal radiation on mixed convective (go–mos2/h2o–c2h6o2) hybrid nanofluid flow past a permeable cone. *Indian Journal of Physics*, 96(12):3567–3578.

Pattnaik, P. K., Parida, S. K., Mishra, S. R., Abbas, M. A., and Bhatti, M. M. (2022). Analysis of metallic nanoparticles (cu, al2o3, and swcnts) on magnetohydrodynamics water-based nanofluid through a porous medium. *Journal of Mathematics*, 2022(1):3237815.

Peng, Y., Khaled, U., Al-Rashed, A. A. A. A., Meer, R., Goodarzi, M., and Sarafraz, M. M. (2020). Potential application of response surface methodology (rsm) for the prediction and optimization of thermal conductivity of aqueous cuo (ii) nanofluid: A statistical approach and experimental validation. *Physica A: Statistical Mechanics and its Applications*, 554:124353.

Pop, I. and Ingham, D. B. (2001). Chapter 2 - mixed convection boundary-layer flow along a vertical flat plate. In Pop, I. and Ingham, D. B., editors, *Convective Heat Transfer*, pages 45–85. Pergamon, Oxford.

Prasannakumara, B. C., Gnaneswara Reddy, M., Thammanna, G. T., and Gireesha, B. J. (2018). Mhd double-diffusive boundary-layer flow of a maxwell nanofluid over a bidirectional stretching sheet with soret and dufour effects in the presence of radiation. *Nonlinear Engineering*, 7(3):195–205.

Preston, J. H. (1946). *The Boundary Layer Flow Over a Permeable Surface Through which Suction is Applied*. ARC-9364. H.M. Stationery Office, London.

Puliti, G., Paolucci, S., and Sen, M. (2011). Nanofluids and their properties. *Applied Mechanics Reviews*, 64(3):030803.

Rafique, K., Mahmood, Z., Khan, U., Eldin, S. M., Oreijah, M., Guedri, K., and Khalifa, H. A. E.-W. (2023). Investigation of thermal stratification with velocity slip and variable viscosity on mhd flow of al2o3- cu-tio2/h2o nanofluid over disk. *Case Studies in Thermal Engineering*, 49:103292.

Rahman, M. M., Grosan, T., and Pop, I. (2016). Oblique stagnation-point flow of a nanofluid past a shrinking sheet. *International Journal of Numerical Methods for Heat & Fluid Flow*, 26(1):189–213.

Ramreddy, C., Pradeepa, T., and Srinivasacharya, D. (2015). Numerical study of mixed convection flow of a micropolar fluid towards permeable vertical plate with convective boundary condition. *Journal of Applied Analysis & Computation*, 6:254–270.

Ramzan, M., Kumam, P., Lone, S. A., Seangwattana, T., Saeed, A., and Galal, A. M. (2023). A theoretical analysis of the ternary hybrid nanofluid flows over a non-isothermal and non-isosolutal multiple geometries. *Heliyon*, 9(4):e14875.

Rashid, M., Alsaedi, A., Hayat, T., and Ahmed, B. (2020). Magnetohydrodynamic flow of maxwell nanofluid with binary chemical reaction and arrhenius activation energy. *Applied Nanoscience*, 10:2951–2963.

Rekha, M. B., Sarris, I. E., Madhukesh, J. K., Raghunatha, K. R., and Prasannakumara, B. C. (2022). Activation energy impact on flow of aa7072-aa7075/water-based hybrid nanofluid through a cone, wedge and plate. *Micromachines*, 13(2):302.

Reza, M. and Gupta, A. S. (2005). Steady two-dimensional oblique stagnation-point flow towards a stretching surface. *Fluid Dynamics Research*, 37(5):334.

Rieutord, M. (2015). *Fluid Dynamics: An Introduction*. Springer Cham, Heidelberg, New York, Dordrecht, London.

Rizwana, R., Hussain, A., and Nadeem, S. (2020). Slip effects on unsteady oblique stagnation point flow of nanofluid in a view of inclined magnetic field. *Mathematical Problems in Engineering*, 2020(1):6580409.

Roberts, P. H. (1967). *An Introduction to Magnetohydrodynamics*, volume 6. Longmans, London.

Rohsenow, W. M., Hartnett, J. P., and Cho, Y. I. (1998). *Handbook of heat transfer*. McGraw-Hill, New York City, New York, U.S., 3 edition.

Rosca, N. C., Grosan, T., and Pop, I. (2012). Stagnation-point flow and mass transfer with chemical reaction past a permeable stretching/shrinking sheet in a nanofluid. *Sains Malaysiana*, 41(10):1271–1279.

Rosseland, S. (1931). *Astrophysik*. Springer, Berlin, Heidelberg.

Rott, N. (1990). Note on the history of the reynolds number. *Annual Review of Fluid Mechanics*, 22(1):1–12.

Sahoo, R. R. and Kumar, V. (2020). Development of a new correlation to determine the viscosity of ternary hybrid nanofluid. *International Communications in Heat and Mass Transfer*, 111:104451.

Schlichting, H. and Gersten, K. (2017). *Boundary-Layer Theory*. Springer, Berlin, Heidelberg, 9 edition.

Selvakumar, P. and Suresh, S. (2012). Use of al2o3-cu/water hybrid nanofluid in an electronic heat sink. *IEEE Transactions on Components, Packaging and Manufacturing Technology*, 2(10):1600–1607.

Shahsavar, A., Sepehrnia, M., Maleki, H., and Darabi, R. (2023). Thermal conductivity of hydraulic oil-go/fe3o4/tio2 ternary hybrid nanofluid: Experimental study, rsm analysis, and development of optimized gpr model. *Journal of Molecular Liquids*, 385:122338.

Shampine, L. F., Kierzenka, J., and Reichelt, M. W. (2000). Solving boundary value problems for ordinary differential equations in matlab with bvp4c. *Tutorial notes*, 2000:1–27.

Shanmugapriya, M., Sundareswaran, R., Kumar, P. S., and Rangasamy, G. (2022). Impact of nanoparticle shape in enhancing heat transfer of magnetized ternary hybrid nanofluid. *Sustainable Energy Technologies and Assessments*, 53:102700.

Shatnawi, T. A. M., Abbas, N., and Shatanawi, W. (2022). Mathematical analysis of unsteady stagnation point flow of radiative casson hybrid nanofluid flow over a vertical riga sheet. *Mathematics*, 10(19):3573.

Sheikholeslami, M. and Ganji, D. D. (2016). Chapter 1 - magnetohydrodynamic and ferrohydrodynamic. In Sheikholeslami, M. and Ganji, D. D., editors, *External Magnetic Field Effects on Hydrothermal Treatment of Nanofluid*, pages 1–47. William Andrew Publishing, New York, U.S.

Shvydkii, V., Kiselev, E., Dudko, V., and Solnceva, E. (2018). Mathematical model of heat exchange and approximate methods of solution of radiation transfer equation in the melting furnace tank. *KnE Engineering*, 3(5):25–34.

Siddiqui, F. R., Tso, C. Y., Chan, K. C., Fu, S. C., and Chao, C. Y. H. (2019). On trade-off for dispersion stability and thermal transport of cu-al2o3 hybrid nanofluid for various mixing ratios. *International Journal of Heat and Mass Transfer*, 132:1200–1216.

Siegel, R. and Howell, J. R. (1992). *Thermal Radiation Heat Transfer*. Hemisphere Publishing Corporation, Washington, D.C., U.S., 3 edition.

Singh, S. K. (2022). Review on the stability of the nanofluids. In Rushd, S. and Ismail, M. A., editors, *Pipeline Engineering*, chapter 8. IntechOpen, Rijeka.

Soid, S. K., Ishak, A., and Pop, I. (2018). Mhd flow and heat transfer over a radially stretching/shrinking disk. *Chinese Journal of Physics*, 56(1):58–66.

Streeter, V. L. and Wylie, E. B. (1975). *Fluid Mechanics*. Civil and mechanical engineering series. McGraw-Hill, New York City, New York, U.S.

Stuart, J. T. (1959). The viscous flow near a stagnation point when the external flow has uniform vorticity. *Journal of the Aerospace Sciences*, 26(2):124–125.

Sulochana, C., Aparna, S. R., and Sandeep, N. (2020). Magnetohydrodynamic mgo/cuo-water hybrid nanofluid flow driven by two distinct geometries. *Heat Transfer*, 49(6):3663–3682.

Sundar, L. S., Chandra Mouli, K. V. V., Said, Z., and Sousa, A. C. M. (2021). Heat transfer and second law analysis of ethylene glycol-based ternary hybrid nanofluid under laminar flow. *Journal of Thermal Science and Engineering Applications*, 13(5):051021.

Sundar, L. S., Sharma, K. V., Singh, M. K., and Sousa, A. C. M. (2017). Hybrid nanofluids preparation, thermal properties, heat transfer and friction factor–a review. *Renewable and Sustainable Energy Reviews*, 68:185–198.

Suresh, S., Venkitaraj, K. P., Selvakumar, P., and Chandrasekar, M. (2011). Synthesis of al2o3–cu/water hybrid nanofluids using two step method and its thermo physical properties. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, 388(1):41–48.

Suresh, S., Venkitaraj, K. P., Selvakumar, P., and Chandrasekar, M. (2012). Effect of al2o3–cu/water hybrid nanofluid in heat transfer. *Experimental Thermal and Fluid Science*, 38:54–60.

Takabi, B., Gheitaghy, A. M., and Tazraei, P. (2016). Hybrid water-based suspension of al2o3 and cu nanoparticles on laminar convection effectiveness. *Journal of Thermophysics and Heat Transfer*, 30(3):523–532.

Tamada, K. (1979). Two-dimensional stagnation-point flow impinging obliquely on a plane wall. *Journal of the Physical Society of Japan*, 46(1):310–311.

Tilley, B. S. and Weidman, P. D. (1998). Oblique two-fluid stagnation-point flow. *European Journal of Mechanics-B/Fluids*, 17(2):205–217.

Tiwari, R. K. and Das, M. K. (2007). Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer*, 50(9):2002–2018.

Tokuda, T., Tsuruda, R., Hara, T., Hou, Z., Kobayashi, H., Tanaka, K., Takarada, W., Kikutani, T., Hinestroza, J. P., Razal, J. M., et al. (2022). Planar or biaxial stretching of poly (ethylene terephthalate) fiber webs prepared by laser-electrospinning. *Materials*, 15(6):2209.

Tooke, R. M. and Blyth, M. G. (2008). A note on oblique stagnation-point flow. *Physics of Fluids*, 20(3):033101.

Turcu, R., Darabont, A., Nan, A., Aldea, N., Macovei, D., Bica, D., Vekas, L., Pana, O., Soran, M.-L., Koós, A., and Biro, L. (2006). New polypyrrole-multiwall carbon nanotubes hybrid materials. *Journal of Optoelectronics and Advanced Materials*, 8(2):643–647.

Turkyilmazoglu, M. (2020). Single phase nanofluids in fluid mechanics and their hydrodynamic linear stability analysis. *Computer Methods and Programs in Biomedicine*, 187:105171.

Ukueje, W. E., Abam, F. I., and Obi, A. (2022). A perspective review on thermal conductivity of hybrid nanofluids and their application in automobile radiator cooling. *Journal of Nanotechnology*, 2022(1):2187932.

Ullah, A. Z., Guo, X., Gul, T., Ali, I., Saeed, A., and Galal, A. M. (2023). Thin film flow of the ternary hybrid nanofluid over a rotating disk under the influence of magnetic field due to nonlinear convection. *Journal of Magnetism and Magnetic Materials*, 573:170673.

Veldman, A. E. P. (2008). Boundary layers in fluids. *Lecture Notes in Applied Mathematics, University of Groningen*.

Versteeg, H. K. and Malalasekera, W. (2007). *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*. Pearson Education Limited, Harlow.

Vinutha, K., Sajjan, K., Madhukesh, J. K., and Ramesh, G. K. (2023). Optimization of rsm and sensitivity analysis in mhd ternary nanofluid flow between parallel plates with quadratic radiation and activation energy. *Journal of Thermal Analysis and Calorimetry*, 149:1595–1616.

Wahid, N. S., Arifin, N. M., Khashi'ie, N. S., Pop, I., Bachok, N., and Hafidzuddin, M. E. H. (2022a). Hybrid nanofluid stagnation point flow past a slip shrinking riga plate. *Chinese Journal of Physics*, 78:180–193.

Wahid, N. S., Arifin, N. M., Pop, I., Bachok, N., and Hafidzuddin, M. E. H. (2022b). Mhd stagnation-point flow of nanofluid due to a shrinking sheet with melting, viscous dissipation and joule heating effects. *Alexandria Engineering Journal*, 61(12):12661–12672.

Waini, I., Ishak, A., and Pop, I. (2020). Hybrid nanofluid flow and heat transfer over a permeable biaxial stretching/shrinking sheet. *International Journal of Numerical Methods for Heat & Fluid Flow*, 30(7):3497–3513.

Waini, I., Ishak, A., and Pop, I. (2021). Hybrid nanofluid flow over a permeable non-isothermal shrinking surface. *Mathematics*, 9(5):538.

Waini, I., Ishak, A., and Pop, I. (2022). Multiple solutions of the unsteady hybrid nanofluid flow over a rotating disk with stability analysis. *European Journal of Mechanics - B/Fluids*, 94:121–127.

Wang, C. Y. (1985). The unsteady oblique stagnation point flow. *The Physics of Fluids*, 28(7):2046–2049.

Wang, C. Y. (2008). Similarity stagnation point solutions of the navier–stokes equations–review and extension. *European Journal of Mechanics-B/Fluids*, 27(6):678–683.

Wang, C. Y. (2015). Uniform flow over a bi-axial stretching surface. *Journal of Fluids Engineering*, 137(8):084502.

Weidman, P. D., Kubitschek, D. G., and Davis, A. M. J. (2006). The effect of transpiration on self-similar boundary layer flow over moving surfaces. *International Journal of Engineering Science*, 44(11):730–737.

Wilks, G. and Bramley, J. S. (1981). Dual solutions in mixed convection. *Proceedings of the Royal Society of Edinburgh: Section A Mathematics*, 87(3–4):349–358.

Wu, J. H., Yen, M. S., Wu, C. P., Li, C. H., and Kuo, M. C. (2013). Effect of biaxial stretching on thermal properties, shrinkage and mechanical properties of poly (lactic acid) films. *Journal of Polymers and the Environment*, 21:303–311.

Xuan, Y. and Roetzel, W. (2000). Conceptions for heat transfer correlation of nanofluids. *International Journal of Heat and Mass Transfer*, 43(19):3701–3707.

Xuan, Z., Zhai, Y., Ma, M., Li, Y., and Wang, H. (2021). Thermo-economic performance and sensitivity analysis of ternary hybrid nanofluids. *Journal of Molecular Liquids*, 323:114889.

Yaseen, M., Kumar, M., and Rawat, S. K. (2021). Assisting and opposing flow of a mhd hybrid nanofluid flow past a permeable moving surface with heat source/sink and thermal radiation. *Partial Differential Equations in Applied Mathematics*, 4:100168.

Yashkun, U., Zaimi, K., Sufahani, S. F., Eid, M. R., and Ferdows, M. (2023). Flow and heat transfer of a nanofluid through a porous medium due to stretching/shrinking sheet with suction, magnetic field and thermal radiation. *Applied Mathematics-A Journal of Chinese Universities*, 38(3):373–391.

Yih, K. A. (1998). The effect of uniform suction/blowing on heat transfer of magnetohydrodynamic hiemenz flow through porous media. *Acta Mechanica*, 130(3):147–158.

Yu, W. and Xie, H. (2012). A review on nanofluids: preparation, stability mechanisms, and applications. *Journal of Nanomaterials*, 2012(1):435873.

Zaimi, K., Ishak, A., and Pop, I. (2012). Boundary layer flow and heat transfer past a permeable shrinking sheet in a nanofluid with radiation effect. *Advances in Mechanical Engineering*, 4:340354.

Zaimi, K., Ishak, A., and Pop, I. (2014). Flow past a permeable stretching/shrinking sheet in a nanofluid using two-phase model. *PLOS ONE*, 9(11):e111743.

Zainal, N. A., Nazar, R., Naganthran, K., and Pop, I. (2021). Heat generation/absorption effect on mhd flow of hybrid nanofluid over bidirectional exponential stretching/shrinking sheet. *Chinese Journal of Physics*, 69:118–133.

Zainal, N. A., Nazar, R., Naganthran, K., and Pop, I. (2022). Unsteady stagnation point flow past a permeable stretching/shrinking riga plate in al2o3-cu/h2o hybrid nanofluid with thermal radiation. *International Journal of Numerical Methods for Heat & Fluid Flow*, 32(8):2640–2658.

Zeeshan, A., Hussain, D., Asghar, Z., Bhatti, M. M., and Duraihem, F. Z. (2023). Thermal optimization of mhd nanofluid over a wedge by using response surface methodology: Sensitivity analysis. *Propulsion and Power Research*, 12(4):556–567.

Zhu, A., Ali, H., Ishaq, M., Junaid, M. S., Raza, J., and Amjad, M. (2022). Numerical study of heat and mass transfer for williamson nanofluid over stretching/shrinking sheet along with brownian and thermophoresis effects. *Energies*, 15(16):5926.