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Statistical Modeling of Gold Price Data Using Generalized Extreme Value Distribution: An Inference Based on Parametric and Nonparametric Bootstrap Confidence Interval

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ABSTRACT

This study applies Extreme Value Theory (EVT) to model and quantify the risk associated with gold price fluctuations in the Malaysian market. Using daily gold price data from February 2010 to May 2023, the monthly maxima of negative daily log returns are modeled using the Generalized Extreme Value (GEV) distribution, with parameters estimated using Maximum Likelihood Estimation (MLE). Value-at-Risk (VaR) estimates for high quantiles are obtained from the fitted GEV model. To quantify the uncertainty of these estimates, the parametric and nonparametric bootstrap methods are used for constructing confidence intervals (CIs). Simulation study conducted across varying return periods (10, 20, 50, and 100 months), sample sizes (50, 100, 150, and 200), and GEV shape parameters (0.1, 0.2, 0.3, and 0.4), reveals that the nonparametric bootstrap method generally outperforms its parametric counterpart. This superiority is particularly evident for larger sample sizes and longer return periods, as demonstrated by narrower confidence intervals and lower error metrics. Applied to the Malaysian gold price data, the analysis yields VaR estimates ranging from 3.16% for a 10-month return period to 5.96% for a 100-month return period, with corresponding probabilities of exceedance decreasing from 10% to 1%. These results highlight the potential for significant losses in gold investments over longer time horizons with correspondingly decreasing probabilities of occurrence, while also demonstrating the effectiveness of EVT and bootstrap techniques in capturing and quantifying the uncertainty associated with extreme market events.

Keywords: Value-at-Risk, Parametric Bootstrap, Nonparametric Bootstrap

INTRODUCTION

Gold, symbolized as Au from the Latin Aurum, has been a treasured commodity for millennia. Its enduring value and consistent price appreciation over time have cemented its status as a favored long-term investment. However, like all investments, gold prices are subject to fluctuations, making it crucial to model and understand gold price dynamics, particularly gold price returns. In the realm of financial risk management, Value-at-Risk (VaR) has emerged as a critical statistical measure. VaR quantifies the maximum potential loss on an investment over a specified period at a given confidence level. When dealing with extreme market conditions, such as those often observed in gold markets, Extreme Value Theory (EVT) methods become particularly relevant for calculating VaR. The study of extreme events in gold price returns is vital due to the heavy-tailed nature of their distribution, as evidenced by previous research. Pagan and Schwert (1990) found substantial evidence of heavy tails in gold returns, while more recent studies by Khan et al. (2021)

and Ali et al. (2020) confirmed these findings in different contexts, applying EVT approaches to gold price risk analysis.

The importance of studying extreme events in gold price returns is further emphasized by various studies in the field. Jang (2007) applied an extreme value theory approach to analyze the extreme risk of gold prices, highlighting the effectiveness of EVT in capturing rare but significant price movements. Additionally, Chen and Liu (2017) examined the macroeconomic determinants of gold prices using a panel approach, providing insights into the factors influencing extreme price fluctuations. These studies underscore the need for robust statistical methods in analyzing gold price dynamics. In this study, we analyze daily gold price data to assess the risk associated with gold investments. Our methodology involves calculating daily returns from the gold price data and then extracting the monthly maxima of these returns. This approach allows us to focus on the extreme movements in gold prices, which are crucial for accurate risk assessment. We then fit the Generalized Extreme Value (GEV) distribution to these monthly maxima, leveraging the strengths of Extreme Value Theory in modeling rare events.

In assessing the risk associated with gold investments, it's crucial to consider return periods and their associated return levels, which are intimately connected to VaR. In the context of our gold price return data, the return level represents the magnitude of a loss that is expected to be exceeded on average once every return period. For instance, a 100-month return level corresponds to a loss magnitude that we expect to be surpassed, on average, once every 100 months. This return level is equivalent to the VaR at a confidence level of 99% (1 - 1/100) for a 1-month holding period. Thus, by estimating return levels for various return periods, we can derive VaR estimates for different confidence levels, providing a comprehensive risk profile for gold investments.

To address the uncertainty inherent in VaR estimates, we employ bootstrap methods. These resampling techniques allow for the estimation of standard errors, bias, and confidence intervals for statistical estimators. The bootstrap method, introduced by Efron in the 1970s, has become a powerful tool for assessing the uncertainty of statistical estimates. DiCiccio and Efron (1996) discuss various methods for constructing bootstrap confidence intervals, providing insights into their theoretical properties and practical implementation. Scholz (2007) further investigates the small sample properties of bootstrap methods, which is particularly relevant when dealing with extreme value data where sample sizes are often limited. Two primary bootstrap approaches are commonly employed: parametric and nonparametric. The parametric bootstrap generates pseudo-data based on fitted model parameters, while the nonparametric bootstrap resamples directly from the observed data without assuming a specific parametric model. The performance of these bootstrap methods can vary depending on the underlying data characteristics and sample sizes. Previous studies, such as those by Kysely (2009) and Konietschke et al. (2015), have suggested that parametric bootstrap methods may outperform their nonparametric counterparts, particularly for heavy-tailed distributions common in financial data.

This study aims to comprehensively assess the performance of parametric and nonparametric bootstrap methods in quantifying the uncertainty of VaR estimates for the gold price return data. We will conduct a simulation study comparing these methods across various scenarios, including different return periods (10, 50, and 100 months), sample sizes (50, 100, 150, 200), and shape parameters of the GEV distribution (0.1, 0.2, 0.3, 0.4). By identifying the superior bootstrap method through our simulation study, we will then apply it to the real gold price data. This application will provide robust VaR estimates and associated confidence intervals, offering valuable insights for risk management in gold investments.

The remainder of this paper is structured as follows: Section 2 outlines the methodology and design for simulation studies. Section 3 presents our findings, including the potential maximum loss of gold investments at various return periods and the quantification of VaR uncertainty through bootstrap confidence intervals. We also evaluate how these results depend on the tail bahaviour and sample size. Finally, Section 4 concludes the study, summarizing our key findings and their implications for gold price risk assessment.

MATERIALS AND METHODS

Data Description

The data used in this study are the daily gold prices in the Malaysia bullion market over the period from February 2010 until May 2023. The data is taken from the following website: https://www.bnm.gov.my/kijang-emas-prices. The scatterplot of daily gold price of study period plot is shown in Figure 1. For extreme value analysis, we will consider the monthly maxima of negative daily log returns of gold price of weight one oz.

Figure 1: Time series plot of daily gold prices from the Malaysia Bullion Market

Generalized Extreme Value Distribution

The significant changes in gold prices can be evaluated by analyzing the daily returns of gold prices per troy ounce. In the context of risk management, particular emphasis is on negative returns (losses) due to their critical significance. Define

$$X_i = -(\ln P_i - \ln P_{i-1})$$

where X_i is a negative log returns of gold prices observed between day i and i-1 which follow the unknown cumulative distribution function $F(x) = \Pr(X_i \leq x)$, P_i and P_{i-1} represent the gold prices of day i and day i-1. Let $X_{(n)}$ represent the monthly maxima of negative movements in the daily log returns of gold prices, that is

$$X_{(n)} = \max(X_1, X_2, \ldots, X_n).$$

The distribution of $X_{(n)}$ can be written as

$$\Pr\left(X_{(n)} \leq x\right) = \Pr\,(X_1 \leq x, X_2 \leq x, \dots, X_n \leq x)$$

$$= \prod_{i=1}^{n} \Pr\,(X_i \leq x)$$

$$= F^n(x).$$

In practice, the parent distribution F is often unknown. Suppose μ_n and $\sigma_n > 0$ are the sequences of constants. Allowing a linear renormalization of the variable $X_{(n)}$ such that

$$Z = \frac{X_{(n)} - \mu_n}{\sigma_n}$$

leads to limit distribution of Z converge to a non-degenerate distribution function as $n \to \infty$ given by

$$G(z) = exp\left\{-\left[1 + \xi\left(\frac{z-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}, \quad (1)$$

defined on the $\left\{z: 1 + \frac{\xi(z-\mu)}{\sigma} > 0\right\}$ where the parameter satisfies $-\infty < \mu < \infty, \sigma > 0$ and $-\infty < \mu < \infty$. The distribution given by Eq. (1) is known as generalized extreme value distribution (GEV). The use of the GEV distribution in modeling extreme events has a rich history in statistical literature. Gumbel (1958) laid the foundation for the statistical theory of extremes, which has since been widely applied in various fields, including finance. Smith (1990) further developed the theory, providing a comprehensive overview of extreme value analysis techniques. These seminal works have paved the way for modern applications of EVT in financial risk management.

Maximum Likelihood Estimation

Maximum likelihood estimation (MLE) is a widely used method for parameter estimation in statistical modeling, including for GEV distribution. Myung (2003) provides a comprehensive tutorial on MLE, explaining its theoretical foundations and practical applications. The method's efficiency and consistency make it particularly suitable for estimating parameters in extreme value analysis. Let $Z_1, Z_2, \dots, Z_n$ be the independent series of monthly maxima of negative daily log returns. The log-likelihood of GEV is given by

$$\ell(\mu, \sigma, \xi) = -n \log \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{n} log\left[1 + \xi\left(\frac{z_i-\mu}{\sigma}\right)\right] - \sum_{i=1}^{n} log\left[1 + \xi\left(\frac{z_i-\mu}{\sigma}\right)\right]^{-1/\xi} \quad (2)$$

provided that $1 + \xi\left(\frac{z_i-\mu}{\sigma}\right) > 0$, for $i = 1,2, \dots., n$. The parameter estimates of the GEV model can be obtained by calculating the partial derivatives of the log-likelihood function in Eq. (2) with respect to μ, σ, and ξ, setting these derivatives equal to zero and solving the resulting system of equations using numerical optimization.

Estimation of Value-at-Risk and Uncertainty Quantification

Once the parameters of the GEV model have been estimated, the primary focus shifts to assessing Value at Risk (VaR), a statistical measure of the risk of loss. VaR is a financial metric used to estimate the potential loss in the value of an asset (e.g., gold) over a specified time period, under normal market conditions, and at a given confidence level (Jorion, 2007).

To estimate VaR, it is crucial to derive the extreme quantiles of the monthly maxima of negative daily log returns, which can be obtained by inverting Eq. (1) and it is given by

$$z_p = \mu - \frac{\sigma}{\xi}\left[1 - \{-log(1-p)\}^{-\xi}\right]$$

where $G(z_p) = 1 - p$, $0 < p < 1$. Generally, z_p is the return level associated with the return period $1/p$. In the context of monthly maxima of negative daily log returns data, z_p can be interpreted as the value that is expected to be exceeded, on average, once every $1/p$ months. This can be related to VaR, which represents a threshold of loss that is expected not to be exceeded with a certain level of confidence over a given time period.

The uncertainty of the VaR estimate is measured through the confidence interval. Bootstrap-based confidence intervals are widely used to measure uncertainty arising from parameter estimates and have applications across many fields, including finance (Jorion, 2007), economics (Davidson, 2004), environmental science (Efron & Tibshirani, 1993), and biostatistics (Davison & Hinkley, 1997). Two bootstrap-based confidence interval methods, i.e., parametric (P) and nonparametric (NP), are used to construct the confidence interval for VaR. In this study, the performance of these two bootstrap methods will be compared to determine which is more effective in quantifying the uncertainty of VaR estimates. A simulation study will be conducted to evaluate the accuracy of these methods. The bootstrap method that demonstrates superior performance in the simulation will then be applied to evaluate the uncertainty of the VaR estimate in gold price data.

Simulation Design

To compare the performance of parametric (P) and nonparametric (NP) bootstrap methods in quantifying the uncertainty of VaR estimates, a simulation study is conducted with the following steps:

i. **Synthetic Data Generation**: Synthetic datasets are generated from parameter values obtained by fitting the Generalized Extreme Value (GEV) model to the monthly maxima of negative daily log returns.

ii. **Bootstrap Sampling:**
   a. For the P bootstrap, 1000 bootstrap samples, each are generated from parameter values obtained by fitting the Generalized Extreme Value (GEV) model to the synthetic data in step i.
   b. For the NP bootstrap, 1000 bootstrap samples are generated by resampling directly from the synthetic data in step i.

iii. **VaR Estimation:** VaR is estimated for each bootstrap sample generated in step ii.

iv. **Performance Metrics:** To assess the accuracy of each method, bias, mean square error (MSE), and root mean square error (RMSE) is calculated for the VaR estimates.

v. **Confidence Interval Construction:** The bootstrap percentile method is applied to construct confidence intervals (CIs) for VaR estimates for both P and NP bootstrap methods. The CIs are based on the percentiles of the bootstrapped VaR estimates. The length of the CIs is used to measure uncertainty, with shorter CIs indicating more precise estimates.

RESULTS AND DISCUSSION

Descriptive Statistics

The descriptive statistics of the monthly maxima of negative daily log returns are summarized in Table 1. The mean value of 0.002569 indicates that, on average, the maximum monthly loss is about 0.26%. The standard deviation of 0.0126 reflects moderate variability in these monthly maximum losses. The distribution exhibits heavy tails and significant right skewness, with a kurtosis of 22.35 and skewness of 3.44. These characteristics suggest frequent occurrence of extreme losses, with some particularly large losses skewing the distribution to the right.

Before fitting the GEV model, the stationarity of the series was tested using the Augmented Dickey-Fuller (ADF) test at the 5% significance level. The p-value of the test is 0.01754, which is less than 0.05, leading to the rejection of the null hypothesis of non-stationarity. Therefore, we can conclude that the series is stationary.

Table 1: Descriptive statistics of monthly maxima of negative daily log returns

Sample Size	160
Minimum	0.002569
Maximum	0.111994
Mean	0.018431
Standard Deviation	0.012599
Skewness	3.438986
Kurtosis	22.349550

Model Fitting and Model Diagnostics

The GEV model was fitted to the monthly maxima of negative daily log returns, with parameters estimated using the MLE method. The estimated location parameter, $\mu = 0.013136$ and the scale parameter, $\sigma = 0.006771$. Additionally, the estimated shape parameter, $\xi = 0.1638102$ where a positive value indicates a right-heavy tail in the distribution. This suggests that extreme negative log returns are more likely to occur than would be expected under a light-tailed distribution.

To evaluate the goodness of fit for the fitted GEV model, we employed the Anderson-Darling (AD) test, the Kolmogorov-Smirnov (KS) test, and Quantile-Quantile (Q-Q) plots. The results indicate that the p-values from both tests are greater than 0.05, specifically, 0.7471 for the AD test and 0.3271 for the KS test. Consequently, we fail to reject the null hypothesis, suggesting that the monthly maxima of negative log returns follow a GEV distribution. Additionally, the Q-Q plot shown in Figure 2 is nearly linear, further supporting that the GEV distribution provides a good fit for the monthly maxima of negative daily log returns.

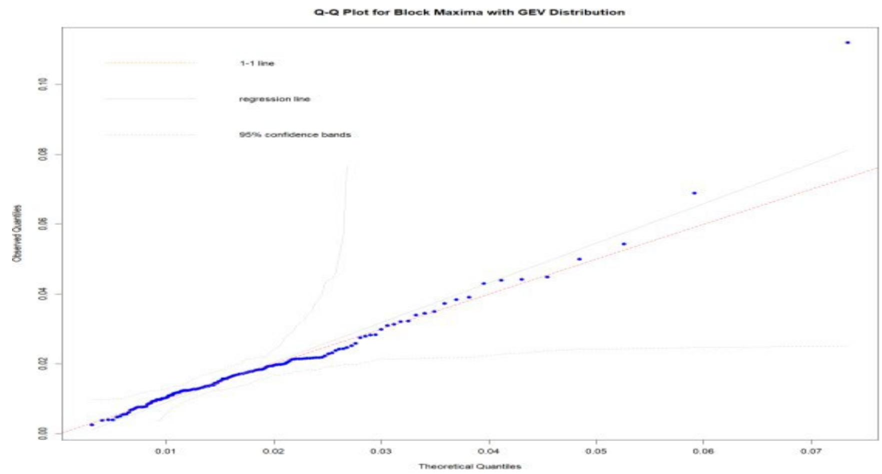

Figure 2: QQ plot of Monthly maxima of negative daily log returns of gold price data

Simulation Result

Tables 2 present the results of the simulation study comparing parametric (P) and nonparametric (NP) bootstrap methods for quantifying the uncertainty of Value at Risk (VaR) estimates. Both methods generally demonstrate improved performance with increasing sample size, as evidenced by lower bias, Mean Square Error (MSE), and Root Mean Square Error (RMSE). However, the nonparametric bootstrap method appears to have a slight edge overall, outperforming its parametric counterpart in approximately 64% of the scenarios examined. This advantage is particularly pronounced for larger sample sizes (150 and 200) and longer return periods (50-months and 100-months). Conversely, the parametric method shows some strengths in scenarios with shorter return periods and smaller sample sizes, although this trend is not consistent across all cases.

Table 2: Bias, MSE and RMSE of VaR estimates at various return period, sample size and tail behaviour for sample generated from parametric and nonparametric bootstrap

Return Period	Sample size	Shape	Nonparametric (NP)			Parametric (P)		
			bias	MSE	RMSE	bias	MSE	RMSE
10-months	50	0.1	0.00015	0.00001	0.00282	0.00020	0.00001	0.00319
		0.2	0.00337	0.00004	0.00665	0.00323	0.00004	0.00645
		0.3	0.00486	0.00004	0.00605	0.00476	0.00004	0.00603
		0.4	0.00380	0.00008	0.00872	0.00378	0.00008	0.00909
	100	0.1	0.00044	0.00001	0.00231	0.00046	0.00001	0.00238
		0.2	0.00375	0.00002	0.00447	0.00371	0.00002	0.00444
		0.3	0.00159	0.00001	0.00368	0.00176	0.00002	0.00402
		0.4	0.00058	0.00002	0.00494	0.00077	0.00002	0.00491
	150	0.1	0.00062	0.00000	0.00178	0.00063	0.00000	0.00190
		0.2	0.00283	0.00001	0.00379	0.00286	0.00002	0.00399
		0.3	0.00371	0.00002	0.00417	0.00380	0.00002	0.00427
		0.4	0.00153	0.00001	0.00315	0.00158	0.00001	0.00332
	200	0.1	0.00053	0.00000	0.00178	0.00046	0.00000	0.00168
		0.2	0.00171	0.00001	0.00278	0.00177	0.00001	0.00284

		0.3	0.00154	0.00001	0.00304	0.00156	0.00001	0.00305
		0.4	0.00352	0.00002	0.00443	0.00347	0.00002	0.00427
20-months	50	0.1	0.00619	0.00006	0.00757	0.00625	0.00006	0.00792
		0.2	0.00035	0.00005	0.00692	0.00013	0.00006	0.00767
		0.3	0.01578	0.00036	0.01908	0.01656	0.00043	0.02070
		0.4	0.02379	0.00085	0.02912	0.02450	0.00093	0.03051
	100	0.1	0.00742	0.00007	0.00817	0.00723	0.00006	0.00809
		0.2	0.00379	0.00003	0.00563	0.00377	0.00003	0.00560
		0.3	0.02115	0.00052	0.02284	0.02081	0.00050	0.02236
		0.4	0.01593	0.00035	0.01871	0.01615	0.00035	0.01864
	150	0.1	0.00724	0.00006	0.00766	0.00725	0.00006	0.00779
		0.2	0.00195	0.00002	0.00497	0.00199	0.00002	0.00483
		0.3	0.01958	0.00043	0.02069	0.01974	0.00044	0.02107
		0.4	0.00991	0.00012	0.01093	0.00985	0.00012	0.01111
	200	0.1	0.00372	0.00002	0.00433	0.00369	0.00002	0.00431
		0.2	0.00471	0.00003	0.00533	0.00459	0.00003	0.00511
		0.3	0.01325	0.00020	0.01402	0.01308	0.00019	0.01385
		0.4	0.00765	0.00008	0.00906	0.00782	0.00008	0.00903
50-months	50	0.1	0.01568	0.00030	0.01726	0.01571	0.00032	0.01797
		0.2	0.02402	0.00074	0.02720	0.02370	0.00071	0.02672
		0.3	0.04422	0.00255	0.05051	0.04554	0.00291	0.05391
		0.4	0.06634	0.00641	0.08009	0.06777	0.00691	0.08311
	100	0.1	0.01714	0.00033	0.01807	0.01729	0.00034	0.01833
		0.2	0.01788	0.00039	0.01966	0.01748	0.00037	0.01928
		0.3	0.03083	0.00108	0.03279	0.03149	0.00114	0.03382
		0.4	0.04630	0.00269	0.05188	0.04618	0.00258	0.05079
	150	0.1	0.01666	0.00029	0.01717	0.01640	0.00029	0.01708
		0.2	0.01795	0.00035	0.01881	0.01797	0.00036	0.01898
		0.3	0.01536	0.00027	0.01648	0.01441	0.00024	0.01548
		0.4	0.02944	0.00095	0.03074	0.02953	0.00098	0.03133
	200	0.1	0.01248	0.00017	0.01306	0.01262	0.00017	0.01316
		0.2	0.01425	0.00024	0.01547	0.01434	0.00025	0.01567
		0.3	0.01683	0.00043	0.02071	0.01787	0.00049	0.02214
		0.4	0.02717	0.00084	0.02900	0.02772	0.00085	0.02924
100-months	50	0.1	0.02662	0.00097	0.03116	0.02403	0.00078	0.02786
		0.2	0.05787	0.00435	0.06593	0.05913	0.00474	0.06887
		0.3	0.07372	0.00745	0.08632	0.07649	0.00875	0.09356
		0.4	0.11999	0.02251	0.15004	0.12065	0.02283	0.15110
	100	0.1	0.02558	0.00072	0.02691	0.02569	0.00074	0.02721
		0.2	0.02798	0.00094	0.03062	0.02847	0.00098	0.03135
		0.3	0.07465	0.00682	0.08260	0.07224	0.00596	0.07720
		0.4	0.07800	0.00770	0.08774	0.07948	0.00762	0.08729
	150	0.1	0.02220	0.00053	0.02296	0.02211	0.00053	0.02310
		0.2	0.02834	0.00087	0.02955	0.02810	0.00087	0.02943
		0.3	0.07845	0.00677	0.08226	0.07917	0.00699	0.08360

		0.4	0.04868	0.00258	0.05075	0.04930	0.00273	0.05229
	200	0.1	0.01962	0.00041	0.02032	0.01979	0.00042	0.02055
		0.2	0.00326	0.00010	0.00988	0.00345	0.00011	0.01028
		0.3	0.06801	0.00497	0.07053	0.06797	0.00497	0.07052
		0.4	0.04614	0.00239	0.04890	0.04712	0.00243	0.04933

Figures 3-6 illustrate the width of confidence intervals (CIs) for both 90% and 95% levels, spanning return periods of 10, 20, 50, and 100 months, with sample sizes ranging from 50 to 200, and shape parameters from 0.1 to 0.4. Consistently, the nonparametric method demonstrates superior performance, evidenced by narrower CIs, particularly for larger sample sizes and longer return periods. This advantage becomes more pronounced as the shape parameter increases, suggesting enhanced efficacy in capturing uncertainty for more extreme events. While both methods exhibit improved precision with increasing sample size, the nonparametric approach maintains a clear edge, especially in scenarios with 50-month and 100-month return periods and shape parameters of 0.3 and 0.4. The parametric method occasionally performs comparably for smaller sample sizes and lower shape parameters but lacks the consistent robustness of its nonparametric counterpart across diverse scenarios.

Based on the comprehensive analysis of the simulation study results, the nonparametric bootstrap method is recommended as the superior approach for quantifying the uncertainty of VaR estimates, particularly for larger sample sizes and longer return periods, due to its consistently better performance across a majority of scenarios and its apparent robustness to varying data characteristics. Therefore, nonparametric bootstrap will be used to measure the uncertainty of VaR estimates of monthly maxima of negative daily log returns gold price dataset.

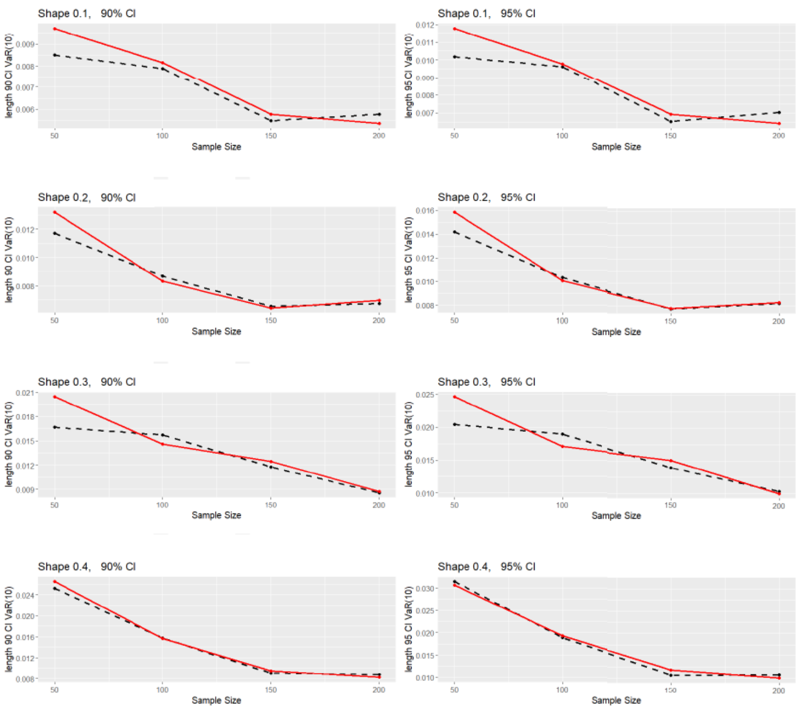

Figure 3: Width of confidence interval (CI) of VaR estimate at 10-months return period at $n = 50, 100, 150, 200$ and $\xi = 0.1, 0.2, 0.3, 0.4$. The left panel is 90% CI and right panel is 95% CI. Black dashed line represents width of CI by nonparametric approach while red solid line represents width of CI by parametric approach

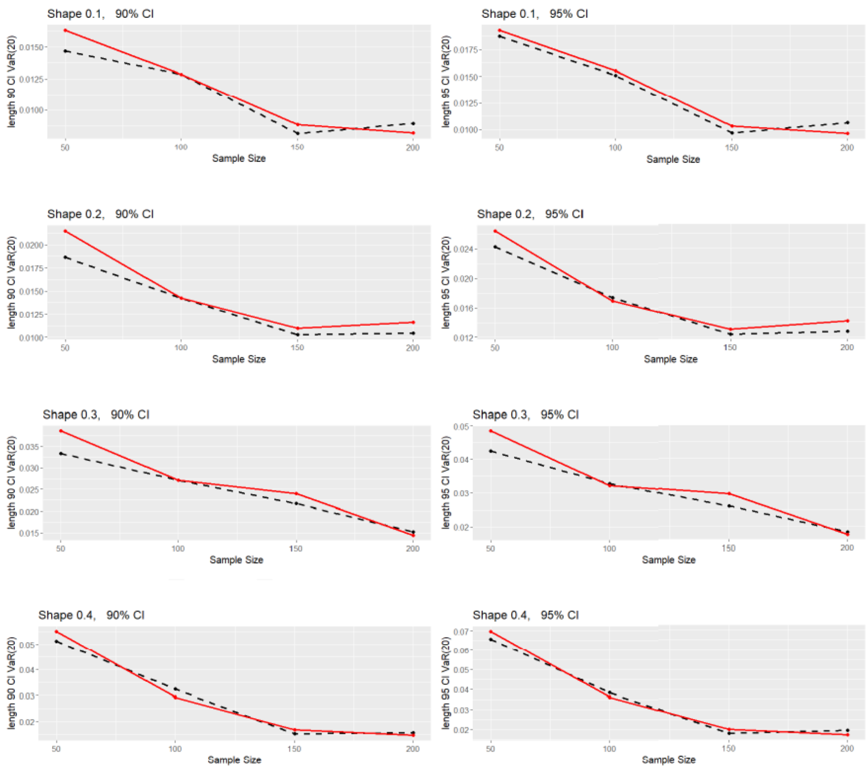

Figure 4: Width of confidence interval (CI) of VaR estimate at 20-months return period at $n = 50, 100, 150, 200$ and $\xi = 0.1, 0.2, 0.3, 0.4$. The left panel is 90% CI and right panel is 95% CI. Black dashed line represents width of CI by nonparametric approach while red solid line represents width of CI by parametric approach

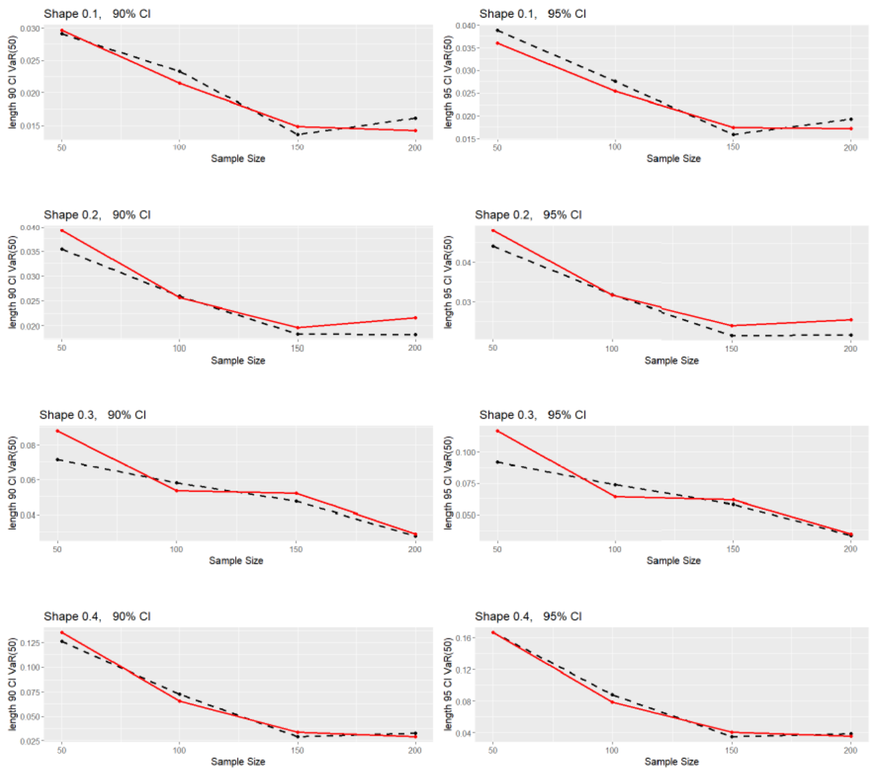

Figure 5: Width of confidence interval (CI) of VaR estimate at 50-months return period at $n = 50, 100, 150, 200$ and $\xi = 0.1, 0.2, 0.3, 0.4$. The left panel is 90% CI and right panel is 95% CI. Black dashed line represents width of CI by nonparametric approach while red solid line represents width of CI by parametric approach

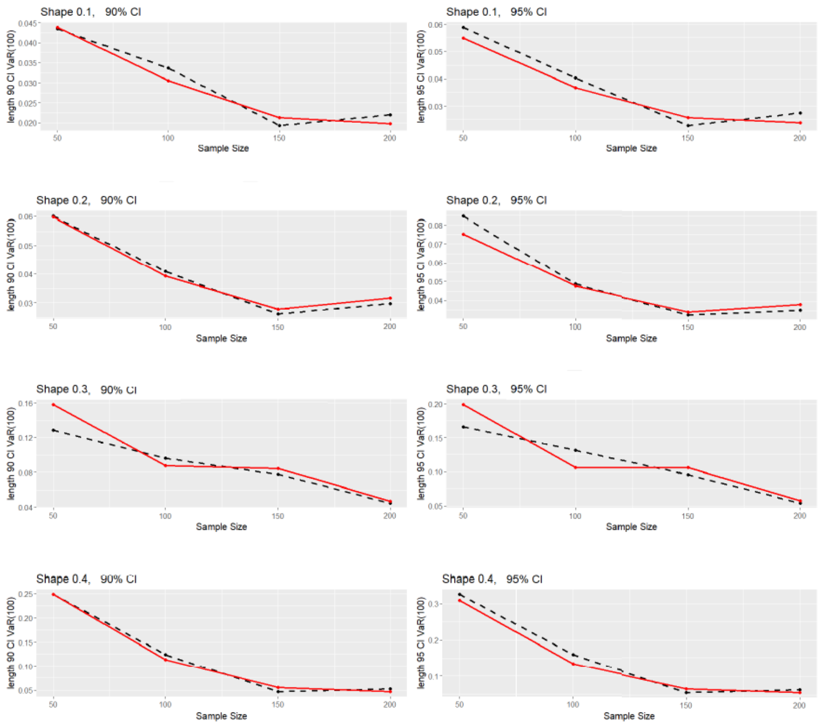

Figure 6: Width of confidence interval (CI) of VaR estimate at 100-months return period at $n = 50, 100, 150, 200$ and $\xi = 0.1, 0.2, 0.3, 0.4$. The left panel is 90% CI and right panel is 95% CI. Black dashed line represents width of CI by nonparametric approach while red solid line represents width of CI by parametric approach

VaR Estimates for Gold Price Data

Table 3 presents the VaR estimates with 90% and 95% confidence intervals constructed using nonparametric bootstraps for monthly maxima of negative daily log returns of gold price datasets. The table shows that for a 10-month return period, the estimated VaR is 3.16%, meaning there is a 10% probability that losses will exceed this value over any given 10-month period. This suggests that moderate losses of this magnitude occur relatively frequently. As the return period increases to 20 months, the estimated VaR rises to 3.9%, with the probability of exceedance dropping to 5%. This indicates that while the potential loss is slightly larger, the chances of it occurring become less frequent. For a 50-month return period, the VaR estimate further increases to 5.01%, and the probability of exceedance falls to 2%, showing that while the losses may become more severe, such extreme losses are rarer over this longer horizon. Finally, for the 100-month return period, the estimated VaR is 5.96%, with only a 1% chance of the loss exceeding this value. This reflects the risk of very large losses occurring over a long time frame, but with a very low likelihood. Overall, as the return period lengthens, the VaR estimates increase, indicating higher potential losses, while the probability of these extreme losses occurring decreases, highlighting the infrequent but severe nature of extreme events over extended periods.

The 95% CIs are consistently wider than the 90% CIs across all return periods. This is expected, as a higher confidence level necessitates a broader range of values to ensure that the true parameter is captured with greater assurance. For example, at the 10-month return period, the 90% CI is (2.835%, 3.546%), while the 95% CI expands to (2.781%, 3.613%). This pattern continues across all return periods, highlighting the trade-off between confidence and precision.

Table 3: Estimate of VaR with 90% and 95% confidence interval using nonparametric bootstrap

Return Period	Estimated VaR	90% CIs	95% CIs	Probability of Exceedances
10-month	0.03156	(0.02835, 0.03546)	(0.02781, 0.03613)	0.1
20-month	0.03904	(0.03365, 0.04514)	(0.03269, 0.04705)	0.05
50-month	0.05013	(0.04095, 0.05996)	(0.03942, 0.06221)	0.02
100-month	0.05962	(0.04652, 0.07431)	(0.04462, 0.07826)	0.01

CONCLUSION

This study has effectively demonstrated the application of Extreme Value Theory (EVT) in modeling and quantifying risk associated with gold price fluctuations in the Malaysian market. By analyzing monthly maxima of negative daily log returns using the Generalized Extreme Value (GEV) distribution, we found that EVT techniques successfully capture the heavy-tailed nature of gold price movements. Our findings align with those of Pratiwi et al. (2019), who similarly applied the GEV distribution for VaR analysis on gold prices. Their study, like ours, demonstrated the effectiveness of EVT techniques in capturing the extreme risk characteristics of gold price movements. The consistency of these results across different markets and time periods reinforces the robustness of EVT approaches in financial risk management, particularly for commodities like gold that can exhibit significant price volatility. Our comprehensive comparison of parametric and nonparametric bootstrap methods revealed that the nonparametric approach generally outperforms in quantifying uncertainty in Value-at-Risk (VaR) estimates, particularly for larger sample sizes and longer return periods. This was evidenced by narrower confidence intervals and lower error metrics produced by the nonparametric method.

The VaR estimates obtained from our analysis show increasing potential losses over longer time horizons, ranging from 3.16% for a 10-month period to 5.96% for a 100-month period, with correspondingly decreasing probabilities of occurrence. This highlights the infrequent but potentially severe nature of extreme events in the gold market.

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