ANALYSIS, DESIGN & COST COMPARISONS OF SIMPLY SUPPORTED AND CONTINUOUS BRIDGES

BY

ARDAL HASAN ESMAIL

A project report /thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in structural engineering and construction in the department of civil engineering
Faculty of Engineering
Universiti Putra Malaysia

FACULTY OF ENGINEERING
UNIVERSITI PUTRA MALAYSIA
2001

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ACKNOWLEDGEMENT

would like to take this opportunity to express my sincere thanks and utmost gratitude to my supervisor Dr. Mohd Saleh Jaafar for his able guidance, understanding and moral support extended to me through out the course of this study. His pleasant manners and ever lasting patience were a source of inspiration during the dark and frustrating hours of uncertainty and despair. I will remain ever grateful to him for his innumerable kindnesses.

would also like to thank my examiners Dr. Waleed Abdul Malik Thanoon and Dr. Anvar Ashrabov for critically examining the thesis and their valuable constructive criticism.

This acknowledgement would be utterly incomplete if I don't mention the remarkable support and help extended to me by my friends Hider Aad, Faris Jasim, Dr. Manoj Kulshreshtha, Rabee Habash and Wahab Hussain, for whom I express my most sincere thanks and heart felt appreciation.

ABSTRACT

This study was undertaken to compare the design and cost of simply supported and continuous bridges. Type of bridge selected is deck girder bridge where the girders are precast, prestressed of pretension system. All the spans in a given bridge were of equal length. Three different spans namely 20, 30 & 40m were considered. Different load cases were considered and analyzed using Finite Element Method to identify both, the critical load cases, in which the maximum forces occur, and the maximum design forces on which the design is based. Design of pretensioned SY- beams, slabs and diaphragms were carried out either using ready-made packages or manually. The cost of the bridges was estimated manually. The effect of temperature differences & non-uniform support settlements on the design and cost of these bridges were examined at seven levels of temperature differences and six settlement conditions.

It was observed that for a given span, moments in both simply supported and continuous bridges were maximum at similar loaded spans. If the bridges are designed for the primary force effects induced due to dead and live loads only, the continuous types are of lower cost compared to the simply supported types. The cost difference between the two types decreases as bridge span increases. In a continuous beam deck, non-linear temperature distribution across the deck depth and differential settlement of supports, cause additional sagging & hogging moments as well as shear forces, resulting in higher costs, and the economical superiority of continuous types vanishes at a certain temperature change or support settlement level. The effect of differential settlement is more pronounced than that of temperature changes.

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Introduction

1.1 - Bridge:

Bridge is a structure facilitating a communication route for carrying road traffic or other moving loads over a depression or obstruction such as river, stream, channel, road or railway. The communication route may be a railway track, a tramway, a roadway, a footpath, a cycle track or a combination of them. [20]

An ideal bridge meets the following requirements:

(i) It serves the intended function with utmost safety and convenience.

(ii) It is aesthetically sound.

(iii) It is economical. [20]

1.2 - Bridge engineering:

When it is necessary to build a bridge, the question arises: What kind of bridge is it necessary to build? From a design standpoint, there may be many possibilities. Thus the creative capability of the designer plays a large role in answering the question posed above. [2]

The creativity of the bridge designer must of course be grounded in the discipline of engineering. It is also necessary to have a technical mastery of the materials used to build bridges before the design process can begin. It is also important for the designer to perfect the methods of bridge building, thus advancing the art of bridge engineering. [2]

Bridge engineering, which began with stone and wooden structures as early as the first century, has undergone a dramatic evolution in terms of analysis and use of materials. [1] Today it is considered a science. However, about 100 years ago it was hardly worthy to be termed an art, and 150 years ago it was no better than a trade. But while bridge building as a learned profession is thus of relatively recent origin, it must not be thought that previous centuries made no contributions to our knowledge of bridge construction. [2]

The significance of this study is that it shows, in addition it's own objectives, how advanced analytical techniques can be used to analyze a bridge structure and simplify its overall design. The resulting validity will underscore the goal of modern practice in producing economies. It follows, therefore, that engineers should have a range of options from which to choose, and this should encompass bridge types, design philosophies, and construction procedures. [1]

1.3 - Components of a bridge:

A bridge is subdivided into:

a) Superstructure,

b) Substructure, and

c) Foundation

Bridge deck system is the part of superstructure directly carrying the vehicular loads. It is furnished with balustrades or parapets, crash barriers, highway surfacing, footpaths, traffic islands, railway tracks on ties, expansion joints and drainage systems.

Substructure comprises piers, columns or abutments, capping beams and bearings. Foundation consists of reinforced concrete footings, spread foundations, rafts bearing directly on soil or rock and capping stabs supported on piles, wells and caissons. [18]

The superstructure or the bridge deck system can be any one or a combination of the following:

- Slabs,
- Coffered slabs,
- Grids,
- Beams,
- Girders,
- Cantilevers,
- Frames,
- Trusses and arches,
- Cables,
- Suspenders and cable-stayed. [18]

Deck surface members may be classified into the three groups, which may be of:

(a) Precast, (b) Cast-in situ, and, (c) Composite construction. They may be of conventional steel reinforcement, partially or fully prestressed or composite construction. [18]

1.4 - Classification of bridges:

Bridges can be classified into various types depending upon the following factors: [20]

- Materials used for construction: Under this category, bridges may be classified as timber bridges, masonry bridges, steel bridges, reinforced cement concrete bridges, pre-stressed bridges and composite bridges.
- Alignment: Under this the bridge can be classified as straight or a skew bridge.

- Location of bridge floor: Under this category, bridges can be classified as deck, semi-through or through bridges.
- Purpose: Under this the bridge can be classified as a aqueduct, viaduct, highway bridge, railway, bridge and footbridge etc.
- Nature of superstructure action: Under this the bridges may be classified as portal frame bridges, truss bridges, balanced cantilever bridges and suspension bridges.
- Position of high flood level: Under this the bridges may be classified as submersible and non-submersible bridges.
- Life: Under this the bridges may be classified as permanent and temporary bridges.
- Loadings: Road bridges and culverts have been classified according to the loadings they are designed to carry.
- Fixed or movable: For navigable channels where permanent and sufficient clear waterway cannot be provided, the movable bridges used are swinging bridges, bascule bridges & lift bridges.
- Span length: Under this category the bridges can be classified as culverts (span less than 8 m), minor bridges (span between 8 to 30 m), major bridges (span above 30 m) and long span bridges (span above 120 m).
- Degree of redundancy: Under this the bridges can be classified as determinate bridges and indeterminate bridges.
- Type of connection: Under this category the steel bridges can be classified as pinned connected, riveted or welded bridges. [20]

1.5 - Development of bridge types:

It is said that the history of bridges is the history of civilization. However, achieving progress in bridge engineering was not an easy task. Bridges, as most other engineering structures, began with the "cut and try" process. Some less kind people say the "try and fail" process. [2] The pioneers used empirical methods. They made some intelligent guesses as to the strength required and built the bridge accordingly. Many centuries passed before man created the five basic types of bridges:

- The beam,
- The cantilever,
- The arch,
- The suspension, and
- The truss.

The first four types were copied from nature long before recorded history began. [2]

The natural example of the simple beam bridge is that of a fallen tree spanning a stream. The next step was to use a stone slab as a bridge quite probably, primitive man discovered the principle of the cantilever bridge at a very early stage of bridge development. He made use of a cantilever to construct longer spans than he was able to build with simple beams. Timber beams or stone slabs projecting out one above the other represented such bridges. [2]

Natural bridges of stone have also been formed, where the action of water has worn away rock until only an arch was left, high above the riverbed. [2]

The suspension or cable bridge is illustrated in nature by the swinging vine, utilized by animals and people to pass from one tree to another over a stream. In its simplest form a suspension bridge consists only of cables and unstiffened roads way. In primitive suspension bridges, the roadway was often laid on top of the cables. But this position was inconvenient, and bridge builders discovered that a level roadway could be obtained by suspending the roadway from the iron chain cable. [2]

The first suspension bridge using this system was erected in Italy in the sixteenth century. Since the beginning of the nineteenth century, flat iron bars were used for cables. Finally, the Truss-type bridge belongs almost exclusively to modern civilization. [2]

In the fifteenth century Leonardo da Vinci was the first to investigate the strength of beams and the forces in triangular structures in his design for a timber truss bridge. [2]

1.6 - Types of reinforced concrete bridges:

In general, a reinforced concrete bridge structure may consist of: [1]

- Deck slabs,
- T-beams (deck girders),
- Through and box girders,
- Rigid frames, and
- Flat slab types

Combinations of these with precasting or prestressing produce additional structural forms and enhance bridge versatility. A major advantage in the use of concrete is the broad variety of structural shapes and forms. In the selection of the proper type of bridge, however, cost is usually the determining criterion. Occasionally, the selection is complicated by factors such as the ratio of dead to live load, appearance, depth constraints and available headroom, limited construction time, labor costs, and difficulties in form work because of the support height or because of traffic maintenance requirements during construction. In this case steel bridges may be more cost-effective. [1]

1.7 - Precast beam bridges:

These widely used structures consist of precast-prestressed I-beams, T-beams, and box girders, which may be either pretensioned or posttensioned. Precast I-beams may be built with cast-in-place decks. With a precast, prestressed T-beam as with an I-beam, the flange must be connected with cast-in-place concrete. Precast, prestressed box sections may be placed side by side to form a bridge span. If necessary, they may be posttensioned transversely. Precast, prestressed beams are used mainly for spans up to about at locations where erection of false work is impossible or not desirable. Such beams are economical for mass fabrication. For longer spans it is necessary to provide heavy equipment for erection and / or transporting purposes. [2]

1.8 - Bridge analysis

1.8.1 - General:

Structural analysis is the process by which the structural engineer determines the response of a structure to specified loads or actions. This response is usually measured by establishing the forces and deformations throughout the structure. A given method of structural analysis is commonly expressed as a mathematical algorithm. However, it is based on information gained through the application of engineering mechanics theory, laboratory research, model and field experimentation, experience, and engineering judgment. [29]

The earliest demands for sophisticated analysis, coupled with some serious limitations on computational capability, led to a host of special techniques for solving a corresponding set of special problems. These so-called classical methods incorporated some ingenious innovations and served the needs of the structural engineer very well for many years. However, the advent and subsequent development of the digital computer increased computational capabilities by several orders of magnitude and thus obviated the need for special techniques. The ingenious specializations of the classical methods were replaced by the sweeping generalities of the modem matrix methods. [29]

The transition from the classical methods to the modem matrix methods has triggered some revolutionary changes in structural engineering and in the education of structural engineers. Although matrix methods have become the foundation of modem structural analysis as it is employed in the practice of structural engineering, classical methods continue to play a vital role in the educational process because they introduce the fundamentals of structural analysis. By either classical or matrix methods, the analysis process can be a part of preliminary design, final design, or construction, as was described in the preceding section. However, it is important to note that structural analysis plays a limited role in the structural design process and an even smaller role in the overall design process. Furthermore, the role that it plays is entirely supportive of the design process. [29]

1.8.2 - Methods of analysis:

Bridge analysis generally requires the solution of number of linear simultaneous equations, which depends on the method of analysis. Some methods avoid simultaneous equations by using iterative or successive correction techniques in order to reduce the amount of computation, and are suitable when the calculations are made by hand or by a hand-held or small desk calculator. [28]

In office practice, the design of bridges utilizes computers and many versatile software packages. Special computer programs have been developed, ranging from simple formula applications to elaborate analyses. With rapidly improving and expanding computer technology, the most precise but complex analytical techniques become routine options. The designer should be cautioned, however, that a computer program is only a tool, and hence the designer should clearly understand the basic assumption of the program and all output should be verified. Thus, any method of analysis that satisfies equilibrium and compatibility and has stress-strain relationships implanted in the process is acceptable. [1]

Computer-aided analysis, hand computation methods and charts based on some approximations and idealizations provided convenient methods of load distribution, which are reasonably accurate for design purposes. Many computer-aided methods have been developed with the advent of digital computers and are in use although some of these methods are highly numerical and expensive. Different techniques commonly in use for the analysis of bridge decks of various types and configurations are: [27]

- Courbon's method
- Orthotropic plate theory
- Finite difference method
- Method of harmonic analysis
- Grillage and space frame analogy
- Folded plate analysis
- Finite element method
- Finite strip method [27]

The main factors, which govern and influence the choice of analytical techniques, are:

1) Form of construction or type of deck
2) Plan-geometry of platform
3) Support conditions [27]

Each of the above techniques is more suited to a particular type or types of bridge decks depending upon the closeness of the actual structure with the assumptions of the method. It may be evident that one particular type of bridge deck can be analyzed by more than one method and in such cases, the choice rests with the designer depending upon the facility, time available, economy and of course his familiarity with the method. [27]

1.8.3 - Finite element analysis of bridge decks:

The finite element method employs an assemblage of discrete two or three-dimensional members. The elements are connected at nodal points, which possess an appropriate number of degrees of freedom. Material properties, often more than one, can be incorporated which will truly represent various components of the bridge deck. Static and dynamic problems can be easily tackled by this single method, thus making in the most powerful technique. [18]

The bridge is idealized as an assemblage of discrete parts, known as elements. The next stage is to evaluate the element properties. This is followed by the structural analysis of the element assemblage. Forces, stresses, moments, displacements, strains, etc. are calculated. Where a computer program is interactive, those results are plotted which would show the exact behavior of various components of the bridge. [18]

Normally, suitable graphical techniques are associated with any finite element program package. These are to be used to interpret meaningful results. [18]

1.9 - Bridge design

1.9.1 - General:

Bridge design should be based on relevant specifications and should demonstrate compliance with applicable standards to ensure credible results. Yet, optimum solutions can be obtained only when the designer understands the assumptions and limitations of analysis. In retrospect, codes are changed continuously but bridge behavior may not, and this difference is important in choosing the contents and the subject matter. In the same context, predicting bridge behavior from computer models has become common practice, and the results confirm the value of testing in verifying structural response. [1]

A critical aspect of the design of concrete bridges is their articulation which, if neglected, may lead to excessive local damage. The structure should be designed to ensure that movement due to: [4]

- Loading
- Ground deformation
- Thermal expansion or contraction
- Creep, and
- Shrinkage

does not induce stress levels in excess of permissible values. Stresses induced in a structure due to shrinkage and thermal movement or settlement of supports are dependent on the modulus of elasticity of the concrete and thus will be modified due to creep. [4]

British design standards adopt limit state philosophy, where the primary limit states for concrete bridges are: collapse, and serviceability. Partial safety factors for loadings and materials will be incorporated. At present the principal design standards are those for loading and stresses. [16]

Loading is covered by Departmental Standard 37/88 (Loads for Highway Bridges) and consists of:

- **Type HA**, an equivalent lane loading, which is the normal design loading, and
- **Type HB**, an abnormal unit loading to be used when specified by the appropriate authority.

Permissible stresses for reinforced highway structures are covered by BS 5400: part 4: Section 6 (Steel, concrete and composite Bridges: Code of practice for design of concrete bridges), which stipulates the use of elastic theory with a modular ratio of 15 for assessing the strength of members. Ultimate strength calculations are not required as a factor of safety is incorporated in the permissible stress levels. Crack width calculations are required as a serviceability check. [16]

A somewhat different approach is adopted for prestressed highway structures, where a mixture of elastic and ultimate load methods is given. The elastic analysis is based on stress limits in the order of 21 N/mm^2 compression and zero tension for type HA loading. [16]

The downward movement of a footing, approach pavement, or structure due to deformations and / or changes in the soil properties is known as settlement. Settlement can be initiated by a number of factors, which include, but are not limited to: [4]

- Overloading the earth at the site
- Lowering the water table
- Vibrations from live loads or seismic loads

- Loading embankments
- Changes in soil properties

Of particular concern to the bridge engineer are differerential settlements where a foundation will move downward in an uneven fashion. Such settlements can induce additional forces resulting in cracking and instability at superstructure elements. [4]

The effects of thermal forces on a structure are significant and should not be underestimated by the designer. Like the adverse effects, which result from uneven settlement, structures can suffer from uneven temperature distribution along concrete depth causing high thermal forces. [4]

1.9.2 - Design methods:

In bridge engineering, there are two principal methods of design in use today. The names used to define these design methods vary depending on the structural material being used, the design code being referenced, or even the era of a publication. For the purposes of this study, we will classify the two design methods as: [4]

- ***Working stress design***
- ***Limit states design***

For most of the century, the working stress design approach was the standard by which bridges and other structural engineering projects were designed. By the 1970's, however, limit states design began to gain acceptance by the general engineering community. What are these two approaches to design and how do they differ? Is one better than the other? To answer these questions, it is first necessary to understand the concepts behind each approach. The following offers both a background and overview of these two design methods and how they apply to the design of structures in general and bridges in particular: [4]

1.9.2.1 - Working stress design:

Working stress design is an approach in which structural members are designed so that unit stresses do not exceed a predefined allowable stress. The allowable stress is defined by a limiting stress divided by a factor of safety, so that, in general, working stress is expressed in the form of: [4]

$f_{actual} \leq f_{allowable}$

For a beam, this actual stress would be defined by:

$f_{actual} = \frac{M.C}{I}$, Where

M = Maximum Moment

C = Distance to the Neutral Axis from the Extreme Fiber

I = Moment of inertia of the Beam Cross Section

And the allowable stress could be given by:

$f_{allowable} = \frac{fy}{FS}$, Where

fy = Minimum Yield Stress, and

FS = Factor of Safety

The allowable stress could also be defined by some other controlling criterion such as the buckling stress for steel, compressive strength of concrete, etc. Thus, the allowable stress can be thought of as a fraction of some failure stress for a given material like steel or concrete. Under working stress approach, the actual stresses are representative of stresses due to the service or working loads that a structure is supposed to carry. The entire structure is designed to fall well within the elastic range of the material the element or component is constructed with. [4]

1.9.2.2 - Limit states design:

The limit states design method was, in part, developed to address the drawbacks to the working stress approach mentioned above. This approach makes use of the plastic range for the design of structural members and incorporates load factors to take into account the inherit variability of loading configurations. [4]

A limit state can be defined as a condition representing structural usefulness. As mentioned previously, working stress suffered from the inability of the factor of safety to adequately address the variable nature of loading conditions. One of the advantages of the limit states approach is that it takes into account this variance by defining limit states, which address strength and serviceability. Bridge designer can think of these terms in the following way: [4]

- Strength is the limit state which defines the safe operation and adequacy of the structure. The criterion, which is used to define this are: yielding, plastic strength, fatigue, buckling, overturning, etc.
- Serviceability is the limit state, which defines the performance and behavior of the structure. Some serviceability criterion are: deflection, vibration, drift, etc. [4]

From the above, it is easy to see why limit states design codes, place a great deal of importance on the strength limit state, since this is the one that is concerned with public safety for the life, limb and property of human beings. This is why the strength limit state is also often referred to as the safety limit state, obviously, the limit states for strength will vary depending on the type of member being designed, its material properties, and the given loading condition. [4]

Therefore, like working stress design, limit states design methods vary depending on the material being used and its related design specification. In general, we can define the limit states equation as:

Strength provided $\geq$ Strength required (Axial Force, Shear, or bending caused by factored loads).

The strength provided is defined by the specification applicable to the design of the member. The strength required is computed using conventional analysis methods and multiplying computed values by appropriate load factors. This can be translated symbolically into an equation whose form is: [4]

$\phi S_n \geq \sum \psi_i L_i$, Where

ϕ = Strength reduction factor

S_n = Nominal strength

Li = Service load acting on the member

ψ_i = Load factor pertaining to uncertainty of L_i

Thus, the right half of the equation above represents the sum of individual loads; each multiplied by its specific load factor. If we were simply considering dead and live loads, the strength required would be dead load times some factor plus live load times another factor. Specific values for load factors are provided by the applicable design codes. [4]

1.10 - Economics:

A comparison of bridge costs is meaningless unless the figures are related to similar design standards, economic climate and site conditions. [16]

There is no one form of design which would be always most economical. It is only by comparing a few designs that the economic design can be found in a particular set of conditions. However, sometimes the quantities of concrete and steel expressed per square meter of deck area are quoted as indicative of economy although these figures are not the only ones which govern the cost of bridge. The results of different research reports on cost estimates and comparative economics of bridge structures indicate that **the most economical schemes appears to be consisting of pre-cast pre-stressed girders simply supported for self-weight and continuous over piers for finishings and live load by appropriately cast-in-situ reinforced deck slab & diaphragms, (i.e. semi-continuous deck).** [5]

Experience of reinforced concrete bridges constructed in the early years of the last century has shown that jointless construction can last for 60 years or more. In cases where damage occurred it appears to have been due mainly to fatigue processes. Such damage may have been less expensive to repair than articulated bridges having failed expansion joints and consequential corrosion. [3]

1.11 - Scope of the study:

This study deals with the analysis, design and cost comparison of two types of bridges (in terms of support condition), namely: simply supported & continuous bridges. It also deals with the effect of temperature and differential settlement on the design and cost comparison. The specific details outlining the scope of this study are presented below:

1) Span length: 20m, 30m & 40m
2) Bridge width: Dual carriageway of tow lanes with walkway at both sides & crash barrier at the center.
3) Material used: Reinforced concrete
4) Type of bridge: Deck girder bridge with precast, pretentioned girders
5) Bridge alignment: Right bridge
6) Support condition: Simply supported & continuous.
7) Deck type: Solid, cast in situ slab with precast beams.
8) Type of analysis: Elastic.
9) Scope of analysis: Dead & live load cases, temperature case & settlement case.
10) Method of analysis: Finite Element Method.
11) Scope of design: Pretentioned Y-beams, slabs & diaphragms.
12) Coast analysis: For comparison purposes not to evaluate the whole bridge.
13) Standard Specifications: BS 5400, BS 8110, BD 37/ 88.

1.12 - Objectives:

It is generally believed that continuous bridge is more economical than the simply supported because the magnitudes of primary forces induced due to dead & live loads are less in continuous type resulting in a lesser material and labor consumed. However, it is not very definite if this relative economy of continuous bridge will be there in all cases, or all bridge span lengths, or not.

Moreover, the design of these two bridges will be influenced by different load cases. Some of these cases will be critical for design. The design of bridges should be based on these critical cases, which need to be identified.

It is well recognized that the temperature differences existing in concrete structures will cause additional stresses, which need to be accounted for in the design of continuous bridges.

Similarly, differential settlement among different supports will have significant effect on the design. Both these factors will tend to increase the stresses resulting into a higher cost for continuous bridges.

It may be noted that the design of simply supported bridge is not influenced by temperature differences and differential settlement of supports. In such a situation, it is quite natural that the economic superiority of continuous bridges over the simply supported bridges may be adversely affected due to these factors.

In view of the above, it is important to examine and quantify the influence of the bridge span, temperature differences and support settlement on the relative economics of these two types of bridges.

Therefore, **this study was undertaken with the following specific objectives:**

1) **Comparison of the design and cost of simply supported and continuous bridges for different span ranges, and**
2) **Examine the effect of temperature and support settlement on the design and cost of continuous bridges.**

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