EFFECTS OF CENTRIFUGE LOADING ON ROCK SLOPE STABILITY USING NUMERICAL MODELLING

By

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Chair : Zainuddin Bin Md Yusoff, PhD
Faculty : Engineering

The shear strength of the rupture surface is often assumed to be the cause of a rock slope's stability. Natural slopes often feature discontinuous rupture surfaces composed of fractures and joints separated by massive rock blocks. In such cases, the strength of the rupture surface is composed of three components: friction, cohesion, and tensile strength. The structure of the rock slope, on the other hand, has a significant influence on rock slope stability. While the influence of the shear strength components, cohesion, and friction on slope stability is well established, little study has been conducted on the role of joint spacing in rock slope stability, which affects the shear and tension strengths of the rock layers in the rock slope.

This study aims to determine the effect of joint spacing on rock slope when it fails in a toppling mode. The rock slope was constructed with a joint spacing of 10 mm based on a laboratory case study. In that case study, the joint set dips deeply into the slope surface, which results in a flexural toppling failure. In this research, rock slope with toppling failure is described as three various sizes of thin slabs of rock moving out of the slope and finally creating a rupture surface. These three joint spacings are smaller, actual, and bigger than the one joint spacing in the case study. Slip between the thin rock layers and tensile rupture across the slabs are both involved in the toppling process. Since there is only one joint set in that case study, only flexural toppling is addressed; as a result, another joint set is added to the rock slope as a secondary joint set to make the model more realistic. On the other hand, the secondary joint set dips in the opposite direction of the main joint set in order to investigate another kind of toppling.

A mixture of cement, sand, and water was employed to create synthetic rock slope specimens. Taguchi and Response Surface Methodology (RSM) combined approaches were used to build the appropriate rock sample to run the other tests (such as the direct shear test) and verify the components to get the best results while decreasing the number of tests and the expense. The resultant combination was then used to create rock layers of varying thicknesses for geotechnical centrifuge testing and numerical modeling based on a distinct element framework.

This study demonstrates that combining Taguchi and RSM techniques is an appropriate strategy for optimization. The experimental findings are within -0.69-0.90 percent of the model's expected values. In the numerical modeling approach, the impact of joint spacing is larger in the middle than in the crest. In the first model representing flexural toppling, failure occurred between 37 and 48 g, with a maximum displacement of two millimeters at the crest and 1.2 millimeters in the middle of the slope. While model B with block toppling failed in the range of 11 to 17 g and with displacements of 0.04 to 0.07 mm at the crest and 0.04 to 0.05 mm in the middle, its gravity loading was much lower. This study shows that the discontinuities inside the slabs of rock slopes are critical in developing the rupture surface. The spacing of rock slabs has a minor influence on toppling slopes, while adding a secondary joint set has a significant role in controlling the slope's stability. In addition, the created rupture surface influences the deformations in the crest and middle of the slope surface. The observed deformation patterns, the propagation of the rupture surface, and the initial condition of collapse were all in good agreement when the results of distinctive element modeling were compared to the results of laboratory scales. The findings suggest that the structure inside the rock slope significantly impacts the stability of toppling slopes.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KESAN PEMBATAN ENTRIF TERHADAP KESTABILAN CERUN BATU MENGGUNAKAN PEMODELAN NUMERIK

Oleh

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Kekuatan ricih permukaan pecah sering diandaikan sebagai punca kestabilan cerun batuan. Cerun semulajadi selalunya mempunyai permukaan pecah terputus yang terdiri daripada patah dan sendi yang dipisahkan oleh bongkah batu besar. Dalam kes sedemikian, kekuatan permukaan pecah terdiri daripada tiga komponen: geseran, kohesi, dan kekuatan tegangan. Struktur cerun batuan pula mempunyai pengaruh yang signifikan terhadap kestabilan cerun batuan. Walaupun pengaruh komponen kekuatan ricih, kohesi dan geseran ke atas kestabilan cerun sudah mantap, sedikit kajian telah dijalankan tentang peranan jarak sendi dalam kestabilan cerun batuan, yang mempengaruhi kekuatan ricih dan tegangan lapisan batuan dalam cerun batuan.

Kajian ini bertujuan untuk menentukan kesan jarak sendi pada cerun batuan apabila ia gagal dalam mod toppling. Cerun batu itu dibina dengan jarak sambungan 10 mm berdasarkan kajian kes makmal. Dalam kajian kes itu, set sambungan merendam jauh ke dalam permukaan cerun, yang mengakibatkan kegagalan tumbang lentur. Dalam penyelidikan ini, cerun batuan dengan kegagalan tumbang digambarkan sebagai tiga pelbagai saiz papak batu nipis yang bergerak keluar dari cerun dan akhirnya mencipta permukaan pecah. Tiga jarak sambungan ini lebih kecil, sebenar dan lebih besar daripada satu jarak sambungan dalam kajian kes. Gelinciran antara lapisan batuan nipis dan pecah tegangan merentasi papak kedua-duanya terlibat dalam proses tumbang. Oleh kerana hanya terdapat satu set sendi dalam kajian kes itu, hanya toppling lenturan ditangani; akibatnya, satu lagi set sambungan ditambah pada cerun batu sebagai set sambungan sekunder untuk menjadikan model lebih realistik. Sebaliknya, set sambungan sekunder menurun ke arah yang bertentangan dengan set sendi utama untuk menyiasat satu lagi jenis topping.

Campuran simen, pasir, dan air digunakan untuk menghasilkan spesimen cerun batuan sintetik. Pendekatan gabungan Taguchi dan Response Surface Methodology (RSM) digunakan untuk membina sampel batu yang sesuai untuk menjalankan ujian lain (seperti ujian ricih langsung) dan mengesahkan komponen untuk mendapatkan hasil terbaik sambil mengurangkan bilangan ujian dan perbelanjaan. Gabungan yang terhasil kemudiannya digunakan untuk mencipta lapisan batuan dengan ketebalan yang berbeza-beza untuk ujian emparan geoteknikal dan pemodelan berangka berdasarkan rangka kerja elemen yang berbeza.

Kajian ini menunjukkan bahawa menggabungkan teknik Taguchi dan RSM adalah strategi yang sesuai untuk pengoptimuman. Penemuan eksperimen adalah dalam lingkungan -0.69-0.90 peratus daripada nilai jangkaan model. Dalam pendekatan pemodelan berangka, impak jarak sambungan lebih besar di tengah berbanding di puncak. Dalam model pertama yang mewakili tumbang lentur, kegagalan berlaku antara 37 dan 48 g, dengan anjakan maksimum dua milimeter pada puncak dan 1.2 milimeter di tengah cerun. Walaupun model B dengan blok tumbang gagal dalam julat 11 hingga 17 g dan dengan anjakan 0.04 hingga 0.07 mm pada puncak dan 0.04 hingga 0.05 mm di tengah, beban gravitinya jauh lebih rendah. Kajian ini menunjukkan bahawa ketakselanjaran di dalam papak cerun batuan adalah kritikal dalam membangunkan permukaan pecah. Jarak papak batuan mempunyai pengaruh kecil pada cerun yang meruntuhkan manakala penambahan set sambungan sekunder mempunyai peranan penting dalam mengawal kestabilan cerun. Di samping itu, permukaan pecah yang dicipta mempengaruhi ubah bentuk pada puncak dan tengah permukaan cerun. Corak ubah bentuk yang diperhatikan, perambatan permukaan pecah, dan keadaan awal keruntuhan semuanya dalam persetujuan yang baik apabila keputusan pemodelan unsur tersendiri dibandingkan dengan keputusan skala makmal. Penemuan menunjukkan bahawa struktur di dalam cerun batu memberi kesan ketara kepada kestabilan cerun yang runtuh.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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LIST OF ABBREVIATIONS

ANOVA	Analysis of variance
C	Cement
LVDT	Linear Variable Differential Transformer
^{o}C	degree Celsius
RSM	Response Surface Methodology
S	Sand
UCS	Uniaxial Compressive Strength
UPM	Universiti Putra Malaysia
W	Water

CHAPTER 1

INTRODUCTION

1.1 Background

Rock slopes are extraordinarily heterogeneous and correlated with several unknown variables such as the stress condition, the structures within the rock mass, and the strength parameters. Those three variables influence the nature and failure process of the rock slopes. Depending on the loading process and stress path in the field or the laboratory, the rock may fail in shear or tension. There are two significant rock slopes: structurally controlled slopes, like planer, wedge, circular, and toppling failures, and non-structurally controlled slopes. Usually, slopes that are structurally controlled collapse due to shear sliding over one or more persistent discontinuities. In comparison, failure is a complex process in the non-structurally regulated slopes and requires failure in both discontinuity and intact material (Li et al., 2014, 2015; Robertson, 1970; Terzaghi, 1962).

The notion that a single discontinuity controls slope failure is a simple approach to researching rock slopes that is only relevant to small-scale slopes, while the continuity is restricted for large slopes unless there is a fault or other large rock formation before failure. Many research types have addressed this issue and concluded that rock joints in large-scale slopes are seldom continuous and that intact rock persists between the joint segments (Einstein et al., 1983; Nichol et al., 2002; Park, 2005; Terzaghi, 1962). These researchers proposed that the fracture surface in the rock slopes moved through the intact rock to create a kinematically permissible break surface. This mechanism can occur progressively, and surface rupture extends from the current rock joints through the intact rock. Some stepping is necessary to develop a kinematically permissible rupture surface. The concentrations of tensile stress at the rock bridges and tips of the joints are essential for this stepping (Ladanyi & Archambault, 1980; Li et al., 2009; Shen et al., 1995). According to some studies, these rock bridges fail under tension, and tensile strength plays a key role in the failure process of rock slopes in regions with low confinement stress (Einstein et al., 1983; Lajtai, 1969a; Shen et al., 1995).

Lajtai (1969a) found fascinating direct shear results for bridges with rock-like content and claimed that the bridges collapsed via a variety of processes depending on the normal stress level. Additionally, he utilized a nonlinear failure envelope to illustrate the impact of normal load on the rock's shear strength. The friction and cohesion did not mobilize simultaneously under direct shear stress; instead, the cohesion mobilized first, followed by the friction, allowing gradual failure to occur inside the rock mass(Lajtai, 1969a). Rock bridges must withstand all stresses due to the direct shear strength of a single plane of weakness in which the joints are open; nevertheless, in closed joints, depending on the degree of mobilization, friction may or may not provide an additional source of strength. The process of non-simultaneous brittle-strength parameter mobilization was also observed in an excavation of a circular test tunnel in a massive brittle rock, which resulted in failure around the tunnel, a brittle-failure process characterized by a loss of cohesion as friction was mobilized (Martin, 1997). Hajiabdolmajid and Kaiser (2002) utilized a "brittleness index" to evaluate the Frank Slide. They applied the idea of cohesion weakening and friction strengthening at various plastic strain levels in a continuum modeling framework. According to these experts, frictional strength begins at zero and is mobilized as plastic strain rises, while cohesiveness is gradually dissolved as damage to the rock mass increases. They developed a model for the direct shear test and compared their findings to those of Lajtai. These findings demonstrated clearly that the jointed rock mass has a non-linear failure envelope. Additionally, the Hoek-Brown failure criteria indicated a non-linear failure envelope for the rock mass, in contrast to the Mohr-Coulomb criterion's linear failure envelope (Hoek, Evert and Brown, 1980). Although rock bridges across discontinuities often fail in tension and shear failure occurs as a secondary failure, the impact of tensile strength on rock slopes is seldom considered, owing to a lack of sufficient failure criteria and tools for accounting for the tensile strength effect (Einstein et al., 1983; Shen et al., 1995; Sjöberg, 1999). The inclination of the bridge between preexisting fractures results in a variety of rock failures. Failures may originate at the tips of pre-existing fractures and propagate toward the center of the bridge, or they might originate in the center of the bridge and propagate toward the tips. This thesis presents new modeling techniques by varying the distance between the bridges in order to account for the impact of tensile strength on rock slope instability. It also investigates whether this modeling method accurately captures the behavior and instability of rock slopes.

1.2 Statement of the Problem

Rock masses are heterogeneous and consist of intact blocks as well as structural planes such as faults, bedding planes, and joints, which are seldom continuous. Terzaghi (1962) and Einstein et al. (1983) indicated that discontinuities' persistence is limited in nature, and there was a need for a complex interaction between pre-existing joints and the brittle propagation of fractures across the intact rock to have a rupture surface.

In rock slope, the rupture surface happens when a stable rock slope deteriorates into a minimally stable state and ultimately fails as a result of external factors; including geometrical changes (unloading toe, loading crest), shock, and/or internal factors; such as promoting progressive failure, progressive rock mass degradation due to cracking, decrease mobilized strength within the rock and/or increase driving stresses (Hajiabdolmajid & Kaiser, 2002). Time-dependent processes, including brittle strength deterioration and progressive failure, are more likely to have been major contributors to the collapse of the slope (Eberhardt et al., 2004). Friction, cohesion, and tensile strength are the strength components of rock masses. The time-dependent deterioration of the strength barely influences the friction angle, whereas its cohesion and tension are vulnerable. Slippage takes place along in- and out-dipping joints, and for any given natural slope exposed to flexural or block toppling, variables affecting the tensile strength of the intact rock, such as weathering and the persistence of cross-cutting joints, may have a major effect on the slope's performance (Alzo’ubi et al., 2010). Tensile strength is less than cohesiveness in natural rock masses. Tensile strength is lost before cohesion if cohesion and tension both deteriorate at the same rate (Alzo’ubi, 2009).

Lajtai (1969b) studied the system of direct shear tests in-depth and their advantages and disadvantages, explaining its compliance with the natural loading conditions. He contended that the major principal stress on the bridges in non-continuous joints is tensile stress, even though the whole stresses are compression, and that the development of tension cracks inside rock bridges arises from such tensile stresses (Figure 1.1). In flexural toppling, after a joint slip is activated at more joints, the rock columns at the toe start to be compressed, which establishes a space for small rotation of the columns. These block rotations will result in a tensile bending failure at the base of the toppling columns.

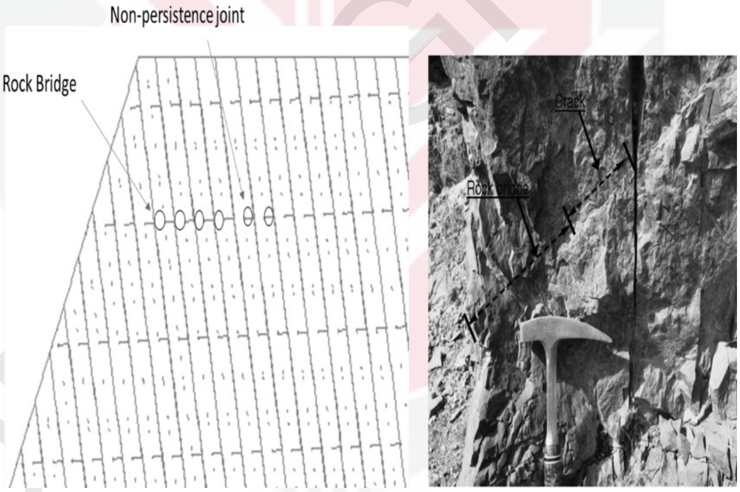

Figure 1.1: Definition of rock bridge and crack in a rock column poses the risk of flexural toppling

A rock mass can be represented as blocks of random size bounded to each other with cohesion, stress, and friction. When the ties between the blocks are breached, failure occurs. Once stresses exceed the strength of the rock, the discrete element method allows failure to be triggered and spread throughout the rock mass. In nature, rocks consist of complicated grain forms that interlock throughout the rock mass. While the rock block is subjected to stresses, localized tensile stress in the direction of the least principal stress may result in failure, especially in the case of low-confinement stress.

Cundall and Strack (1979) introduced the discrete numerical modeling approach. The discrete element approach is improved with a new degree of freedom by a Voronoi tessellation generator that generates random blocks within the rock mass. This technique can generate blocks of random size to replicate rock mass heterogeneity and induce tensile stress (Alzo’ubi et al., 2010; Alzo’ubi, 2009). The strength is related to both the rock bridge and the joint section in non-persistent jointed rock masses. In the conventional architecture method, limit equilibrium, the strength is assumed to be controlled by the joints, and the bridge's strength is ignored. Consequently, the designs dependent on this method are conservative since a small bridge will substantially contribute to the strength of the rock mass.

Furthermore, the tensile strength is critical for the flexural and block toppling mechanism and for controlling the toppling failure load. However, recent studies indicate that the deformation pattern seems to be indifferent to the tensile strength (Alzo’ubi et al., 2010; Lian et al., 2018). On the other hand, increasing or decreasing the rock layers' thicknesses can affect the rock blocks' strength, resulting in changing the failure pattern. The rock mass's internal deformation, brittleness, and ductility result in a complex failure process. Einstein et al. (1983) and Shen et al. (1995) also concluded that rock bridges collapse under tension, and the second phenomenon is a shear failure.

The tensile strength of the intact rock in weathered rock is greatly influenced by the degree of weathering and cross-cutting joint persistence (Alzo’ubi et al., 2010; Aydin & Basu, 2006). Cut slopes in rock masses begin to erode immediately after excavation owing to stress release and weathering, according to a case study on a rock slope in a humid tropical environment in Sabah, Malaysia. In Malaysia, toppling failures are uncommon. However, rock slope failures such as slide, wedge, and toppling have been documented in the Cameron Highlands. Given the significance of rock layer strength and its interaction with weathering through time, the impact of its significance should not be overlooked. The degradation is a time-dependent process that is affected by the local climate, rock mass, history, and environment. The quantity of degradation per time unit ('the weathering intensity rate') is not constant throughout time but is more prominent while the mass is less weathered and decreases as weathering progress (Tating et al., 2013). The findings of their investigation indicate that the optimal relationship between intact rock strength and exposure duration is through a logarithmic function. This conclusion demonstrates the significance of the effects of tensile strength deterioration on the rocky slopes, and in order to determine the stability of the rock slopes, this impact should not be ignored, especially in tropical areas.

Rock slope failure mechanisms should be investigated, considering the influence of tensile fracture on the kinematics of rock failure and the strength obtained from the bridges and joints. These processes are investigated by analyzing the effect of introducing an additional degree of freedom throughout rock mass to create initiation, spread, and coalescence of fracture initiation among preexisting discontinuous joints or along the discontinuous basal slip surface. Field experiments and computational models have demonstrated that fracture initiation and distribution in brittle rocks are critical factors in progressive failures and typically contribute to catastrophic failure. For this cause, the simulation of fracture propagation and rock mass collapse is significant in modeling the failure phase.

In rock slope modeling, in addition to the factors described in rock tensile strength, the mechanical characteristics of discontinuities, such as shear strength, its geometrical aspects, including the angle of joints, joint spacing, and persistence of joints, have a substantial impact on the kind of failure and its stability. Changing the thickness of rock layers may lead the model to become totally continuous even if the density of the joint formed by its spacing is increased or decreased; this behavior, half of which is controlled by a change in the rock's shear and tensile strength, is essential. In addition, the continuity of a joint set, measured in terms of a percentage, is also of significance. It has been proven that these joint sets do not form a totally continuous sequence in nature. Naturally, this discontinuity level can remain the same even when the distance between the joints changes. However, it is critical to determine whether or not the amount of shear and tensile strength, which is responsible for the change in the propagation of the fracture surface, will also remain the same.

1.3 Scope of the Thesis

This thesis focused on exploring the processes and mechanisms of rock slope failure by developing a centrifuge model with the assistance of a distinct element approach capable of modeling the damage process within rock masses and capturing the effect of tensile and shear strength degradation on rock slope instability.

1.4 Objectives of the Study

The main aim of this research is to explore the behavior of a rock slope stability built with synthetic rock layers dip-in and -out of the slope under gravity loading with different joint spacing by using geotechnical centrifuge and numerical modeling. To achieve this aim, the following objectives were focused:

a) To obtain the effect of synthetic rock specimens’ composition on uniaxial compressive strength
b) To evaluate the state of flexural and block toppling failures under gravitational loading with different joint spacing
c) To determine the extent of joint spacing variations impact on the flexural toppling and block toppling failures

1.5 Limitations of the Study

This research focused only on the stability of rock slopes that had a tendency for two forms of toppling failure: flexural and block toppling. The findings of these two forms were acquired from a limited number of tests with the aim of having the joint spacing effect while being subjected to centrifugal force. It was necessary to conduct a large number of experiments using a variety of different types of geometry on the layered rock. The most significant restrictions might be broken down into three categories dealing with different aspects of the research: sample preparation, physical modeling, and numerical modeling.

Regarding the preparation of synthetic rock samples and joints, it should be noted that synthetic rock has been made to resemble natural rock as closely as possible. In contrast, in nature, real rock undergoes different changes and transformations, including weathering, which can be different, particularly in a tropical region such as Malaysia, where the amount of rainfall and the percentage of humidity are very high and have a significant impact on the resistance. In addition, the surface of the joints in this model has been smoothed and connected with materials such as grease to increase its shear resistance. However, the nature of the surface sections prevented them from being perfectly smooth, and the sole purpose of these cross sections was to control the joints in modeling. Adding additional materials, such as glue or even concrete, would significantly increase the shear strength of the joints, but it would also lengthen the modeling process and make it uncontrollable.

One of the most important aspects of physical modeling was the model's size and weight. Increasing the model's weight would make it unstable when the model was spinning, resulting in incorrect results. Using professional devices such as a stroboscope and a camera would help collect images showing the model's changes during each phase of gravity loading, and its absence would result in just estimating the centrifuge loading.

In numerical modeling, when the model was impacted by loading and failed, the amount of displacement in various parts of the sample might be the same as its physical type, but the propagation of failure in the actual case may vary from the propagation of failure in numerical modeling. Creating an internal flaw in the intact rock might mitigate this disparity to some degree, but it would significantly increase the modeling time.

1.6 Organization of Thesis

The thesis is organized into five chapters. This chapter (Chapter 1) presents the thesis's problem, the research objectives, and the organization of the thesis adapted to support the thesis hypothesis.

Chapter 2 discusses the current state of knowledge in rock slope engineering. The discussion covers different rock types of toppling failures in experimental studies and the methods of analysis.

The proposed method of rock slope analysis is introduced in Chapter 3. Synthetic specimens of intact rocks and rock slabs are used to identify the specimen's properties and simulate the rock slope stability.

In Chapter 4, several kinds of geotechnical centrifuge-numerical methods on rock slopes are simulated to evaluate rock slope stability. The centrifuge-numerical tests will examine the discrete element model to capture the rupture surface of the rock slope. In this chapter, the deformation pattern is observed in the laboratory as compared to DEM.

Finally, Chapter 5 provides the current research's findings and suggestions. This chapter contains an outline of the research's findings.

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