education sciences

MDPI

Article

Investigating the Impact of the Stratified Cognitive Apprenticeship Model on High School Students' Math Performance

Ruimei Wang [1], Nurul Nadwa Zulkifli [1,2,*] and Ahmad Fauzi Mohd Ayub [1,3]

[1] Institute for Mathematical Research, Universiti Putra Malaysia, Serdang 43400, Selangor, Malaysia; gs59190@student.upm.edu.my (R.W.); afmy@upm.edu.my (A.F.M.A.)
[2] Faculty of Humanities, Management & Science, Universiti Putra Malaysia, Sarawak Campus, Bintulu 97008, Sarawak, Malaysia
[3] Faculty of Educational Studies, Universiti Putra Malaysia, Serdang 43400, Selangor, Malaysia
* Correspondence: nurulnadwa@upm.edu.my

Abstract: This study assessed the impact of a cognitive apprenticeship model (CAM)-based stratified teaching module on the mathematical proficiency of high school students. The stratified cognitive apprenticeship model teaching module (SCTM) first involves grouping students based on their mathematical abilities. Students with higher performance are placed in one class, while those with lower scores are placed in another. Instruction for each group is then conducted using the cognitive apprenticeship model, tailoring the teaching approach to align with the specific needs and abilities of each group. A quasi-experimental design was adopted and 150 students were recruited. This study compared the outcomes of a control group, which was instructed using conventional teaching methods (CI), with those of two experimental groups—one instructed using a stratified cognitive teaching method (SCTM)-based on the CAM—and another instructed using the CAM alone. Students' performance was evaluated based on a mathematics test including the following dimensions: knowing and understanding, investigating, communication, and application (of mathematical knowledge to real-life problems). The data were analyzed using an analysis of covariance (ANCOVA). The results indicated that students instructed using the SCTM outperformed their peers in mathematical achievement, thereby validating SCTM's effectiveness as a comprehensive educational strategy for mathematics education at the senior high school level.

Keywords: cognitive apprenticeship model; mathematics education; stratified teaching

check for updates

Citation: Wang, R.; Zulkifli, N.N.; Mohd Ayub, A.F. Investigating the Impact of the Stratified Cognitive Apprenticeship Model on High School Students' Math Performance. *Educ. Sci.* **2024**, *14*, 898. https://doi.org/10.3390/educsci14080898

Academic Editor: Myint Swe Khine

Received: 2 July 2024
Revised: 13 August 2024
Accepted: 15 August 2024
Published: 17 August 2024

1. Introduction

The rise of "shadow education" in China [1] underscores a significant deficiency in the public education system's ability to cater to diverse educational needs, particularly for students requiring special attention. "Shadow education", also known as private tutoring or supplementary education, includes cram schools, private one-to-one tutoring, and exam preparatory courses [2]. Despite Chinese governmental regulatory efforts, the proliferation of extracurricular tutoring points to an increasing demand for personalized education, a need that the current public school system has failed to meet adequately, has become a global phenomenon [3,4]. Within the Chinese context, this situation is exacerbated by the predominant teacher-centric approach adopted in schools, in which instruction often involves teachers' lectures and students passively taking notes [5]. This traditional method of teaching, known as conventional instruction (CI), typically emphasizes rote memorization and repetitive practice [6].

This traditional model struggles to implement individualized instruction because of teachers' insufficient skills and pedagogical knowledge, leading to a uniform educational experience that fails to address students' diverse learning needs [7]. China's teaching-centered education has been widely criticized [8]. Accordingly, Xue and Li have advocated student-centered education [9]. Student-centered learning, also known as learner-centered education, shifts the focus of instruction from the teacher to the student to develop learner autonomy and independence [10].

In light of these challenges, stratified teaching has emerged as a promising alternative. Stratified teaching classifies students on the basis of their knowledge, talent, and potential. It customizes teaching tactics for each group to ensure the best possible progress for learners [11]. Sun [12] and Zhu et al. [13] emphasized the potential of stratified teaching in addressing the challenges posed by the uniformity of traditional teaching methods in mathematics. This approach acknowledges the varying mathematical foundations of students and aims to tailor their education according to their levels and interests. However, the implementation of this approach in China, particularly in senior high school mathematics, must be improved because of the persistent dominance of traditional teacher-centric methods [5]. The importance of mathematics in national development underscores the need for more collaborative and engaging teaching methods that consider the factors influencing mathematics performance, such as student-teacher cooperation [14]. China's mathematics curriculum is changing to accommodate students' demands and foster the development of inventive abilities [15].

The necessity to implement more captivating and interactive instructional approaches in China's stratified educational system is apparent. The notion of 'cognitive apprenticeship,' initially introduced by Collins et al. [16] integrates traditional apprenticeship methods with formal education to enhance students' cognitive development. In the United States, combines conventional apprenticeship techniques with formal education to augment students' cognitive capacities, analytical reasoning, problem-solving, and proficiency in intricate tasks that require both knowledge and expertise [17]. This pedagogical method entails students' active engagement in professional communities to acquire knowledge. Studies have shown that students who receive instruction using CAM achieve higher test scores than those taught using traditional methods [18]. CAM prioritizes the implementation of techniques such as modeling, coaching, and scaffolding to assist students in comprehending the procedure of accomplishing tasks, thereby fostering their autonomous cognition and self-drive [19]. If demonstration by an expert induces fear in students, it increases their anxiety. Meanwhile, class size should be reduced to accurately monitor students' cognitive processes [20].

Integrating CAM into a stratified teaching framework offers a promising solution to enhance students' mathematics performance. This integration seeks to address the challenges of the one-dimensional teaching approach prevalent in Chinese schools and to foster a more dynamic and inclusive educational environment. This study aimed to evaluate the effectiveness of a stratified teaching model based on CAM in enhancing high school students' mathematics performance in China. CAM comprises an interactive and contextual approach, thus making the stratified cognitive teaching method (SCTM) a potentially effective way to teach high school mathematics in China.

1.1. Research Questions

To investigate the effectiveness of different instructional strategies in high school mathematics education, this study addresses several key questions. These questions aim to compare the impacts of the cognitive apprenticeship model (CAM), the stratified cognitive apprenticeship teaching module (SCTM), and conventional instruction (CI) on various aspects of students' mathematical abilities. The research questions guiding this study are as follows:

1. What are the effects of the cognitive apprenticeship model (CAM), the stratified cognitive apprenticeship teaching module (SCTM), and conventional instruction (CI) on the overall mathematics performance of ninth-grade high school students in the post-test and delayed post-test?
2. How do CAM, SCTM, and CI impact students' ability to acquire and apply factual information ("knowing") and to make sense of and use this information in various contexts ("understanding")?

3. What are the effects of CAM, SCTM, and CI on students' skills in investigating mathematical patterns and relationships?
4. How do CAM, SCTM, and CI influence students' mathematical communication skills and their ability to apply mathematical knowledge to real-life situations?

1.2. Overview of Key Teaching Methods

To effectively implement and evaluate these educational strategies, this study examined several key teaching methods:

Conventional instruction (CI): CI refers to the traditional teaching method commonly used in classrooms. This approach typically involves a teacher-centered model in which the instructor delivers lectures and students passively receive information. The focus is on rote memorization and standardized testing, with a limited emphasis on interactive or individualized learning experiences [8].

Cognitive apprenticeship model (CAM): CAM is an educational framework that integrates traditional apprenticeship methods with formal education. Collins et al. [16] identified six core features of CAM, which are illustrated in Figure 1 as adapted by Kurt [21].

Modeling: The teacher demonstrates the task and provides a clear example of how to perform it.

Coaching: The teacher observes and provides guidance and feedback when the student attempts the task.

Scaffolding: The teacher offers support as needed to help students accomplish a task.

Fading: The teacher gradually removes support as the student becomes more proficient.

Articulation: The student articulates their knowledge and reasoning processes.

Reflection: The student reflects on their performance and compares it with that of the expert to understand their strengths and areas for improvement.

Stratified cognitive apprenticeship model teaching module (SCTM): SCTM is an innovative instructional approach that combines the differentiated instruction of stratified teaching with the experiential learning principles of CAM. This model was specifically designed to address varying levels of comprehension and interest in mathematics among high school students.

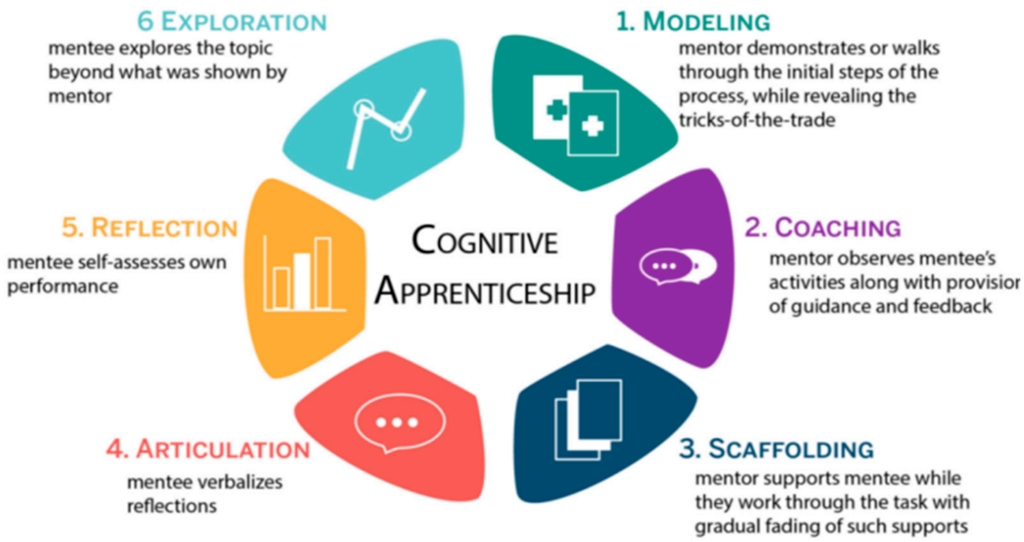

Figure 1. The six core strategies of the cognitive apprenticeship framework (adapted from Collins et al. [14] by Kurt [21]).

For the SCTM, students were categorized using a straightforward stratification technique, which required the creation of two tiers of classrooms, depending on their pre-test scores. The highest achievers (50% of the sample) were assigned to the high-performance tier, while the remaining half were assigned to the low-performance tier. Consequently, the educational goals and curricula were tailored for each tier. Figure 2 shows a flowchart of the development of the SCTM in this study. For the SCTM teaching group, higher-performance classes typically covered content at a higher level, whereas lower-performance classes grasped it at a lower level. Correspondingly, the students were allowed to choose homework tasks based on their understanding and skill levels. Each class level then implemented cognitive apprenticeship model (CAM) teaching.

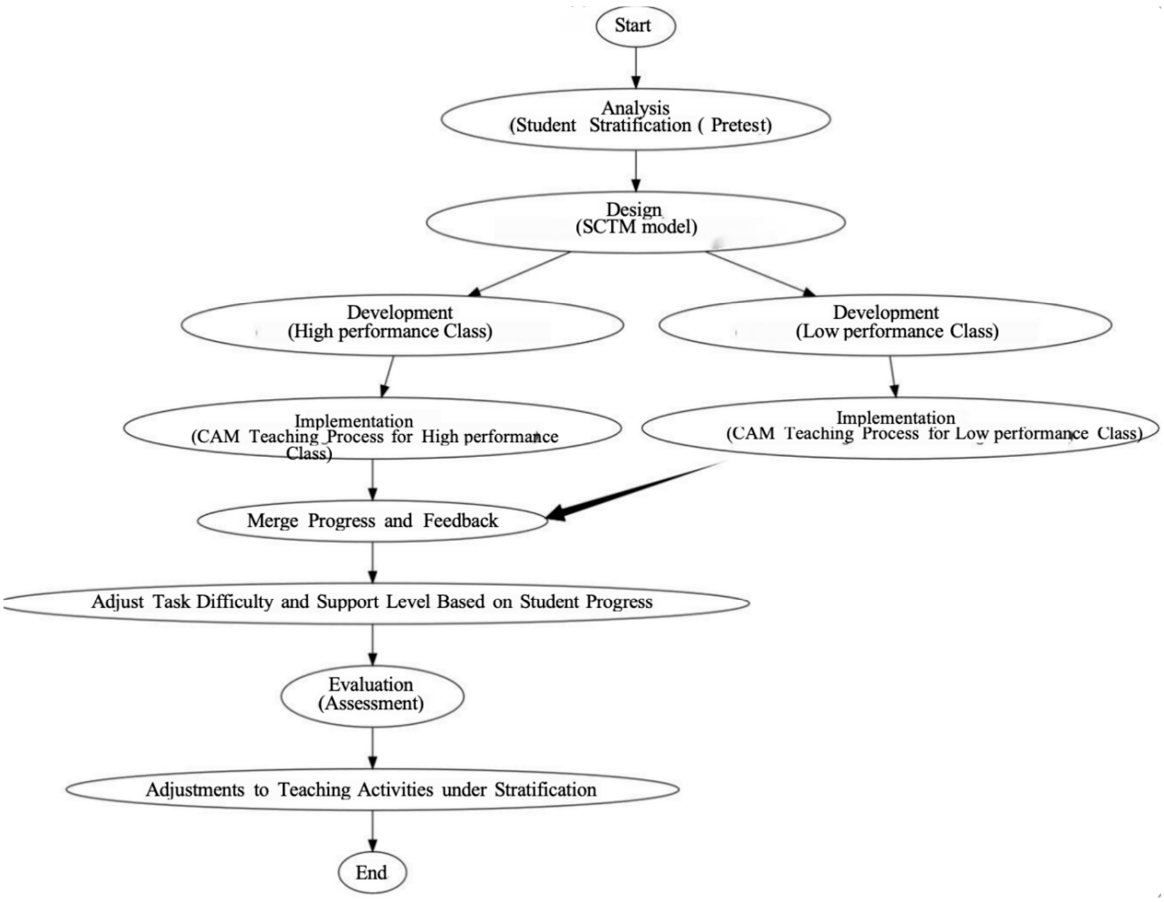

Figure 2. Flowchart of the SCTM development process.

In the CAM group, the teachers tailored the instruction based on the student's mastery of the material per lecture, ensuring that all students received the same content, and then strictly enforced CAM teaching.

The CI group was instructed using a teacher-directed methodology, where the teacher strictly followed the lesson plan and objectives while the students took notes and practiced.

1.3. Instruments and Measurements

In this study, we evaluate students' mathematics performance using the comprehensive criteria set forth by the International Baccalaureate Organization (IBO), which include knowing and understanding, communication, investigation, and application. These criteria provide a robust framework for assessing the effectiveness of different instructional strategies in fostering students' holistic development in mathematics.

Proficient IB-certified educators devised the pre-, post-, and delayed post-tests following a systematic scientific method to ensure the validity and reliability of the tests. The construction process included several key steps:

1. Item development: Test items were designed to align with the specific criteria set forth by the IBO, covering knowing and understanding, communication, investigation, and

application. Items were crafted to assess both factual knowledge and the ability to apply this knowledge in various contexts.
2. Expert review: University experts reviewed the initial set of items to ensure content validity. This review process involved assessing the relevance and clarity of each item and providing feedback for refinement.
3. Pilot study: A pilot study was conducted with a sample of students to validate the difficulty coefficients and the reliability of the tests. Data from this study were used to make necessary adjustments to the test items.
4. Scoring criteria: All test papers were developed to meet specific scoring criteria established by the IBO. The questions were designed by an experienced IB mathematics examiner and question setter with 20 years of expertise in the field.

1.3.1. Knowing and Understanding

According to the Stanford Encyclopedia of Philosophy, "knowing" refers to possessing beliefs or information about a particular subject, which includes being conscious and gaining knowledge, information, or abilities through formal or self-directed education. Meanwhile, "understanding" involves profound exploration beyond the sheer collection of knowledge. This is the capacity to elucidate, situate, and grasp information or concepts. Comprehension entails a profound understanding of the importance of knowledge, incorporating it into a more comprehensive framework, and utilizing it in many situations. This study examines the correlation between knowing and understanding and the 2023 standards set by the IBO. It focuses on students' capacity to choose and proficiently use mathematical techniques to solve familiar and unfamiliar issues in different situations [22].

1.3.2. Investigating Patterns

Pattern analysis is essential in the fields of psychology and social science. It plays a critical role in identifying and comprehending trends in behaviors, events, and other phenomena as they unfold over time [23]. In mathematics, the term "investigating patterns" refers to the process of recognizing and analyzing consistent or regular occurrences of numbers, forms, or equations. This involves studying sequences, geometric figures, algebraic structures, and other elements to detect patterns and establish links. Its goal is to reveal underlying mathematical principles or theorems [7]. This study used the IBO criteria [22] to examine these patterns. These guidelines focus on student proficiency in employing mathematical techniques to uncover intricate patterns and articulate them as general principles while verifying the established rules.

1.3.3. Communication

Communication encompasses the ability to organize and connect mathematical concepts using different modes of communication [24]. This involves effectively expressing mathematical ideas, critically assessing others' mathematical reasoning, and using precise mathematical language. The evaluation of communication in this study aligns with IBO's guidelines [22], which emphasize the use of appropriate mathematical terminology, seamless movement between different forms of representation, and the expression of coherent and logical concepts.

1.3.4. Application

The application of mathematics in real-life situations involves incorporating real-world scenarios into educational programs and evaluations to encourage the practical utilization of mathematical principles [25]. This study's evaluation of the application (of mathematics in real-life contexts) adheres to IBO's criteria [22]. It assesses students' ability to identify pertinent elements of real-life situations, select and employ suitable mathematical methods, and provide valid explanations for the accuracy and significance of the obtained solutions.

Proficient IB-certified educators devised pre-, post-, and delayed post-tests, and their validity was verified and refined by university experts. A pilot study was conducted to validate the difficulty coefficients and the reliability of the tests. All the test papers met the scoring criteria.

2. Methods

2.1. Research Design

This study employed a quasi-experimental design appropriate for educational research. This design allowed for a comparison between the groups exposed to different teaching strategies. This study included a randomized sample of 150 9th-grade students from an international baccalaureate (IB) high school in China, renowned for its all-encompassing and thorough education, which includes a varied and applicable IB mathematics curriculum (International Baccalaureate Organization [22]. An IB high school was selected due to its rigorous academic standards and emphasis on comprehensive, inquiry-based learning [26]. The IB mathematics curriculum provides a robust framework for assessing the effectiveness of different instructional strategies [27]. Additionally, the IB program's focus on critical thinking, problem-solving, and real-world application of knowledge ensures that the students are engaged in a high level of academic rigor [28], making it an ideal environment to evaluate the impact of the cognitive apprenticeship model (CAM) and the stratified cognitive apprenticeship model teaching module (SCTM)

Of the 150 students selected to participate in this study, these students were divided into 6 classes, with each class consisting of 25 students. Two classes were randomly allocated to Treatment Group 1 (CAM group), two classes to Treatment Group 2 (the SCTM group), and the remaining two classes were assigned to the conventional teaching (CI) group. The unit of analysis for this study was therefore the class, rather than individual students, allowing for a comparison between the instructional strategies at the class level.

The control group consisted of two classes, Class A and Class B, with 25 students each, and each was taught using traditional Chinese methods focusing on teaching, practice, and memorization. Treatment Group 1 (CAM group) also comprised two randomly assigned classes, Class C and Class D, each with 25 students, where a single teacher implemented the CAM strategy to assess its effectiveness compared to traditional methods. Both the CAM and CI groups were randomly assigned to ensure that the two classes within each group had similar predicted performance levels based on pre-test scores.

In Treatment Group 2 (SCTM group), students were divided based on their pre-test scores into Class E (high-performance class) and Class F (low-performance class), each with 25 students. The SCTM approach was applied, with differentiated learning objectives tailored for each subgroup to evaluate the model's impact on diverse student needs.

It is important to note that different teachers were assigned to each instructional method to avoid any teacher effect. Specifically, one teacher was assigned to teach both classes in the CI group, another teacher was assigned to teach both classes in the CAM group, and a third teacher was assigned to teach both classes in the SCTM group. This ensured consistency within each instructional method, and a total of three teachers were involved in the study. To maintain the integrity of each method, classroom observations were conducted to ensure teachers adhered to their designated instructional methods. This protocol, detailed in Table 1, systematically documented aspects such as the use of stratified teaching and CAM strategies, group collaboration, feedback, and task differentiation. The classroom observation form captured critical elements like content delivery, student interactions, and feedback effectiveness, verifying the implementation of stratified teaching methods.

It was justified that at the initial stage, each group (CI, SCTM, and CAM) had an equal number of lower and higher-performing students, with approximately 50 students in each group. Therefore, these three groups were balanced in terms of lower and higher-performing students. Additionally, a pretest was used as a covariate to control for bias due to differences in student abilities across these three groups.

Table 1. Class observation form.

Criterion	Yes/No
Teaching class	
Teaching content	
Stratified or not	
Use CAM or not	
Whether there is group cooperation	
Whether there is feedback	
Whether there are tasks at different levels	

Both teachers rigorously applied the CAM and SCTM approaches as trained. It was essential that all students were provided the same learning opportunities, such as solving the same problems and answering the same questions, to ensure that only the instructional approach varied. The control group and CAM group did not have lower and upper-level classes. This division was unique to the SCTM group to evaluate differentiated instruction based on pre-test performance. The pre-test was implemented after dividing the students into control and treatment groups.

As shown in Figure 3, the quasi-experimental design used in this study, including the pre-test, post-test, and delayed post-test stages, along with the grouping of students and the different instructional methods (CI, CAM, SCTM) applied.

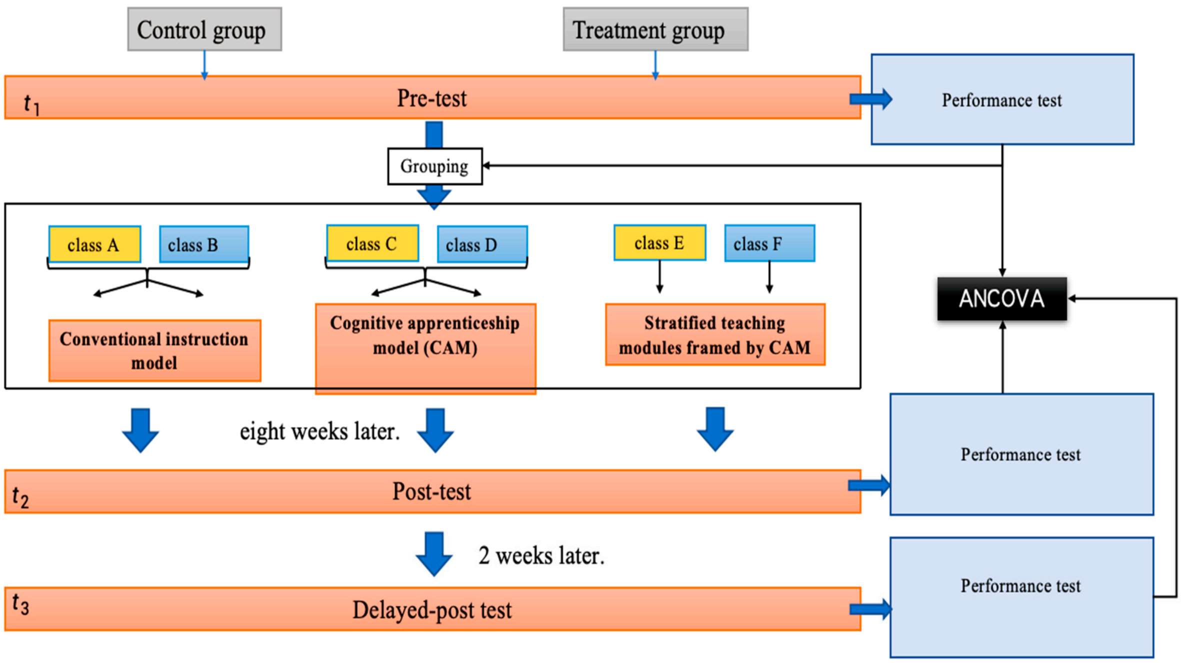

Figure 3. Experimental design of the study.

An 11-week mathematics teaching experiment was conducted in this study, consisting of an 8-week teaching experiment, followed by a 2-week gap, and then a 1-week delayed post-test period. Before the experiment began, the students took a pre-test ($t = t_1$) to determine their scores on the four dimensions of learning ability: knowing and understanding, investigating patterns, communication, and application.

Following eight weeks of instruction, the middle year program (MYP) post-test ($t = t_2$) was administered to the three groups of students, and scores were obtained across the four dimensions of learning ability. Two weeks later, the MYP-delayed post-test ($t = t_3$) was administered to all three groups of students. Scores on the four dimensions of learning ability were recorded for each group. Although a two-week gap between the post-test and the delayed post-test might seem short, previous research has demonstrated the effectiveness of such intervals in assessing retention and understanding [29–31]. The chosen interval allows for a practical balance between assessing immediate retention and minimizing external factors that might influence long-term memory.

Cronbach's alpha measures internal consistency reliability, with values ranging from 0 to 1; higher values indicate higher reliability [32]. As shown in Table 2, Cronbach's alpha coefficients for all three variables were relatively high, with values of 0.910, 0.897, and 0.900 for the pre-, post-, and delayed post-tests, respectively. This indicated that the items within each set of variables were highly consistent, suggesting that the measures were reliable and internally consistent. Overall, these results provide evidence that the measures used in this study are reliable and consistent, thus increasing confidence in the validity of the findings.

Table 2. Reliability analysis of the performance tests.

Variables	Number of Items	Cronbach's α Coefficient
Pre-test	17	0.910
Post-test	17	0.897
Delayed post-test	17	0.900

The initial versions of the three test papers (pre-, post-, and delayed post-tests) were selected from a school question bank. The examiners were IB teachers who reviewed all tests. Before the study, the students were selected to complete the tests, which allowed us to investigate their reliability. To revise the project, item analysis was conducted to determine the difficulty and discrimination of the items.

2.2. Research Procedures

The research process began with a pilot study conducted at a high school, involving 100 9th-grade students. The students were divided into a control group using traditional teaching methods and an experimental group using the new stratified cognitive apprenticeship model teaching module (SCTM). The pilot study helped refine the research methods and ensured the validity and reliability of the data.

The formal study involved 150 students, divided into three groups: CI, CAM, and SCTM. The data collection procedure lasted 11 weeks. Initially, a pre-test was administered, followed by an eight-week teaching experiment, and concluded with a post-test and a delayed post-test after two weeks to measure immediate and long-term learning outcomes.

An analysis of covariance (ANCOVA) was employed to control for pre-test differences and assess the effectiveness of the interventions on the post-test and delayed post-test scores. This approach allowed for comparing the adjusted means of mathematics performance across the three instructional strategies, distinguishing between their short-term effectiveness and long-term impact.

To understand both the immediate and long-term impacts of the instructional strategies, two types of tests were used: post-tests to measure immediate learning outcomes and delayed post-tests to assess the retention of knowledge over time. This dual approach helps in distinguishing between the short-term effectiveness and the lasting impact of the interventions on students' mathematics performance.

3. Results and Discussion

This section analyzes the effects of CAM, SCTM, and CI strategies on students' performance in mathematics, focusing on post-test and delayed post-test results.

3.1. ANCOVA Assumptions

For accurate and reliable application of ANCOVA, it is crucial to meet assumptions such as the linear relationship between the dependent variable and the covariate and homogeneity of regression slopes [29]. Tests conducted to examine these assumptions indicated significant interactions between the group and the covariate for performance tests, Table 3 shows the results of the regression slope homogeneity test for interaction between the group and the covariate. The significance values (Sig.) for both the post-test and the Delayed Post-Test are less than 0.05, indicating significant interaction effects between the group and the covariate.

Table 3. Regression slope homogeneity test for interaction between group and covariate across different tests.

Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Post-test	7031.974	3	2343.991	42.108	0.000
Delayed post-test	4796.471	3	1598.824	67.868	0.000

3.2. Math Performance Analysis

Comparative analysis is essential to discern the efficacy of diverse instructional approaches. Table 4 integrates the descriptive statistics with the ANCOVA results, demonstrating the impact of different instructional strategies on students' mathematics post-test scores. After adjusting for pre-test scores, ANCOVA revealed significant differences in the post-test scores of the three instructional strategy groups (F [2,146] = 63.137, $p < 0.001$). Indicating the significant impact of different instructional strategies on students' mathematics performance. Specifically, the performance of the SCTM group was the best, followed by that of the CAM group, whereas the CI group performed the worst.

Table 4. Descriptive statistics and ANCOVA results.

Group	Mean	Standard Deviation	Sample Size	F(2,146)	*p*-Value	Mean Difference of the CI Group	Mean Difference of the CAM Group
CI group	48.66	7.73	50	63.137	<0.001	-	-
CAM group	57.81	7.43	50	-	-	9.192	-
SCTM group	65.40	7.15	50	-	-	16.745	7.552

Table 5 presents the results of the post hoc pairwise comparisons, analyzing the specific differences among the instructional strategy groups.

Table 5. Results of post hoc pairwise comparisons.

Comparison	Mean Difference (I-J)	Standard Error	*p*-Value
CI group vs. CAM group	−9.192	1.496	<0.001
CI group vs. SCTM group	−16.745	1.492	<0.001
CAM group vs. SCTM group	−7.552	1.495	<0.001

Subsequent pairwise comparisons confirmed significant differences between the three instructional strategy groups. The SCTM group significantly outperformed the CAM and CI groups in this regard. These results again emphasize the effectiveness of SCTM in enhancing students' mathematics achievement.

This analysis examined the effectiveness of different instructional strategies (CI, CAM, and SCTM) on students' long-term retention of mathematics skills as measured by a delayed post-test.

As shown in Table 6, the ANCOVA results for the delayed post-test (F[2,146] = 101.789, $p < 0.001$) indicated significant differences in long-term mathematics performance among the strategy groups. The SCTM group demonstrated superior performance, indicating that this was the most effective strategy for long-term knowledge retention.

Table 6. Integrated descriptive statistics and ANCOVA results for the delayed post-test.

Group	Mean	Standard Deviation	N	F(2,146)	*p*-Value	Mean Difference of the CI Group	Mean Difference of the CAM Group
CI	47.74	4.49	50	101.789	<0.001	-	-
CAM	55.71	5.20	50	-	-	7.983	-
SCTM	61.53	4.80	50	-	-	13.797	5.813

As shown in Table 7, post hoc comparisons further confirmed the significant differences among the instructional strategies. Notably, the SCTM group's advantage over the CAM and CI groups highlights SCTM's effectiveness in sustaining students' mathematics achievement over time.

Table 7. Post hoc pairwise comparisons for delayed post-test performance.

Comparison	Mean Difference (I-J)	Standard Error	*p*-Value
CI group vs. CAM group	−7.983	0.973	<0.001
CI group vs. SCTM group	−13.797	0.971	<0.001
CAM group vs. SCTM group	−5.813	0.972	<0.001

In Figure 4, it is evident that all groups showed improvement from pre-test to post-test. However, the SCTM group consistently outperformed the other groups in both the post- and delayed post-test stages, indicating that SCTM may have a more substantial impact on both immediate learning and long-term retention of mathematics knowledge. The error bars suggest variability within each group, with overlapping confidence intervals between the CAM and SCTM groups in the post-test phase, which were then separated in the delayed post-test phase, highlighting the potential long-term benefits of SCTM.

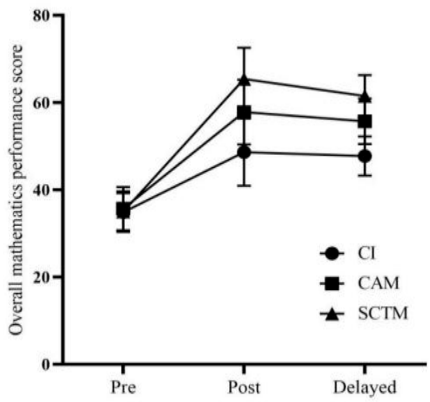

Figure 4. Comparison of student performance scores in mathematics across different instructional strategies.

3.3. Knowing and Understanding Analysis

This section evaluates the impact of the different instructional strategies—CI, CAM, and SCTM—on students' knowledge and understanding in both the post- and delayed post-tests.

As shown in Table 8, the ANCOVA results for both the post- and delayed post-tests exhibited significant differences in knowing and understanding among the groups. The ANCOVA results indicated statistically significant differences among the groups in both tests, with F-values of 33.799 for the post-test and 27.509 for the delayed post-test, and *p*-values less than 0.001 for both. The SCTM group consistently outperformed the CAM and CI groups, indicating SCTM's effectiveness of SCTM in enhancing students' understanding of mathematical concepts both immediately and over time.

Table 8. Descriptive statistics and ANCOVA results for knowing and understanding.

Test	Group	Mean	Standard Deviation	N	F-Value	*p*-Value	Mean Difference of the CI Group	Mean Difference of the CAM Group
Post-test	CI	51.80	7.77	50	33.799	<0.001	-	-
	CAM	59.28	7.13	50	-	-	7.407	-
	SCTM	64.50	8.24	50	-	-	12.687	5.279
Delayed Post-test	CI		50		27.509	<0.001		
	CAM		50					
	SCTM		50					

From Table 9, post hoc comparisons across both tests further confirmed the significant differences between the groups. The performance of the CI group was consistently lower than that of the CAM and SCTM groups, with the SCTM group demonstrating a notable advantage over the CAM group. These results underscore the effectiveness of SCTM in fostering deeper knowledge and understanding among students.

Table 9. Post hoc pairwise comparisons for knowing and understanding.

Test Type	Comparison	Mean Difference (I-J)	Standard Error	*p*-Value
Post-test	CI group vs. CAM group	−7.407	1.557	<0.001
	CI group vs. SCTM group	−12.687	1.550	<0.001
	CAM group vs. SCTM group	−5.279	1.554	0.003
Delayed post-test	CI group vs. CAM group	−7.301	1.710	<0.001
	CI group vs. SCTM group	−12.573	1.702	<0.001
	CAM group vs. SCTM group	−5.272	1.707	0.007

In Figure 5, the mean scores clearly depict that SCTM and CAM led to better student performance than CI, both immediately and in the long term. This aligns with the ANCOVA results mentioned in the previous section, which indicated significant differences among instructional strategies.

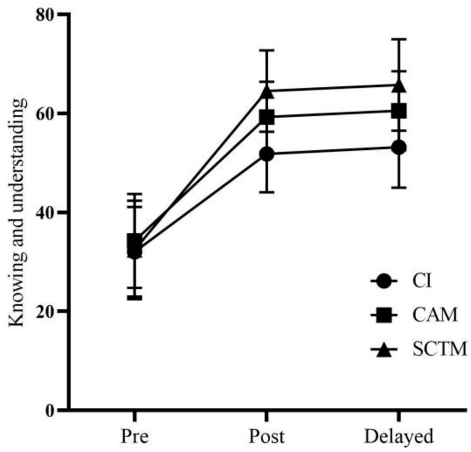

Figure 5. Comparison of students' knowing and understanding scores in mathematics across different instructional strategies.

3.4. Investigating Patterns Analysis

From Table 10, the ANCOVA results for both the post-tests and delayed post-tests indicated significant differences in investigative skills among the instructional strategy groups. SCTM consistently resulted in superior investigative performance, highlighting its effectiveness in promoting deeper engagement and systematic exploration of mathematics. The delayed post-test outcomes affirm the impact of SCTM on students' abilities to investigate mathematical concepts over time.

Table 10. Descriptive statistics and ANCOVA results for investigating performance.

Test Type	Group	Mean	Standard Deviation	N	F(2,146)	*p*-Value	Mean Difference of the CI Group	Mean Difference of the CAM Group
Post-test	CI	54.76	12.82	50	11.816	<0.001	-	-
	CAM	60.52	14.39	50	-	-	−5.799	-
	SCTM	66.46	12.43	50	-	-	−11.077	−5.278
Delayed Post-test	CI	56.36	6.96	50	25.410	<0.001	-	-
	CAM	62.18	9.07	50	-	-	−5.822	-
	SCTM	67.36	7.06	50	-	-	−11.000	−5.178

As shown in Table 11, post hoc comparisons further illustrate the significant performance disparities among the groups. In both the post-tests and delayed post-tests, the advantage of the SCTM group was evident, underscoring the SCTM's capacity to foster investigative competencies. Notably, the consistent performance gap between SCTM and the other two strategies across both tests emphasizes that the stratified and explorative learning process inherent in the SCTM approach is critical for enhancing students' investigative skills in mathematics.

Table 11. Post hoc pairwise comparisons for investigating performance.

Comparison	Post-Test Mean Difference (I-J)	Delayed Post-Test Mean Difference (I-J)	*p*-Value
CI vs. CAM	−5.799	−5.822	<0.001
CI vs. SCTM	−11.077	−11.000	<0.001
CAM vs. SCTM	−5.278	−5.178	0.003

Figure 6 shows that, while all groups started with similar baseline abilities, the SCTM group significantly outperformed the others immediately after instruction and maintained a higher level of skill over time, as indicated by the delayed post-test scores. This visual representation underscores the effectiveness of SCTM in fostering an immediate understanding and long-term retention of mathematical investigation skills.

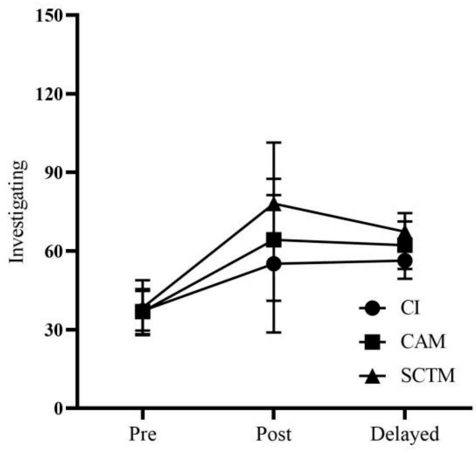

Figure 6. Comparison of student "investigating" scores in mathematics across different instructional strategies.

3.5. Communication Analysis

From Table 12, the ANCOVA results for both the post-tests and delayed post-tests indicated significant differences in students' communication skills among the groups. SCTM consistently led to superior communication performance, highlighting its effectiveness in facilitating students' mathematical expressions. The delayed post-test results further reinforce the sustained impact of the SCTM strategy on students' ability to communicate mathematical ideas over time.

Table 12. Descriptive statistics and ANCOVA results for communication performance.

Test Type	Group	Mean	Standard Deviation	N	F(2,146)	*p*-Value	Mean Difference from CI Group	Mean Difference from CAM Group
Post-test	CI	43.00	13.90	50	30.565	<0.001	-	-
	CAM	56.48	13.56	50	-	-	−13.475	-
	SCTM	63.68	12.65	50	-	-	−20.685	−7.210
Delayed post-test	CI	38.94	13.91	50	36.043	<0.001	-	-
	CAM	53.02	13.56	50	-	-	−13.991	-
	SCTM	61.52	12.67	50	-	-	−22.808	−8.818

From Table 13, post hoc comparisons highlighted significant differences among the groups in their ability to communicate in mathematics, with the CI group performing significantly lower than the CAM and SCTM groups in both post-test and delayed post-test. The mean differences highlight that students in the CI group scored 13.475 and 13.991 points lower than those in the CAM group in the post-test and delayed post-tests, respectively, with *p*-values indicating a high statistical significance (<0.001). The performance gap widened when comparing CI with SCTM, with immediate and delayed mean differences of 20.685 and 22.808 points, respectively. Furthermore, the comparison between the CAM and SCTM groups also showed significant differences, albeit smaller, with immediate and delayed mean differences of 7.210 and 8.818 points, respectively, and a *p*-value of 0.024. The SCTM group's advantage was particularly notable, suggesting that SCTM effectively enhances students' communication skills, likely through its emphasis on the clear expression and understanding of mathematical concepts.

Table 13. Post hoc pairwise comparisons for communication performance.

Comparison	Post-Test Mean Difference (I-J)	Delayed Post-Test Mean Difference (I-J)	*p*-Value
CI vs. CAM	−13.475	−13.991	<0.001
CI vs. SCTM	−20.685	−22.808	<0.001
CAM vs. SCTM	−7.210	−8.818	0.024

In Figure 7, error bars indicate the variability within each group. Despite some overlap, particularly between the CAM and SCTM groups, the SCTM group consistently outperformed the other groups in both the post- and delayed post-test stages, visually affirming the statistical results suggesting the superior effectiveness of SCTM in teaching communication skills in mathematics.

Figure 8 displays the students' performance in application, measured before the intervention, and post- and delayed post-tests to assess retention. The results showed that while all instructional strategies—CI, CAM, and SCTM—led to improved performance from pre- to post-intervention, the SCTM group scored the highest in both the post- and delayed post-test stages. The slight decrease in scores on the delayed test for all groups suggests some attrition of applied skills over time; however, SCTM led to the best long-term application outcomes, as evidenced by its consistently higher scores.

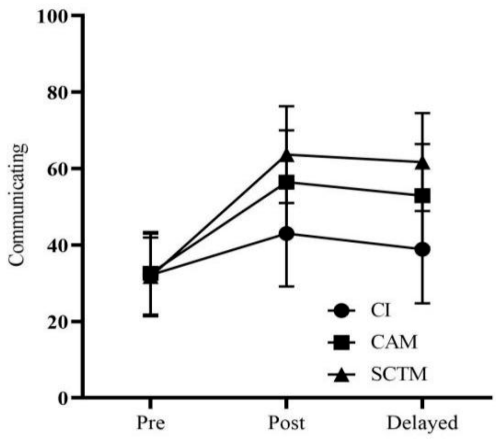

Figure 7. Comparison of student communication scores in mathematics across different instructional strategies.

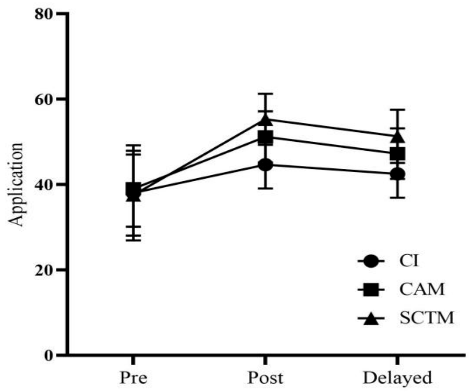

Figure 8. Comparison of students' application scores across different instructional strategies.

3.6. Application Analysis

From Table 14, the ANCOVA results for both the post-tests and delayed post-tests underscored significant differences in the groups' ability to apply mathematics in real-life contexts. SCTM exhibited the highest performance, indicating its effectiveness in promptly and sustainably equipping students with practical application skills.

Table 14. Descriptive statistics and ANCOVA results for application performance.

Test Type	Group	Mean	SD	N	F(2,146)	*p*-Value	Mean Difference of the CI Group	Mean Difference of the CAM Group
Post-test	CI	44.66	5.60	50	41.522	<0.001	-	-
	CAM	51.20	5.98	50	-	-	−6.550	-
	SCTM	55.28	5.98	50	-	-	−10.615	-4.064
Delayed Post-test	CI	42.50	5.61	50	27.384	<0.001	-	-
	CAM	47.24	5.91	50	-	-	−4.748	-
	SCTM	51.30	6.25	50	-	-	−8.796	-4.048

Note. N = sample size; SD = standard deviation.

From Table 15, the post hoc analysis validated the significant variances among the groups in their practical application of mathematics, with CI students showing considerably lower performance than the CAM and SCTM groups in both immediate and long-term contexts. The SCTM group's consistent outperformance highlights SCTM's emphasis on deep, explorative learning and application, proving to be significantly effective in fostering students' abilities to apply mathematical concepts in real-life situations.

Table 15. Post hoc pairwise comparisons for application performance.

Comparison	Post-Test Mean Difference (I-J)	Delayed Post-Test Mean Difference (I-J)	*p*-Value
CI vs. CAM	−6.550	−4.748	<0.001
CI vs. SCTM	−10.615	−8.796	<0.001
CAM vs. SCTM	−4.064	−4.048	0.002

3.7. SCTM's Superiority in Enhancing Understanding and Application

The SCTM group's consistent outperformance, as evidenced by the post- and delayed post-test results, underscores its efficacy in fostering higher average scores and ensuring stability in learning outcomes. This is particularly noteworthy in mathematics education, where the ability to apply learned concepts to new and unpracticed problems is crucial. Unlike the CI group, which relied on rote memorization and formulaic applications, the SCTM group displayed a profound conceptual understanding and an innovative ability to employ mathematical principles.

3.8. Integrating the CAM for Deeper Comprehension

CAM's emphasis on articulation and reflection was evident in the students' ability to effectively understand and communicate their problem-solving processes. Despite the CAM group's improved understanding, limitations in their ability to fully articulate problem-solving steps suggest that while CAM fosters a more profound understanding than traditional methods, its integration with stratified teaching strategies (i.e., the SCTM) might yield the most significant educational benefits. This finding supports Collins et al.'s [16] assertion regarding the value of cognitive apprenticeship in enhancing learning outcomes through active student involvement.

3.9. Critique of CI

Traditional teaching methodologies that prioritize factual knowledge over conceptual understanding may lead to superficial learning outcomes characterized by a lack of flexibility in applying knowledge to novel problems. Such approaches, as highlighted by the performance of the CI group, underscore the necessity of teaching methods that encourage deep engagement with the material and the development of adaptable problem-solving skills.

3.10. The Role of Expert Guidance and Authentic Tasks

This study reaffirms the importance of expert guidance and the utilization of authentic tasks in education, as emphasized by CAM and SCTM. These approaches facilitate a deep understanding of complex concepts and enhance students' ability to apply this knowledge to real-world scenarios [17]. Such strategies are instrumental in moving beyond the limitations of traditional educational practices, offering a more effective means of preparing students for the challenges of real-life problem-solving.

4. Conclusions and Educational Implications

This study integrated the potential drawbacks of stratified teaching with the advantages of CAM, leading to favorable outcomes. Moreover, experimental data indicated that SCTM significantly improved students' academic performance. The integration of these methodologies not only enhances the learning experience but also ensures that diverse student needs are met more effectively.

A comparative analysis of teaching methodologies illustrates the superior efficacy of SCTM, which combines the strengths of CAM with a structured and differentiated approach to learning. This model addresses diverse learning needs and promotes a deeper understanding and application of mathematical concepts, thereby significantly enhancing students' performance. These findings contribute to the ongoing discourse on educational strategies, supporting a shift toward more nuanced student-centric approaches to teaching and learning in mathematics [33–35]. Such a shift is essential for fostering problem-solving skills, and the ability to apply knowledge in various contexts, which are crucial for students' success in the 21st century.

SCTM significantly improves students' mathematical performance, highlighting the need for dynamic, student-centric teaching approaches in modern education and supporting a move away from traditional, uniform teaching methods. This study contributes to the growing body of evidence in favor of differentiated and student-focused teaching strategies, providing a practical framework for educators and theoretical basis for further academic inquiries into educational methodologies. The success of SCTM demonstrates that when teaching methods are tailored to students' individual strengths and weaknesses, educational outcomes can be substantially improved.

This study plays a pivotal role in understanding and enhancing teaching strategies, particularly in mathematics education. This underscores the importance of adapting teaching methods to suit diverse learning needs, and offers valuable insights for educators, policymakers, and educational researchers. Future research should continue to explore the long-term impacts of SCTM and similar models on various student populations, ensuring that these innovative approaches can be refined and widely implemented to benefit all learners.

Author Contributions: Data curation, R.W.; Formal analysis, R.W.; Investigation, R.W.; Methodology, R.W.; Project Administration, R.W.; Resources, R.W.; Software, R.W.; Visualization, R.W.; Writing—original draft, R.W.; Supervision, N.N.Z., A.F.M.A.; Analysing; Writing—review & editing, R.W., N.N.Z., A.F.M.A. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: The study was conducted in accordance with the Declaration of Helsinki and approved by Ethics Committee of Universiti Putra Malaysia, Secretariat of JKEUPM, Unit of Ethics Research (Level 5), Office of the Deputy Vice Chancellor (Research & Innovation) (JKEUPM ref no: JKEUPM-2023-128, 28 February 2023).

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: Due to privacy and ethical restrictions, the data from this study cannot be publicly available. However, anonymized data can be provided upon reasonable request, subject to approval by the ethics committee.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Yang, M.; Bray, M. Chinese parents' motivations for sending their children to extracurricular classes (ECCs): Evidence from five case studies. *Front. Educ. China* **2018**, *13*, 238–266.
2. Yu, J.; Zhang, R. A Review of Shadow Education. *Sci. Insights Educ. Front.* **2022**, *11*, 1579–1593. [CrossRef]
3. Bray, M. Private Supplementary Tutoring: Comparative Perspectives on Patterns and Implications. Compare. *J. Comp. Int. Educ.* **2006**, *36*, 515–530. [CrossRef]
4. Mori, I.; Baker, D. The origin of universal shadow education: What the supplemental education phenomenon tells us about the postmodern institution of education. *Asia Pac. Educ. Rev.* **2010**, *11*, 36–48. [CrossRef]
5. Yan, D. Hierarchical Interactive Teaching Model and Its Practical Exploration in High School Mathematics Teaching. Ph.D. Thesis, Southwest University, Chongqing, China, 2016.
6. Chaika, O. *Bridging the Gap: Traditional vs. Modern Education (A Value-Based Approach for Multiculturalism). Lifelong Learning—Education for the Future World*; Gomez Paloma, F., Ed.; IntechOpen: London, UK, 2024. [CrossRef]
7. Chen, L. Pattern recognition in mathematical sequences. *J. Math. Theory* **2021**, *44*, 204–219.
8. Wang, D.Q. *The Art of Teaching*; Sichuan University Press: Chengdu, China, 2010.
9. Xue, E.; Li, J. What is the value essence of "double reduction" (Shuang Jian) policy in China? A policy narrative perspective. *Educ. Philos. Theory* **2023**, *55*, 787–796. [CrossRef]

10. Student-Centered Learning. Wikipedia. 19 June 2024. Available online: https://en.wikipedia.org/wiki/Student-centered_learning (accessed on 29 July 2024).
11. Gong, P.; Xiao, Y. United States: The Evolution of "Stratified Teaching". *Curric. Teach. Mater. Teach. Method* **2004**, *11*, 96.
12. Sun, W.Z. Discussion on high school mathematics stratified teaching under the new curriculum standard concept. *China Extra-Scholast. Educ.* **2014**. Available online: https://www.doc88.com/p-1032175039926.html (accessed on 29 July 2024).
13. Zhu, X.; Zhang, Y.; Yang, L.; Liu, Z. Developing stratified teaching to improve teaching quality. In Proceedings of the International Conference on Arts, Humanity and Economics, Management (ICAHEM 2019), Wuhan, China, 21–22 December 2019; Atlantis Press: Paris, France, 2020; pp. 34–37. [CrossRef]
14. Walde, G.S. Hierarchical linear model to examine determinants of students' mathematics performance. *J. Phys. Conf. Ser.* **2019**, *1176*, 042088. [CrossRef]
15. Gong, S.Q.; Zhu, Z. China's Mathematics Education in 2035. *Middle Sch. Math. Mag.* **2021**, *2021*.
16. Collins, A.; Brown, J.S.; Newman, S.E. Cognitive apprenticeship: Teaching reading, writing, and mathematics crafts. In *Knowing, Learning, and Instruction: Essays in Honor of Robert Glaser*; Resnick, L.B., Ed.; Lawrence Erlbaum Associates Inc.: Hillsdale, NJ, USA, 1989; pp. 453–494.
17. Cakmakci, G.; Aydeniz, M.; Brown, A.; Makokha, J. Situated Cognition and Cognitive Apprenticeship Learning. In *Science Education in Theory and Practice, An Introductory Guide to Learning Theory*; Springer: Cham, Switzerland, 2020; pp. 293–310. [CrossRef]
18. Foster, K. The Effects of Math Literacy Utilizing a Reading Apprenticeship Framework on Math Achievement of Analytic Geometry Students. Ph.D. Thesis, Mercer University, Macon, GA, USA, 2020.
19. Bruin, L.R. The Use of Cognitive Apprenticeship in the Learning and Teaching of Improvisation: Teacher and Student Perspectives. *Res. Stud. Music Educ.* **2019**, *41*, 261–279. [CrossRef]
20. Brill, M.; Weidemann, S.; BOSTI Associates. *Disproving Widespread Myths about Workplace Design*; Kimball International: Jasper, IN, USA, 2001.
21. Kurt, S. ADDIE Model: Instructional Design. *Educ. Technol.* **2017**. Available online: https://educationaltechnology.net/the-addie-model-instructional-design/#addie-stages (accessed on 14 August 2024).
22. International Baccalaureate (IB). The Impact of an IB Education. 6 June 2023. Available online: https://www.ibo.org/ (accessed on 14 August 2024).
23. Miller, T. Investigating Patterns in Clinical Research: Methodologies and Applications. *Clin. Psychol. Rev.* **2021**, *33*, 115–129.
24. Rohid, N.; Suryaman, S.; Rusmawati, R.D. Students' mathematical communication skills (MCS) in solving mathematics problems: A case in Indonesian context. *Anatol. J. Educ.* **2019**, *4*, 19–30. [CrossRef]
25. Smith, C.; Morgan, C. Curricular orientations to real-world contexts in mathematics. *Curric. J.* **2016**, *27*, 24–45. [CrossRef]
26. Conley, D.T. The Common Core State Standards: Insight into their development and purpose. *Educ. Res.* **2014**, *43*, 14–16.
27. Mayer, R.E. *The Cambridge Handbook of Multimedia Learning*, 2nd ed.; Cambridge University Press: Cambridge, UK, 2020. [CrossRef]
28. Smith, C.; Kathotia, V.; Ward-Penny, R.; Howson, O.; Wermelinger, M. Mathematical and data literacy: Competencies and curriculum implications at the intersection of mathematics, data science, statistics and computing. *R. Soc.* **2023**. Available online: https://royalsociety.org/-/media/policy/projects/maths-futures/intersection-mathematics-data-statistics-computing.pdf (accessed on 14 August 2024).
29. Rohrer, D.; Pashler, H. Increasing retention without increasing study time. *Curr. Dir. Psychol. Sci.* **2007**, *16*, 183–186. [CrossRef]
30. Cepeda, N.J.; Vul, E.; Rohrer, D.; Wixted, J.T.; Pashler, H. Spacing effects in learning: A temporal ridgeline of optimal retention. *Psychol. Sci.* **2008**, *19*, 1095–1102. [CrossRef] [PubMed]
31. Karpicke, J.D.; Roediger, H.L. The critical importance of retrieval for learning. *Science* **2008**, *319*, 966–968. [CrossRef] [PubMed]
32. McNeish, D. Thanks coefficient alpha, we'll take it from here. *Psychol. Methods* **2018**, *23*, 412–433. [CrossRef] [PubMed]
33. Freeman, S.; Eddy, S.L.; McDonough, M.; Smith, M.K.; Okoroafor, N.; Jordt, H.; Wenderoth, M.P. Active learning increases student performance in science, engineering, and mathematics. *Proc. Natl. Acad. Sci. USA* **2014**, *111*, 8410–8415. [CrossRef] [PubMed]
34. National Academies of Sciences, Engineering, and Medicine. *How People Learn II: Learners, Contexts, and Cultures*; National Academies Press: Washington, DC, USA, 2018. [CrossRef]
35. Wan, S.W.-Y. Differentiated instruction: Are Hong Kong in-service teachers ready? *Teach. Teach.* **2017**, *23*, 284–311. [CrossRef]