



**UNIVERSITI PUTRA MALAYSIA**

**TWO AND THREE-POINT BLOCK METHODS FOR SOLVING FIRST  
ORDER ORDINARY DIFFERENTIAL EQUATIONS IN PARALLEL**

**LEE LAI SOON**

**FSAS 2000 6**

**TWO AND THREE-POINT BLOCK METHODS FOR SOLVING FIRST  
ORDER ORDINARY DIFFERENTIAL EQUATIONS IN PARALLEL**

**By**

**LEE LAI SOON**

**Thesis Submitted in Fulfilment of the Requirements for the  
Degree of Master of Science in the Faculty of  
Science and Environment Studies  
Universiti Putra Malaysia**

**May 2000**



## DEDICATION

This book is dedicated to Professor Dr. Mohamed Bin Suleiman for his guidance and motivation throughout my studies. Thank you Prof. And I hope to continue to grow under your tutelage.

To my father, thank you for your support and love. " Pa, I kept my promise. I did it ! ".

Abstract of thesis submitted to the Senate of Universiti Putra Malaysia  
in fulfilment of the requirements for the degree of Master of Science.

**TWO AND THREE-POINT BLOCK METHODS FOR SOLVING FIRST  
ORDER ORDINARY DIFFERENTIAL EQUATIONS IN PARALLEL**

By

**LEE LAI SOON**

**May 2000**

**Chairman : Professor Mohamed Bin Suleiman, Ph.D.**

**Faculty : Science and Environment Studies**

This thesis concerns mainly in deriving new 2-point and 3-point block methods for solving a single equation of first order ODE directly using constant step size in both explicit and implicit methods. These methods, which calculate the numerical solution at more than one point simultaneously, are suitable for parallel implementations. The programs of the methods employed are run on a shared memory Sequent Symmetry SE30 parallel computer. The numerical results show that the new methods reduce the total number of steps and execution time. The accuracy of the parallel block and 1-point methods is comparable particularly when finer step size are used. The stability of the new methods also had been investigated.



A new rectified sequential and parallel algorithms from the existing program for solving systems of ODEs directly with variable step size and order using 2-point block methods is also developed. The results demonstrate the superiority of the new rectify program in terms of the execution times, speedup, efficiency, cost and temporal performance especially with finer tolerances.

Consequently, the new methods developed appear to be a natural approach to solving ODEs on a parallel processor.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains.

**KAEDAH BLOK DUA DAN TIGA-TITIK BAGI MENYELESAIKAN  
PERSAMAAN PEMBEZAAN BIASA PERINGKAT PERTAMA  
SECARA SELARI**

Oleh

**LEE LAI SOON**

**Mei 2000**

**Pengerusi : Profesor Mohamed Bin Suleiman, Ph.D.**

**Fakulti : Sains dan Pengajian Alam Sekitar**

Tumpuan utama tesis ini adalah untuk menerbitkan kaedah baru blok 2-titik dan 3-titik bagi menyelesaikan persamaan pembezaan biasa tunggal secara langsung dengan menggunakan saiz langkah malar dalam kaedah tersirat dan kaedah tak tersirat. Kaedah yang menghitung penyelesaian berangka pada beberapa titik secara serentak ini adalah sesuai untuk implimentasi selari. Semua atur cara dilaksanakan dengan menggunakan Sequent Symmetry SE30, iaitu sebuah komputer selari berkongsi ingatan. Keputusan berangka menunjukkan bahawa kedua-dua kaedah tersebut dapat mengurangkan bilangan langkah dan masa pelaksanaannya. Kejitian

kaedah blok selari dan 1-titik adalah setanding khususnya apabila saiz langkah yang kecil digunakan. Kestabilan kaedah blok baru itu turut dikaji selidik.

Algoritma jujukan dan selari terubahsuai daripada atur cara yang sedia ada bagi menyelesaikan sistem persamaan pembezaan secara langsung dengan saiz langkah dan nilai belakang boleh ubah menggunakan kaedah blok 2-titik turut dibangunkan. Keputusan berangka membuktikan bahawa atur cara terubahsuai ini mempunyai kelebihan dari segi masa pelaksanaan, kecepatan, keberkesanan, kos dan prestasi 'temporal', terutamanya bagi toleransi yang kecil.

Kesimpulannya, kaedah blok baru yang dibangunkan ini merupakan pendekatan natural dalam menyelesaikan persamaan pembezaan menggunakan pemproses selari.

## ACKNOWLEDGEMENTS

I would like to express my most gratitude and sincere appreciation to the Chairman of my Supervisory Committee, Professor Dr. Mohamed Bin Suleiman for his outstanding supervision and continuous support. Special thanks also due to Professor for providing financial support in the form of Graduate Assistantship throughout the duration of my research work in the university. Despite his heavy administrative responsibility as a Chairman of National Accreditation Board, he always found time to go through the work and gave me critical suggestions and guidance.

A significant acknowledgement is also due to the members of the Supervisory Committee, Dr. Mohamed Bin Othman of the Department of Communication Technology and Network and Dr. Fudziah Binti Ismail of the Department of Mathematics for their untiring guidance, valuable advice, support and comments. Their patience and persistent encouragement during the course of my research is instrumental to the completion of this thesis.

Special thanks is given to the Head of Department, Assoc. Prof. Dr. Isa Bin Daud, Assoc. Prof. Dr. Harun Bin Budin, Assoc. Prof. Dr. Bachok Bin Taib, the academic and general staff of the Department of Mathematics and Faculty of Computer Science and Information Technology, Universiti Putra Malaysia, for their assistance in various capacities.

Last but not least, I would like to thank my family and friends for their understanding support and encouragement throughout the course of my study. I like to dedicate this to my father for his support and motivation.





## TABLE OF CONTENTS

	Page
<b>DEDICATION</b> .....	ii
<b>ABSTRACT</b> .....	iii
<b>ABSTRAK</b> .....	v
<b>ACKNOWLEDGEMENTS</b> .....	vii
<b>APPROVAL SHEETS</b> .....	viii
<b>DECLARATION FORM</b> .....	x
<b>LIST OF TABLES</b> .....	xiv
<b>LIST OF FIGURES</b> .....	xx
<b>LIST OF ABBREVIATIONS</b> .....	xxii
 <b>CHAPTER</b>	
<b>I INTRODUCTION</b> .....	1
Introduction .....	1
Objective of the Studies .....	6
Organization of the Studies .....	7
 <b>II INTRODUCTION TO PARALLEL COMPUTING</b> .....	 8
Introduction .....	8
Parallel Computer Architectures .....	9
The Sequent Symmetry SE30 .....	17
Elements of Parallel Programming .....	22
Shared and Private Data .....	23
Process Creation and Termination .....	23
Scheduling .....	24
Synchronization .....	26
Mutual Exclusion .....	26
Identifying .....	29
Data Dependence In Loop .....	29
Control Dependence .....	29
Interprocedural Dependence .....	30
Performance Metrics .....	30
Run Time .....	32
Speedup .....	32
Efficiency .....	33



	Cost .....	34
	Temporal Performance .....	34
	Scalability .....	35
<b>III</b>	<b>INTRODUCTION TO THE NUMERICAL METHODS IN ORDINARY DIFFERENTIAL EQUATION .....</b>	<b>37</b>
	Introduction .....	37
	Linear Multistep Methods .....	39
	Divided Differences .....	46
	Newton Backward Divided Difference Formula .....	47
	Survey of Parallel Algorithms for the Solution of ODEs .....	49
<b>IV</b>	<b>2-POINT AND 3-POINT EXPLICIT BLOCK METHODS FOR FIRST ORDER ODEs .....</b>	<b>54</b>
	Introduction .....	54
	Derivation of 2-Point Block Method .....	57
	Derivation of 3-Point Block Method .....	63
	Stability .....	65
	1-Point Explicit Block Method .....	66
	2-Point Explicit Block Method .....	69
	Problem Tested .....	76
	Numerical Results .....	77
	Discussion .....	97
	Total Number of Steps .....	97
	Accuracy .....	97
	Execution Times .....	99
<b>V</b>	<b>2-POINT AND 3-POINT IMPLICIT BLOCK METHODS FOR FIRST ORDER ODEs .....</b>	<b>101</b>
	Introduction .....	101
	Derivation of 2-Point Block Method .....	101
	Derivation of 3-Point Block Method .....	107
	Stability .....	110
	1-Point Implicit Block Method .....	111
	2-Point Implicit Block Method .....	114
	Problem Tested .....	120
	Numerical Results .....	120
	Discussion .....	139
	Total Number of Steps .....	139
	Accuracy .....	140



	Execution Times .....	141
<b>VI</b>	<b>RECTIFICATION OF THE ALGORITHMS FOR 2-POINT BLOCK METHODS .....</b>	<b>143</b>
	Introduction .....	143
	Improvement of Sequential Algorithms 2-Point Block Methods ....	144
	Improvement of Parallel Algorithms 2-Point Block Methods .....	155
	Problem Tested .....	166
	Performance Metrics .....	168
	Discussion .....	180
	Run Time .....	180
	Speedup .....	181
	Efficiency .....	182
	Cost .....	182
	Temporal Performance .....	183
<b>VII</b>	<b>CONCLUSIONS .....</b>	<b>185</b>
	Summary .....	185
	Future Work .....	188
	<b>BIBLIOGRAPHY .....</b>	<b>189</b>
	<b>APPENDIX</b>	
<b>A</b>	Basic Definition of Parallel Computing .....	194
<b>B</b>	Mathematica Programming for the Explicit Block Methods .....	196
<b>C</b>	Mathematica Programming for the Implicit Block Methods .....	201
<b>D</b>	Sequential Program for the Explicit Block Methods .....	206
<b>E</b>	Parallel Program for the Explicit Block Methods .....	212
<b>F</b>	Sequential Program for the Implicit Block Methods .....	219
<b>G</b>	Parallel Program for the Implicit Block Methods .....	229
<b>H</b>	Original Version of Sequential Program PROGSEQ.C by Omar (1999). .....	240
<b>I</b>	Rectified Version of Sequential Program PROGSEQ.C .....	252
<b>J</b>	Original Version of Parallel Program PROGPAL.C by Omar (1999). .....	263
<b>K</b>	Rectified Version of Parallel Program PROGPAL.C .....	276
<b>VITA</b>	.....	287



## LIST OF TABLES

Table		Page
1	Configuration of Sequent Symmetry SE30 .....	20
2	Parallel Programming Library .....	27
3	Integration Coefficients of the First Point of the 2-Point Explicit Block Method when $f$ is Integrated Once .....	60
4	Integration Coefficients of the Second Point of the 2-Point Explicit Block Method when $f$ is Integrated Once .....	62
5	Integration Coefficients of the Third Point of the 3-Point Explicit Block Method when $f$ is Integrated Once .....	64
6	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 1 of First Order ODEs when $k=3$ .....	79
7	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 1 of First Order ODEs when $k=5$ .....	80
8	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 1 of First Order ODEs when $k=8$ .....	81
9	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 2 of First Order ODEs when $k=3$ .....	82
10	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 2 of First Order ODEs when $k=5$ .....	83
11	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 2 of First Order ODEs when $k=8$ .....	84
12	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 3 of First Order ODEs when $k=3$ .....	85



13	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 3 of First Order ODEs when $k=5$ .....	86
14	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 3 of First Order ODEs when $k=8$ .....	87
15	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 4 of First Order ODEs when $k=3$ .....	88
16	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 4 of First Order ODEs when $k=5$ .....	89
17	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 4 of First Order ODEs when $k=8$ .....	90
18	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 5 of First Order ODEs when $k=3$ .....	91
19	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 5 of First Order ODEs when $k=5$ .....	92
20	Comparison Between the E1P, 2PEB and 3PEB Methods for Solving Problem 5 of First Order ODEs when $k=8$ .....	93
21	The Ratio Steps and Execution Times of the 2PEB and 3PEB Methods to the E1P Methods for solving First Order ODEs when $k=3$ .....	94
22	The Ratio Steps and Execution Times of the 2PEB and 3PEB Methods to the E1P Methods for solving First Order ODEs when $k=5$ .....	95
23	The Ratio Steps and Execution Times of the 2PEB and 3PEB Methods to the E1P Methods for solving First Order ODEs when $k=8$ .....	96
24	Integration Coefficients of the First Point of the 2-Point Implicit Block Method when $f$ is Integrated Once .....	104



25	Integration Coefficients of the Second Point of the 2-Point Implicit Block Method when $f$ is Integrated Once .....	107
26	Integration Coefficients of the Third Point of the 2-Point Implicit Block Method when $f$ is Integrated Once .....	110
27	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 1 of First Order ODEs when $k=3$ .....	121
28	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 1 of First Order ODEs when $k=5$ .....	122
29	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 1 of First Order ODEs when $k=8$ .....	123
30	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 2 of First Order ODEs when $k=3$ .....	124
31	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 2 of First Order ODEs when $k=5$ .....	125
32	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 2 of First Order ODEs when $k=8$ .....	126
33	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 3 of First Order ODEs when $k=3$ .....	127
34	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 3 of First Order ODEs when $k=5$ .....	128
35	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 3 of First Order ODEs when $k=8$ .....	129
36	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 4 of First Order ODEs when $k=3$ .....	130
37	Comparison Between the 11P, 2PIB and 3PIB Methods for Solving Problem 4 of First Order ODEs when $k=5$ .....	131





38	Comparison Between the I1P, 2PIB and 3PIB Methods for Solving Problem 4 of First Order ODEs when $k=8$ .....	132
39	Comparison Between the I1P, 2PIB and 3PIB Methods for Solving Problem 5 of First Order ODEs when $k=3$ .....	133
40	Comparison Between the I1P, 2PIB and 3PIB Methods for Solving Problem 5 of First Order ODEs when $k=5$ .....	134
41	Comparison Between the I1P, 2PIB and 3PIB Methods for Solving Problem 5 of First Order ODEs when $k=8$ .....	135
42	The Ratio Steps and Execution Times of the 2PIB and 3PIB Methods to the I1P Methods for solving First Order ODEs when $k=3$ .....	136
43	The Ratio Steps and Execution Times of the 2PIB and 3PIB Methods to the I1P Methods for solving First Order ODEs when $k=5$ .....	137
44	The Ratio Steps and Execution Times of the 2PIB and 3PIB Methods to the I1P Methods for solving First Order ODEs when $k=8$ .....	138
45	<i>goto</i> Statement .....	144
46	Calculation of Table G (Integration Coefficient) .....	147
47	Errors Computations .....	149
48	Function and Solution Evaluation .....	150
49	EPSELON .....	151
50	Redundant Variable, Expressions and Statements .....	152
51	Typing Errors .....	153
52	Assignment Statements .....	153



53	Variable Declaration .....	154
54	Redundant <i>goto</i> Statement and Subroutine SOLN .....	155
55	<i>goto</i> Statement .....	156
56	Calculation of Table G in Parallel .....	157
57	Errors Computations in Parallel .....	160
58	Function and Solution Evaluation .....	161
59	EPSELON .....	162
60	Redundant Variable, Expressions and Statements .....	163
61	Typing Errors .....	164
62	Assignment Statements .....	164
63	Variable Declaration .....	165
64	Redundant <i>goto</i> Statement and Subroutine SOLN .....	166
65	Comparison Between O2PBVSO and R2PBVSO Methods for Solving Problem 6 Directly .....	170
66	Comparison Between O2PBVSO and R2PBVSO Methods for Solving Problem 7 Directly .....	171
67	Comparison Between O2PBVSO and R2PBVSO Methods for Solving Problem 8 Directly .....	172
68	Comparison Between O2PBVSO and R2PBVSO Methods for Solving Problem 9 Directly .....	173
69	Comparison Between O2PBVSO and R2PBVSO Methods for Solving Problem 10 Directly .....	174





70	The Ratio Steps and Execution Times of the 2PBVSO Method to the 1PVSO Method for Solving Problem 6 to Problem 10 ...	175
71	Comparison between Original Program and Rectified Program of the P2PBVSO for Problem 10 at $TOL = 10^{-11}$ in terms of Run Time, Speedup, Efficiency, Cost and Temporal Performance .....	177



## LIST OF FIGURES

Figure		Page
1	SISD Computer .....	11
2	SIMD Computer .....	12
3	MISD Computer .....	13
4	MIMD Computer .....	14
5	Shared Bus Multiprocessor Clusters .....	16
6	Shared Memory Parallel Computer .....	17
7	Symmetry 5000 Architecture .....	18
8	2-Point Method .....	54
9	3-Point Method .....	55
10	2-Point 3-Block Methods .....	55
11	3-Point 2-Block Methods .....	56
12	Stability Region of the Explicit 1-Point 1-Block Methods...	74
13	Stability Region of the Explicit 1-Point 2-Block Methods...	74
14	Stability .....	75
15	Stability Region of the Explicit 2-Point 2-Block Methods...	75
16	Stability Region of the Implicit 1-Point 1-Block Methods...	118
17	Stability Region of the Implicit 1-Point 2-Block Methods...	118



18	Stability Region of the Implicit 2-Point 1-Block Methods...	119
19	Stability Region of the Implicit 2-Point 2-Block Methods...	119
20	Table G (Integration Coefficients) for Sequential .....	148
21	Table G (Integration Coefficients) for Parallel .....	158
22	Execution Times of Problem 10 at TOL = $10^{-11}$ when tested on both version of P2PBVSO Methods with $p$ processors ( $p = 1, 2$ ).....	177
23	Speedup of Problem 10 at TOL = $10^{-11}$ when tested on both version of P2PBVSO Methods with $p$ processors ( $p = 1, 2$ ) .....	178
24	Efficiency of Problem 10 at TOL = $10^{-11}$ when tested on both version of P2PBVSO Methods with $p$ processors ( $p = 1, 2$ ) .....	178
25	Cost of Problem 10 at TOL = $10^{-11}$ when tested on both version of S2PBVSO and P2PBVSO Methods with $p$ processors ( $p = 1, 2$ ) .....	179
26	Temporal Performance of Problem 10 at TOL = $10^{-11}$ when tested on both version of P2PBVSO Methods with $p$ processors ( $p = 1, 2$ ) .....	179



## LIST OF ABBREVIATIONS

DI	:	Direct Integration
EIP	:	Explicit 1-Point
IIP	:	Implicit 1-Point
IVP	:	Initial Value Problem
MIMD	:	Multiple Instruction Stream, Multiple Data Stream
MISD	:	Multiple Instruction Stream, Single Data Stream
ODEs	:	Ordinary Differential Equations
SIMD	:	Single Instruction Stream, Multiple Data Stream
SISD	:	Single Instruction Stream, Single Data Stream
2PEB	:	2-Point Explicit Block
2PIB	:	2-Point Implicit Block
3PEB	:	3-Point Explicit Block
3PIB	:	3-Point Implicit Block



## CHAPTER I

### INTRODUCTION

#### Introduction

Since the advent of computers, the numerical solution of Initial Value Problem (IVP) for Ordinary Differential Equations (ODEs) has been the subject of research by numerical analysts. IVP manifest themselves in almost all branches of science, engineering and technology. Considerable amount of work is being done to write general purpose codes to produce accurate solutions to most of these problems occurring in practice. Some of the problems given by Atkinson(1989) are as follows :

- (1) The problem of determining the motion of a projectile, rocket, satellite, or planet.
- (2) The problem of determining the charge or current in an electric circuit.
- (3) The problem of the heat conduction in a rod or in a slab.
- (4) The problem of determining the vibrations of a string or membrane.
- (5) The study of the rate of decomposition of radioactive substance or the rate of growth of a population.
- (6) The study of the chemicals reactions.
- (7) The problem of the determination of curves that have certain geometrical properties.

The problems listed above obey certain scientific laws that involve rates of change of one or more quantities with respect to other quantities. These rate of change can be expressed mathematically by derivatives. When the problems are formulated in mathematical equations they will become differential equations.

The available codes for the numerical solution of IVPs for ODEs, to be run on the conventional sequential computers, have already reached a very high level of efficiency, reliability, and portability. Nevertheless, the continuous and dramatic growth in dimension and the increasing complexity of the mathematical models, which are designed in applied research, often make such codes inadequate in term of speed. The continuous progress of the microelectronic technology is not even sufficient. Faster and faster microprocessors do not yet overcome the thresholds imposed by those problem which are computationally very expensive and, at the same time, need a real time response to the user, see Amodio and Trigiante (1993).

In the first computing wave, scientific and business computers were more or less identical: big and slow. And, even if early electronic computers were not very fast, they achieved speeds that easily exceeded human computers.

But the original power users who pioneered computing continued to emphasize speed above all else. Single processor supercomputers achieved unheard of speeds beyond 1000 million instructions per second, and pushed hardware technology to the physical limits of chip building. But soon this trend will come to an end, because there are physical and architectural bounds which limit the computational power that can be achieved with a single processor system.