UNIVERSITI PUTRA MALAYSIA

TWO AND THREE-POINT BLOCK METHODS FOR SOLVING FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS IN PARALLEL

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By

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Thesis Submitted in Fulfilment of the Requirements for the
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DEDICATION

This book is dedicated to Professor Dr. Mohamed Bin Suleiman for his guidance and motivation throughout my studies. Thank you Prof. And I hope to continue to grow under your tutelage.

To my father, thank you for your support and love. " Pa, I kept my promise. I did it ! ".

Abstract of thesis submitted to the Senate of Universiti Putra Malaysia in fulfilment of the requirements for the degree of Master of Science.

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Chairman : Professor Mohamed Bin Suleiman, Ph.D.

Faculty : Science and Environment Studies

This thesis concerns mainly in deriving new 2-point and 3-point block methods for solving a single equation of first order ODE directly using constant step size in both explicit and implicit methods. These methods, which calculate the numerical solution at more than one point simultaneously, are suitable for parallel implementations. The programs of the methods employed are run on a shared memory Sequent Symmetry SE30 parallel computer. The numerical results show that the new methods reduce the total number of steps and execution time. The accuracy of the parallel block and 1-point methods is comparable particularly when finer step size are used. The stability of the new methods also had been investigated.

A new rectified sequential and parallel algorithms from the existing program for solving systems of ODEs directly with variable step size and order using 2-point block methods is also developed. The results demonstrate the superiority of the new rectify program in terms of the execution times, speedup, efficiency, cost and temporal performance especially with finer tolerances.

Consequently, the new methods developed appear to be a natural approach to solving ODEs on a parallel processor.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains.

KAEDAH BLOK DUA DAN TIGA-TITIK BAGI MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA PERINGKAT PERTAMA SECARA SELARI

Oleh

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Mei 2000

Pengerusi : Profesor Mohamed Bin Suleiman, Ph.D.

Fakulti : Sains dan Pengajian Alam Sekitar

Tumpuan utama tesis ini adalah untuk menerbitkan kaedah baru blok 2-titik dan 3-titik bagi menyelesaikan persamaan pembezaan biasa tunggal secara langsung dengan menggunakan saiz langkah malar dalam kaedah tersirat dan kaedah tak tersirat. Kaedah yang menghitung penyelesaian berangka pada beberapa titik secara serentak ini adalah sesuai untuk implimentasi selari. Semua atur cara dilaksanakan dengan menggunakan Sequent Symmetry SE30, iaitu sebuah komputer selari berkongsi ingatan. Keputusan berangka menunjukkan bahawa kedua-dua kaedah tersebut dapat mengurangkan bilangan langkah dan masa pelaksanaannya. Kejituan

kaedah blok selari dan 1-titik adalah setanding khususnya apabila saiz langkah yang kecil digunakan. Kestabilan kaedah blok baru itu turut dikaji selidik.

Algoritma jujukan dan selari terubahsuai daripada atur cara yang sedia ada bagi menyelesaikan sistem persamaan pembezaan secara langsung dengan saiz langkah dan nilai belakang boleh ubah menggunakan kaedah blok 2-titik turut dibangunkan. Keputusan berangka membuktikan bahawa atur cara terubahsuai ini mempunyai kelebihan dari segi masa pelaksanaan, kecepatan, keberkesanan, kos dan prestasi 'temporal', terutamanya bagi toleransi yang kecil.

Kesimpulannya, kaedah blok baru yang dibangunkan ini merupakan pendekatan natural dalam menyelesaikan persamaan pembezaan menggunakan pemproses selari.

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LIST OF ABBREVIATIONS

DI	:	Direct Integration
E1P	:	Explicit 1-Point
I1P	:	Implicit 1-Point
IVP	:	Initial Value Problem
MIMD	:	Multiple Instruction Stream, Multiple Data Stream
MISD	:	Multiple Instruction Stream, Single Data Stream
ODEs	:	Ordinary Differential Equations
SIMD	:	Single Instruction Stream, Multiple Data Stream
SISD	:	Single Instruction Stream, Single Data Stream
2PEB	:	2-Point Explicit Block
2PIB	:	2-Point Implicit Block
3PEB	:	3-Point Explicit Block
3PIB	:	3-Point Implicit Block

CHAPTER I

INTRODUCTION

Introduction

Since the advent of computers, the numerical solution of Initial Value Problem (IVP) for Ordinary Differential Equations (ODEs) has been the subject of research by numerical analysts. IVP manifest themselves in almost all branches of science, engineering and technology. Considerable amount of work is being done to write general purpose codes to produce accurate solutions to most of these problems occurring in practice. Some of the problems given by Atkinson(1989) are as follows :

(1) The problem of determining the motion of a projectile, rocket, satellite, or planet.

(2) The problem of determining the charge or current in an electric circuit.

(3) The problem of the heat conduction in a rod or in a slab.

(4) The problem of determining the vibrations of a string or membrane.

(5) The study of the rate of decomposition of radioactive substance or the rate of growth of a population.

(6) The study of the chemicals reactions.

(7) The problem of the determination of curves that have certain geometrical properties.

The problems listed above obey certain scientific laws that involve rates of change of one or more quantities with respect to other quantities. These rate of change can be expressed mathematically by derivatives. When the problems are formulated in mathematical equations they will become differential equations.

The available codes for the numerical solution of IVPs for ODEs, to be run on the conventional sequential computers, have already reached a very high level of efficiency, reliability, and portability. Nevertheless, the continuous and dramatic growth in dimension and the increasing complexity of the mathematical models, which are designed in applied research, often make such codes inadequate in term of speed. The continuous progress of the microelectronic technology is not even sufficient. Faster and faster microprocessors do not yet overcome the thresholds imposed by those problem which are computationally very expensive and, at the same time, need a real time response to the user, see Amodio and Trigiante (1993).

In the first computing wave, scientific and business computers were more or less identical: big and slow. And, even if early electronic computers were not very fast, they achieved speeds that easily exceeded human computers.

But the original power users who pioneered computing continued to emphasize speed above all else. Single processor supercomputers achieved unheard of speeds beyond 1000 million instructions per second, and pushed hardware technology to the physical limits of chip building. But soon this trend will come to an end, because there are physical and architectural bounds which limit the computational power that can be achieved with a single processor system.