



UNIVERSITI PUTRA MALAYSIA

**SIMULATIONS OF ONSET OF CONVECTION IN A NON-NEWTONIAN
LIQUID INDUCED BY UNSTEADY-STATE HEAT CONDUCTION**

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LIQUID INDUCED BY UNSTEADY-STATE HEAT CONDUCTION**

By

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SIMULATIONS OF ONSET OF CONVECTION IN A NON-NEWTONIAN LIQUID INDUCED BY UNSTEADY-STATE HEAT CONDUCTION

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The onset of convection in an initially static non-Newtonian liquid under Fixed Surface Temperature (FST) and Constant Heat Flux (CHF) boundary conditions was simulated using a CFD package. Steady-state and unsteady-state simulations were successfully conducted for bottom surface heating of shear thinning non-Newtonian liquids. Simulations on Newtonian liquid water and glycerine were conducted to verify the simulation setup.

Fourier's law of heat conduction was used to validate the steady-state simulation results. Simulations conducted for non-Newtonian liquid with Tien *et al.*'s (1969) experimental data were found to agree well with Fourier's law at conduction phase. Tien *et al.*'s definition of non-Newtonian power-law Rayleigh number was found to be inadequate in representing the onset of convection in non-Newtonian liquid. Attempts to determine the Rayleigh number for non-Newtonian liquid using apparent viscosity was successfully carried out. A more realistic critical Rayleigh number for non-Newtonian liquid was successfully determined with local values of Rayleigh number around a convection cell successfully obtained.



For simulations conducted for unsteady-state heat conduction in non-Newtonian liquid, transient heat conduction theory was used to validate the results. Convection was found to occur in a continuous deep fluid bounded by two horizontal rigid surfaces and adiabatic vertical walls. Transient critical Rayleigh number for non-Newtonian liquid under unsteady state heat conduction defined by Tan (1994) was successfully applied. Transient critical Rayleigh number for non-Newtonian liquid was found to vary with flow behavior n of the Power Law model. A more realistic transient critical Rayleigh number for non-Newtonian liquid was successfully determined using apparent viscosity.

Development of thermal plumes in viscous non-Newtonian liquid were found to differ slightly from the development of thermal plumes in non-viscous Newtonian liquid. The Nu_{max} for unsteady-state simulations of Newtonian and non-Newtonian liquid were observed to be 3.8 ± 2.0 for FST cases and 2.7 ± 1.8 for CHF cases.

Effect of boundary condition at interface on onset of transient convection were studied. Velocity boundary condition of a top surface solid were found to be best approximated using top-cooling simulations. Bottom-heating simulations in a deep fluid revealed that the upper interface boundary has the property between a solid and a free surface.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

**SIMULASI PERMULAAN PEROLAKAN DALAM CECAIR BUKAN
NEWTONIAN YANG DIARUHI KONDUKSI HABA TIDAK MANTAP**

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Permulaan perolakan dalam cecair bukan Newtonian yang statik pada mulanya di bawah keadaan Permukaan Suhu Tetap (FST) dan Fluks Haba Tetap (CHF) telah disimulasikan dengan pakej Pengiraan Dinamik Bendalir (CFD). Simulasi untuk keadaan mantap dan keadaan tidak mantap telah berjaya dilaksanakan untuk pemanasan cecair bukan Newtonian dari permukaan bawah. Simulasi ke atas cecair Newtonian air dan glycerine telah dilaksanakan untuk mengesahkan penetapan simulasi.

Teori konduksi haba Fourier telah digunakan dalam pemastian keputusan untuk simulasi keadaan mantap. Simulasi yang dilaksanakan untuk cecair bukan Newtonian dengan data ujikaji Tien *et al.* (1969) telah didapati mematuhi teori Fourier dengan baiknya pada fasa konduksi. Definisi Tien *et al.* untuk nombor Rayleigh cecair bukan Newtonian power-law telah didapati kurang memuaskan dalam pengambaran permulaan perolakan di dalam cecair bukan Newtonian. Percubaan untuk menentukan nombor Rayleigh untuk cecair bukan Newtonian dengan menggunakan kepekatan ketara telah berjaya dilakukan. Suatu nombor Rayleigh untuk cecair bukan Newtonian yang lebih realistik telah berjaya ditentukan

dengan nilai tempatan nombor Rayleigh di sekitar satu sel perolakan berjaya didapati.

Untuk simulasi yang dilaksanakan untuk konduksi haba tidak mantap di dalam cecair bukan Newtonian, teori konduksi haba tidak mantap telah digunakan dalam pemastian keputusan simulasi. Perolakan dapat diperhatikan di dalam cecair berlanjutan dibatasi dua sempadan mendatar yang tegar dan dua dinding tegak adiabatik. Nombor Rayleigh kritikal tidak mantap bagi cecair bukan Newtonian yang didefinisikan oleh Tan (1994) untuk konduksi haba tidak mantap telah berjaya digunakan. Nombor Rayleigh kritikal tidak mantap bagi cecair bukan Newtonian didapati berubah dengan indeks sifat pengaliran cecair n dalam model Power Law. Suatu nombor Rayleigh tidak mantap untuk cecair bukan Newtonian yang lebih realistik telah berjaya ditentukan dengan menggunakan kelikatan ketara.

Perubahan kepulan haba di dalam cecair bukan Newtonian pekat didapati berlainan sedikit dengan perubahan di dalam cecair Newtonian yang cair. Nu_{max} untuk simulasi keadaan tidak mantap cecair Newtonian dan cecair bukan Newtonian didapati berada dalam lingkungan 3.8 ± 2.0 untuk kes-kes FST dan 2.7 ± 1.8 untuk kes-kes CHF.

Kesan keadaan sempadan di permukaan dwihala pada permulaan perolakan telah diselidiki. Keadaan sempadan halaju yang tegar pada permukaan atas didapati paling baik dikaji dengan simulasi penyejukan dari atas. Simulasi pemanasan dari bawah dalam cecair berlanjutan mencadangkan permukaan dwihala atas mempunyai sifat di antara tegar dan tidak tegar.

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LIST OF NOTATIONS

\tilde{a}_c	Dimensionless wave number
B	Constant rate of surface temperature variation (K/s)
C	Heating current (A)
c_p	Specific heat (J/kg.K)
D	Diameter (m)
d	Total depth of liquid layer for steady-state heat conduction (m)
g	Gravitational acceleration (m/s^2)
h	Heat transfer coefficient ($W/m^2.K$)
k	Thermal conductivity (W/m. K)
K	Consistency index ($Pa.s^n$)
l	Liquid depth for unsteady-state heat conduction (m)
n	Flow behaviour index in Power Law model
Nu	Nusselt number
q	Heat flux (W/m^2)
q^0	Constant surface heat flux (W/m^2)
\bar{q}	Total heat conducted per area into the liquid (J/m^2)
Q	Heat input (W)
Ra	Rayleigh number
Ra_{NNc}	Critical transient Rayleigh number for non-Newtonian liquid
Pe	Peclet number
t	Time (s)
T	Temperature (K)
ΔT	Temperature difference between top and bottom surfaces (K)



u	Fluid velocity (m/s)
X	Horizontal length of computational domain (m)
Δx	Computational grid size in horizontal direction (m)
Y	Vertical height of computational domain (m)
Δy	Computational grid size in vertical direction (m)
z	Vertical distance in liquid measured from the bounding surface (m)

Greek Symbols

α	Volumetric coefficient of thermal expansion (K^{-1})
β	(Constant) temperature gradient (K/m); $\beta = \Delta T / d$
δ	Thickness of effective thermal layer (m)
Γ	ratio of thermal conductivity to heat capacity (kg/s.m); $\Gamma = k / c_p$
$\dot{\gamma}$	Strain rate (s^{-1})
κ	Thermal diffusivity (m^2/s)
λ	Wavelength (m)
μ	Viscosity (Pa.s)
μ_{app}	Apparent Viscosity (Pa.s)
ν	Kinematic Viscosity (m^2/s)
ρ	Density (kg/m^3)
σ	Under relaxation factor

Subscripts

c	Critical
0	Initial state
s	Surface
T	temperature dependent
app	apparent
max	Maximum

Abbreviation

CFD	Computational Fluid Dynamics
CHF	Constant Heat Flux
CMC	Carboxy Methyl Celulose
FST	Fixed surface temperature
NN	Non-Newtonian liquid

CHAPTER 1

INTRODUCTION

Studies of natural convection phenomena have been done in general for many years in natural sciences like astrophysics, geology, oceanography, climatology and meteorology. In convection, the hotter and lighter fluid rises while the colder and heavier fluid sinks. Natural convection, or free convection, seems to have been first described by Thomson (1882), but the first quantitative experiment was done by Benard (1900).

Lord Rayleigh (1916) studied the onset of buoyancy convection in a horizontal liquid layer bounded by two free surfaces based on an adverse linear temperature gradient. A dimensionless stability parameter was defined after him, the Rayleigh number. Convection occurs when Rayleigh number exceeded its critical value.

For natural convection induced by a time-dependent and non-linear temperature profile, Tan and Thorpe (1996) developed a new transient Rayleigh number for the deep fluid under various boundary conditions. They proposed that the correct way to begin any stability analysis is to identify the Biot number. They analyzed previous researchers' experimental data by first determine the Biot number for each case (Tan & Thorpe, 1996; 1999a). Critical Rayleigh number were re-calculated and found to be consistent within a range of identified Biot number and wave number.

