STABILITY AND RUPTURE OF LIQUID FILM FLOWING DOWN AN INCLINED PLANE

MUATAZ ALI ATIEH

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STABILITY AND RUPTURE OF LIQUID FILM FLOWING DOWN AN INCLINED PLANE

By

MUATAZ ALI ATIEH

Thesis Submitted in Fulfilment of the Requirement for the Degree of Master of Science in the Faculty of Engineering Universiti Putra Malaysia

October 2001
DEDICATED

To

My Parents, brothers and my sister
Abstract of the thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

STABILITY AND RUPTURE OF LIQUID FILM FLOWING DOWN AN INCLINED PLANE

By

MUATAZ ALI ATIEH

October 2001

Chairman: Dr. Ahmad Tariq Jameel

Faculty: Engineering

Liquid film flowing down inclined or vertical planes find applications in thin film heat and mass transfer, wetted wall columns, liquid drainage in packed columns, surface coating operations, and the like.

The film is modeled as a two-dimensional Newtonian liquid of constant density \( \rho \) and viscosity \( \mu \) flowing down an inclined plane. The liquid film of mean thickness \( h_0 \) is bounded above by a passive gas and laterally extends to infinity (two-dimensional model). Then such a flow can be represented by a two-dimensional Navier-Stokes equation coupled with continuity equation and associated boundary conditions. The body force term in the Navier-Stokes equation is modified by the inclusion of excess intermolecular interactions between fluid film and the solid surface owing to long-range van der Waals force, in addition to gravity force. The modified Navier-Stokes equation
with associated boundary conditions is solved under long wave approximation method to obtain a nonlinear equation of evolution of the film interface.

A nonlinear theory based upon the condition of infinitesimal perturbation on the film surface is derived to obtain the growth coefficient, dominant wavelength (i.e., wavelength corresponding to maximum growth coefficient of the surface instability) and the film rupture time.

The nonlinear equation of evolution is solved numerically in conservative form as part of an initial-value problem for spatially periodic boundary condition on the fixed domain \(0 < x < 2\pi/k\), where \(k\) is a wavenumber. Centered difference in space and the midpoint (Crank-Nicholson) rule in time are employed. The mesh size is taken sufficiently small so that space and time errors are negligible. The nonlinear algebraic equations obtained as a result of finite difference discretization are solved using efficient-numerical technique employing IMSL subroutine DNEQNJ.

The nonlinear simulation shows that the dominant wavelengths (corresponding to minimum time) for film rupture are very close to the prediction of the linear theory for all types of films. There seems to be no influence of surface inclination on the instability of thin films. Inclination dose influence the growth of instability in thick films. The film rupture time increases with increasing film thickness for inclined planes. Increase in the amplitude of perturbation results into decreased time of rupture. The deviations between the predictions of nonlinear and linear theory results are minimum around dominant
wavelength. The linear theory may overestimate or underestimate the time of rupture by several orders of magnitude depending upon thin film parameters. Hence linear theory is inadequate to describe the stability characteristics of inclined films and therefore, the need of a nonlinear approach to the study of inclined film dynamics.
Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
Sebagai memenuhi keperluan untuk ijazah Master Sains.

KESTABILAN DAN PEMECAHAN PADA LAPISAN CECAIR
YANG MENGALIR PADA SATAH CONDONG

Oleh

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Saput cecair yang mengalir ke bawah pada permukaan yang condong/tegak dijumpa kegunaannya dalam pemindahan haba dan jisim dalam kolum dinding dibasahi, pengaliran cecair di dalam kolum padat dan operasi penyaduran permukaan.

Saput dimodelkan sebagai satu cecair Newtonian dua-dimensi dengan ketumpatan malar, \( \rho \) dan kelikatan,\( \mu \) yang mengalir ke bawah satu permukaan condong. Saput cecair dengan ketebalan purata \( h_0 \) disempadankan atas satu gas passive dan sisinya dikembangkan kepada yang keterhadan(model dua-dimensi). Oleh demikian, pengaliran ini boleh diwakili oleh satu persamaan Navier-Stokes 2 – dimensi bersamaan dengan persamaan berterusan dan keadan sempadan yang bersepadan. Kata-kata daya badan dalam persamaan Navier-Stokes adalah diubahsuaikan dengan penglibatan kesalingtindakan antara molekul yang berlebihan antara saput dan permukaan pepejal yang disebabkan oleh daya Van der Waals yang berjarak-panjang. Persamaan Navier-
Stokes yang telah berubahsuai bersamaan dengan keadaan sempadan adalah diselesaikan bawah kaedah pendekatan gelombang jauh untuk mendapat satu persamaan bukan linear bagi pengembangan pada ketidakstabilan saput.

Satu teori bukan linear berasaskan keadaan penggangguan ketidakterhadan pada permukaan saput telah dihasilkan untuk memperolehi pekali penubuhan, jarak gelombang dominant (contohnya: jarak gelombang berkaitandengan pekali penubuhan maximum pada permukaan ketidakstabilan) dan masa perpecahan saput.

Persamaan bukan linear bagi evolusi adalah diselesaikan secara numerical dalam bentuk keabadian sebagai sebahagian daripada satu masalah nilai permula bagi keadaan sempadan perodik yang wujud dalam ruang pada domain yang ditetepkan, \( 0 < x < 2\pi/k \). Pembezaan tengah dalam ruangan dan peraturan titik tengah (Crank –Nicholson)dalam masa digunakan. Saiz mesh yang dipakai adalah cukup kecil supaya ralat ruangan dan masa boleh diabaikan. Persamaan algehra bukan linear diperolehi sebagai keputusan pembezaan finite adalah diselesaikan dengan penggunaan teknik numerical-berkesan, IMSL subroutine DNEQNJ.

Simulasi bukan linear menunjukkan jarak gelembong dominant (berkaitan kepada masa minimum) bagi pemecahan raput adalah sangat dekat dengan jangkaan oleh teori linear bagi semua jenis raput. Ini menunjukkan tiada pengaruh oleh kecondongan oleh ketidakstabilan bagi raput nipis. Kecondongan akan pengaruh penambahan ketidakstabilan dalam raput tebal. Masa pemecahan pemecahan bertambah...
dengan penambahan ketebalan raput bagi ketebalan condong. Penambahan dalam kebesaran penggangguan menyebabkan kekurangan masa pemecahan keputusan
Perbezaan antara jangkaan daripada teori bukan linear dan keputusan teori linear adalah minimum disekitar jarak gelembong dominant. Teori linear mungkin menaksir terlampau atau terkurang masa pemecahan oleh beberapa cara magnitute bergantung pada parameter raput nipis. Sebab itu, teori linear adalah kemungkinan besar adalah kurang tepat untuk menerangkan kestabilan raput tercendong dan oleh demikian perlulah satu pendekatan bukan linear kepada pengajian dinamik raput tercendong.
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Every praise is due to Allah alone, the Merciful and peace be upon His prophet who is forever a torch of guidance and knowledge for humanity as a whole.

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I certify that an Examination Committee met on 11th October 2001 to conduct the final examination of Muataz Ali Atieh on his Master of Science thesis entitled “Stability and Rupture of Liquid Film Flowing Down an Inclined Plane” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be award the relevant degree. Members of the Examination Committee are as follows:

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Date: 13 DEC 2001
DECLARATION

I hereby declare that the thesis is based on my original work except for quotation and citations, which have been duly, acknowledge. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

Muataz Ali Atieh Hussien

Date: 19-10-2001

xii
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEDICATION</td>
<td>ii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>iii</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td>vi</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ix</td>
</tr>
<tr>
<td>APPROVAL</td>
<td>x</td>
</tr>
<tr>
<td>DECLARATION</td>
<td>xiii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xviii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>xxiii</td>
</tr>
</tbody>
</table>

CHAPTER

I INTRODUCTION

Scope of the study 3
Objective of study 4

II LITERATURE REVIEW

Introduction 5
Van der Waals 11
Inclined Plane 15

III METHODOLOGY

Mathematical Formulation 41
Scaling of the hydrodynamic Equations 43
Long-Wave theory 45
Linear Stability analysis 49

IV NUMERICAL SOLUTION OF THE NONLINEAR EVOLUTION EQUATION 51

V RESULTS AND DISCUSSION

Linear Theory 55
Result from Nonlinear Simulation 58
Effect of Inclination 63
Effect of Mean Film Thickness 66
Effect of the amplitude of disturbance on the Rupture Time 69
Comparison of prediction from Nonlinear and Linear Theories 75
Depiction of the growth of instability (film profiles) 79
<table>
<thead>
<tr>
<th>IV</th>
<th>CONCLUSION</th>
<th>84</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RECOMMENDATIONS</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>REFERENCES</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>APPENDIX A</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>APPENDIX B</td>
<td>108</td>
</tr>
<tr>
<td></td>
<td>APPENDIX C</td>
<td>118</td>
</tr>
<tr>
<td></td>
<td>BIODATA OF THE AUTHOR</td>
<td>124</td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>A5</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>A6</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>A7</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td>A8</td>
<td>96</td>
<td></td>
</tr>
</tbody>
</table>

LIST OF TABLES
A9  Rupture time and growth rate as function of wavelength & wavenumber at $\theta=45$ & $h_0=200$ nm from linear theory

A10 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=45$ & $h_0=300$ nm from linear theory

A11 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=60$ & $h_0=10$ nm from linear theory

A12 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=60$ & $h_0=50$ nm from linear theory

A13 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=60$ & $h_0=100$ nm from linear theory

A14 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=60$ & $h_0=200$ nm from linear theory

A15 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=60$ & $h_0=300$ nm from linear theory

A16 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=90$ & $h_0=10$ nm from linear theory

A17 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=90$ & $h_0=50$ nm from linear theory

A18 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=90$ & $h_0=100$ nm from linear theory

A19 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=90$ & $h_0=200$ nm from linear theory

A20 Rupture time and growth rate as function of wavelength & wavenumber at $\theta=90$ & $h_0=300$ nm from linear theory

A21 Rupture time at $h_0=10$ nm and different inclinations from nonlinear equation

A22 Rupture time at $h_0=50$ nm and different inclinations from nonlinear equation

A23 Rupture time at $h_0=100$ nm and different inclinations from nonlinear equation

A24 Rupture time at $h_0=200$ nm and different inclinations from nonlinear equation
A25 Rupture time at $h_o=300\text{nm}$ and different inclinations from nonlinear equation

A26 Rupture Time at $\theta=0$ and different $h_o$

A27 Rupture Time at $\theta=10$ and different $h_o$

A28 Rupture Time at $\theta=45$ and different $h_o$

A29 Rupture Time at $\theta=60$ and different $h_o$

A30 Rupture Time at $\theta=90$ and different $h_o$

A31 Ratio of rupture time as calculated from nonlinear theory and linear theory $h_o=10$

A32 Ratio of rupture time as calculated from nonlinear theory and linear theory $h_o=50$

A33 Ratio of rupture time as calculated from nonlinear theory and linear theory at $h_o=100$

A34 Ratio of rupture time as calculated from nonlinear theory and linear theory at $h_o=200$

A35 Ratio of rupture time as calculated from nonlinear theory and linear theory at $h_o=300$

A36 Ratio of rupture time with different mean film thickness different $\theta=0$ and dominate wavelength

A37 Rupture time as function of wavelength at different $\varepsilon$ with $\theta=0$ & $h_o=10\text{nm}$

A38 Rupture time as function of wavelength at different $\varepsilon$ with $\theta=45$ & $h_o=200\text{ nm}$
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Snapshot of the Reyleight-Taylor instability of a silicone-oil on the underside of a horizontal plane.</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Photographs of a silicone-oil on a nonuniformly heated plate: a dimpled film when the heat flux is sufficiently low; The nearly bare regions that result at larger heat fluxes</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>(a) Synchronous 3D instability of 2D periodic waves. A snapshot taken at the inclination angle of 6.4°, Reynolds number of 72, and imposed perturbation frequency of 10.0 Hz. (b) A herringbone (or checkerboard) pattern due to 3D sub harmonic instability. A snapshot taken at the inclination angle of 4°, Reynolds number of 50.5, and imposed perturbation frequency 14 Hz</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>The various forms of sheet flow down an inclined plane. The marked horizontal lines in each photograph are 5 cm apart: (a) silicone-oil MS200/100 (b) the flow of glycerin down a slope.</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>The physical configuration of thin layer flowing down an inclined plane</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=10$ nm and $\theta=0$</td>
<td>59</td>
</tr>
<tr>
<td>7</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=10$ nm and $\theta=10$</td>
<td>59</td>
</tr>
<tr>
<td>8</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=10$ nm and $\theta=45$</td>
<td>60</td>
</tr>
<tr>
<td>9</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=10$ nm and $\theta=60$</td>
<td>60</td>
</tr>
<tr>
<td>10</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=200$ nm and $\theta=10$</td>
<td>61</td>
</tr>
<tr>
<td>11</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=300$ nm and $\theta=45$</td>
<td>62</td>
</tr>
<tr>
<td>12</td>
<td>Nondimensional time of rupture as function of wavelength at $h_o=300$ nm and $\theta=60$</td>
<td>62</td>
</tr>
</tbody>
</table>
13 Nondimensional time of rupture as function of wavelength at $h_o=300$ nm and $\theta=90$

14 Rupture time as function of wavelength at $h_o=10$ nm and different inclination

15 Rupture time as function of wavelength at $h_o=50$ nm and different inclination

16 Rupture time as function of wavelength at $h_o=100$ nm and different inclination

17 Rupture time as function of wavelength at $h_o=200$ nm and different inclination

18 Rupture time as function of wavelength at $h_o=300$ nm and different inclination

19 Rupture times as function of wavelength at different $h_o$ and $\theta=0$

20 Rupture times as function of wavelength at different $h_o$ and $\theta=10$

21 Rupture times as function of wavelength at different $h_o$ and $\theta=45$

22 Rupture times as function of wavelength at different $h_o$ and $\theta=60$

23 Rupture times as function of wavelength at different $h_o$ and $\theta=90$

24 Rupture Time as function of amplitude of perturbation at $\theta=0$, $h_o=10$ nm & $k_m=0.7$

25 Rupture Time as function of amplitude of perturbation at $\theta=45$, $h_o=10$ nm & $k_m=0.7$

26 Rupture Time as function of amplitude of perturbation at $\theta=90$, $h_o=10$ nm & $k_m=0.7$

27 Rupture Time as function of amplitude of perturbation at $\theta=0$, $h_o=300$ nm & $k_m=0.9$

28 Rupture Time as function of amplitude of perturbation at $\theta=45$, $h_o=300$ nm & $k=0.9$

29 Rupture Time as function of amplitude of perturbation at theta=90, $h_o=300$ nm & $k=0.9$
Nonlinear mode selection ($T_N$ vs. wavelength) at $h_0=10$ nm and $\theta=0$

Nonlinear mode selection ($T_N$ vs. wavelength) at $h_0=200$ nm and $\theta=45$

Ratio of rupture time as function of wavelength at $h_0=10$ nm & different inclination

Ratio of rupture time as function of wavelength at $h_0=50$ nm & different inclination

Ratio of rupture time as function of wavelength at $h_0=100$ nm & different inclination

Ratio of rupture time as function of wavelength at $h_0=200$ nm & different inclination

Ratio of rupture time as function of wavelength at $h_0=300$ nm & different inclination

Ratio of rupture time as function of mean film thickness at dominant wavelength different inclination

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $\theta=0$ and $h_0=10$ nm. The rupture proceeds explosively at $T_N=4.399$

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $h_0=10$ nm. The rupture proceeds explosively at $T_N=4.399$

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $h_0=10$ nm and $\theta=45$. The rupture proceeds explosively at $T_N=4.399$

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $h_0=10$ nm, $\theta=60$ & $k=1.0$. The rupture proceeds explosively at $T_N=13.898$

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1, $h_0=50$ nm, $\theta=60$ & $k=1.0$. The rupture proceeds explosively at $T_N=4.397$

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1, $h_0=100$ nm, $\theta=0$. The rupture proceeds explosively at $T_N=4.4$

Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $\theta=45$ and $h_0=100$ nm. The rupture proceeds explosively at $T_N=4.398$
44 Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $\theta=45$, $h_0=200$ nm and $k=0.9$. The rupture proceeds explosively at $T_N=101.408$.

45 Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $\theta=0$ and $h_0=300$ nm. The rupture proceeds explosively at $T_N=4.458$.

46 Film profile at different times for van der Waals and gravity system. The initial amplitude is 0.1 at $\theta=45$ and $h_0=300$ nm. The rupture proceeds explosively at $T_N=936$.

B1 Nondimensional time of rupture as function of wavelength at $h_0=10$ nm & $\theta=90$.

B2 Nondimensional time of rupture as function of wavelength at $h_0=50$ nm & $\theta=0$.

B3 Nondimensional time of rupture as function of wavelength at $h_0=50$ nm & $\theta=10$.

B4 Nondimensional time of rupture as function of wavelength at $h_0=50$ nm & $\theta=45$.

B5 Nondimensional time of rupture as function of wavelength at $h_0=50$ nm & $\theta=60$.

B6 Nondimensional time of rupture as function of wavelength at $h_0=50$ nm & $\theta=90$.

B7 Nondimensional time of rupture as function of wavelength at $h_0=100$ nm & $\theta=0$.

B8 Nondimensional time of rupture as function of wavelength at $h_0=100$ nm & $\theta=10$.

B9 Nondimensional time of rupture as function of wavelength at $h_0=100$ nm & $\theta=45$.

B10 Nondimensional time of rupture as function of wavelength at $h_0=100$ nm & $\theta=60$.

B11 Nondimensional time of rupture as function of wavelength at $h_0=100$ nm & $\theta=90$.

B12 Nondimensional time of rupture as function of wavelength at $h_0=200$ nm & $\theta=0$. 

xxi
<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B13</td>
<td>Nondimensional time of rupture as function of wavelength at ( h_0 = 300 \text{ nm} ) &amp; ( \theta = 0 )</td>
<td>114</td>
</tr>
<tr>
<td>B14</td>
<td>Nondimensional time of rupture as function of wavelength at ( h_0 = 300 \text{ nm} ) &amp; ( \theta = 10 )</td>
<td>115</td>
</tr>
<tr>
<td>B15</td>
<td>Nondimensional time of rupture as function of wavelength at ( h_0 = 300 \text{ nm} ) &amp; ( \theta = 45 )</td>
<td>115</td>
</tr>
<tr>
<td>B16</td>
<td>Nondimensional time of rupture as function of wavelength at ( h_0 = 300 \text{ nm} ) &amp; ( \theta = 60 )</td>
<td>116</td>
</tr>
<tr>
<td>B17</td>
<td>Nondimensional time of rupture as function of wavelength at ( h_0 = 300 \text{ nm} ) &amp; ( \theta = 90 )</td>
<td>116</td>
</tr>
<tr>
<td>B18</td>
<td>Growth rate as function of wavelength at ( \theta = 0 ) &amp; ( h_0 = 50 \text{ nm} ) from linear theory</td>
<td>117</td>
</tr>
<tr>
<td>B19</td>
<td>Growth rate as function of wavelength at ( \theta = 90 ) &amp; ( h_0 = 30 \text{ nm} ) from linear theory</td>
<td>117</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

A (A') Hamakar constant.
G constant defined in Eq.(3.10)
H, (h) thickness of thin film.
(h_o) mean film thickness.
k Qv wavenumber of perturbations.
k_n neutral wavenumber from linear theory.
k_m dominate wavenumber from linear theory.
n number of spatial grids employed in numerical solutions.
P(p) hydrodynamic pressure inside the film.
P_o pressure in the gas.
S capillary number.
T_r(t) time coordinate.
T_L, (t_L) time of rupture from linear theory.
T_N, (t_N) time of rupture from nonlinear theory.
T_ml, (t_ml) minimum time of rupture from linear theory.
T_mN, (t_mN) minimum time of rupture from nonlinear theory.
U_r(u) x-component of the velocity vector.
W_r(w) z-component of the velocity vector.
X_r(x) spatial coordinate in the longitude direction.
Z_r(z) spatial coordinate in the longitude direction.

Greek Symbols

xxiii
\sigma \quad \text{interfacial tension}

\epsilon \quad \text{amplitude of perturbation}

\zeta, \tau \quad \text{rescaled spatial and time coordinate for longwave approximation.}

\lambda \quad \text{wavelength of perturbation.}

\lambda_n \quad \text{neutral wavelength of perturbation.}

\lambda_m \quad \text{dominate wavelength of perturbation}

\mu \quad \text{dynamic viscosity of the film fluid.}

\nu \quad \text{kinematics viscosity of the film fluid.}

\rho \quad \text{density of the film fluid.}

\Pi \quad \text{disjoining pressure}

\theta \quad \text{angle of deviation of the plane}

\omega, (\omega_0) \quad \text{disturbance growth coefficient.}

\omega_m, (\omega_{mo}) \quad \text{maximum disturbance of the growth coefficient.}

\phi (\Phi) \quad \text{van der Waals force}

\textbf{Subscripts}

H, (h) \quad \text{derivative with respect to the film thickness.}

T, (t) \quad \text{derivative with respect to the time.}

X, (x) \quad \text{derivative with respect to the } X(x).

Z, (z) \quad \text{derivative with respect to the } Z(z).