UNIVERSITI PUTRA MALAYSIA

STABILITY AND RUPTURE OF LIQUID FILM FLOWING DOWN AN INCLINED PLANE

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STABILITY AND RUPTURE OF LIQUID FILM FLOWING DOWN AN INCLINED PLANE

By

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Thesis Submitted in Fulfilment of the Requirement for the
Degree of Master of Science in the Faculty of Engineering
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DEDICATED

To

My Parents, brothers and my sister

Abstract of the thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

STABILITY AND RUPTURE OF LIQUID FILM FLOWING DOWN AN INCLINED PLANE

By

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Faculty: Engineering

Liquid film flowing down inclined or vertical planes find applications in thin film heat and mass transfer, wetted wall columns, liquid drainage in packed columns, surface coating operations, and the like.

The film is modeled as a two-dimensional Newtonian liquid of constant density ρ and viscosity μ flowing down an inclined plane. The liquid film of mean thickness h_o is bounded above by a passive gas and laterally extends to infinity (two-dimensional model). Then such a flow can be represented by a two-dimensional Navier-Stokes equation coupled with continuity equation and associated boundary conditions. The body force term in the Navier-Stokes equation is modified by the inclusion of excess intermolecular interactions between fluid film and the solid surface owing to long-range van der Waals force, in addition to gravity force. The modified Navier-Stokes equation with associated boundary conditions is solved under long wave approximation method to obtain a nonlinear equation of evolution of the film interface.

A nonlinear theory based upon the condition of infinitesimal perturbation on the film surface is derived to obtain the growth coefficient, dominant wavelength (i.e., wavelength corresponding to maximum growth coefficient of the surface instability) and the film rupture time.

The nonlinear equation of evolution is solved numerically in conservative form as part of an initial-value problem for spatially periodic boundary condition on the fixed domain $0< x< 2\pi/k$, where k is a wavenumber. Centered difference in space and the midpoint (Crank-Nicholson) rule in time are employed. The mesh size is taken sufficiently small so that space and time errors are negligible. The nonlinear algebraic equations obtained as a result of finite difference discretization are solved using efficient-numerical technique employing IMSL subroutine DNEQNJ.

The nonlinear simulation shows that the dominant wavelengths (corresponding to minimum time) for film rupture are very close to the prediction of the linear theory for all types of films. There seems to be no influence of surface inclination on the instability of thin films. Inclination dose influence the growth of instability in thick films. The film rupture time increases with increasing film thickness for inclined planes. Increase in the amplitude of perturbation results into decreased time of rupture. The deviations between the predictions of nonlinear and linear theory results are minimum around dominant

wavelength. The linear theory may overestimate or underestimates the time of rupture by several orders of magnitude depending upon thin film parameters. Hence linear theory is inadequate to describe the stability characteristics of inclined films and therefore, the need of a nonlinear approach to the study of inclined film dynamics.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
Sebagai memenuhi keperluan untuk ijazah Master Sains.

KESTABILAN DAN PEMECAHAN PADA LAPISAN CECAIR YANG MENGALIR PADA SATAH CONDONG

Oleh

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Saput cecair yang mengalir ke bawah pada permukaan yang condong/tegak dijumpa kegunaannya dalam pemindahan haba dan jisim dalam kolum dinding dibasahi, pengaliran cecair di dalam kolum padat dan operasi penyaduran permukaan.

Saput dimodelkan sebagai satu cecair Newtonian dua-dimensi dengan ketumpatan malar, ρ dan kelikatan,μ yang mengalir ke bawah satu permukaan condong. Saput cecair dengan ketebalan purata ,h_o disempadankan atas satu gas passive dan sisinya dikembangkan kepada yang keterhadan(model dua-dimensi). Oleh demikian, pengaliran ini boleh diwakili oleh satu persamaan Navier-Stokes 2 – dimensi bersamaan dengan persamaan berterusan dan keadan sempadan yang bersepadan. Kata-kata daya badan dalam persamaan Navier-Stokes adalah diubahsuaikan dengan penglibatan kesalingtindakan antara molekul yang berlebihan antara saput dan permukaan pepejal yang disebabkan oleh daya Van der Waals yang berjarak-panjang. Persamaan Navier-

Stokes yang telah berubahsuai bersamaan dengan keadaan sempadan adalah diselesaikan bawah kaedah pendekatan gelombang jauh untuk mendapat satu persamaan bukan linear bagi pengembangan pada ketidakstabilan saput.

Satu teori bukan linear berasaskan keadaan penggangguan ketidakterhadan pada permukaan saput telah dihasilkan untuk memperolehi pekali penubuhan, jarak gelombang dominant (contohnya : jarak gelombang berkaitandengan pekali penubuhan maximum pada permukaan ketidakstabilan) dan masa perpecahan saput.

Persamaan bukan linear bagi evolusi adalah diselesaikan secara numerical dalam bentuk keabadian sebagai sebahagian daripada satu masalah nilai permula bagi keadaan sempadan perodik yang wujud dalam ruang pada domain yang ditetepkan , ($0<x<2\pi/k$) .Pembezaan tengah dalam ruangan dan peraturan titik tengah (Crank –Nicholson)dalam masa digunakan. Saiz mesh yang dipakai adalah cukup kecil supaya ralat ruangan dan masa boleh diabaikan. Persamaan aigebra bukan linear diperolehi sebagai keputusan pembezaan finite adalah diselesaikan dengan penggunaan teknik numerical-berkesan , IMSL subroutine DNEQNJ.

Simulasi bukan linear menunjukkan jarak gelembong dominant (berkaitan kepada masa minimum) bagi pemecahan raput adalah sangat dekat dengan jangkaan oleh teori linear bagi semua jenis raput. Ini menunjukkan tiada pengaruh ioleh kecondongan oleh ketidakstabilan bagi raput nipis. Kecondongan akan pengaruh penambahan ketidakstabilan dalam raput tebal. Masa pemecahan pemecahan bertambah dengan penambahan ketebalan raput bagi ketebalan condong. Penambahan dalam kebesaran pengganggguan menyebabkan kekurangan masa pemecahan keputusan Perbezaan antara jangkaan daripada teori bukan linear dan keputusan teori linear adalah minimum disekitar jarak gelembong dominant. Teori linear mungkin menaksir terlampau atau terkurang masa pemecahan oleh beberapa cara magnitute bergantung pada parameter raput nipis. Sebab itu, teori linear adalah kemungkinan besar adalah kurang tepat untuk menerangkan kestabilan raput tercendong dan oleh demikian perlulah satu pendekatan bukan linear kepada pengajian dinamik raput tercendong.

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotation and citations, which have been duly, acknowledge. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

Muataz Ali Atieh Hussien

Date: 19 - 10 - 2001

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LIST OF SYMBOLS

A (A′)	Hamakar constant.
G	constant defined in Eq.(3.10)
H, (h)	thickness of thin film.
(h_o)	mean film thickness.
k Qv	wavenumber of perturbations.
k_n	neutral wavenumber from linear theory.
k_m	dominate wavenumber from linear theory.
n	number of spatial grids employed in numerical solutions.
P(p)	hydrodynamic pressure inside the film.
P_o	pressure in the gas.
S	capillary number.
T,(t)	time coordinate.
T_L , (t_L)	time of rupture from linear theory.
T_N , (t_N)	time of rupture from nonlinear theory.
T_{mL} , (t_{mL})	minimum time of rupture from linear theory .
T_{mN} , (t_{mN})	minimum time of rupture from nonlinear theory.
U,(u)	x-component of the velocity vector.
W,(w)	z-component of the velocity vector.
X,(x)	spatial coordinate in the longitude direction.
Z,(z)	spatial coordinate in the longitude direction.

Greek Symbols

σ	interfacial tension
ε	amplitude of perturbation
ζ ,τ	rescaled spatial and time coordinate for longwave approximation.
λ	wavelength of perturbation.
λ_n	neutral wavelength of perturbation.
λ_m	dominate wavelength of perturbation
μ	dynamic viscosity of film fluid.
ν	kinematics viscosity of the film fluid.
ρ	density of the film fluid.
Π	disjoining pressure
θ	angle of deviation of the plane
ω,(ω_o)	disturbance growth coefficient.
ω_m,(ωmo)	maximum disturbance of the growth coefficient.
ϕ (Φ)	van der Waals force

Subscripts

H,(h)	derivative with respect to the film thickness.
T,(t)	derivative with respect to the time .
X,(x)	derivative with respect to the X(x).
Z,(z)	derivative with respect to the Z(z).